Applied Mathematics

Mechanics

Third secondary

Student Book

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Introduction

بسم الله الرحمن الرحيم

We are pleased to introduce this book to show the philosophy on which the academic content has been prepared. This philosophy aims at:

Today's world is witnessing an ongoing scientific development. The future generation is required to get qualified using the developing methods of the future in order to keep matching with the gigantic progress of the different sciences. In regard to apply such a principle, the Ministry of Education is doing the best to improve the curricula through having the learner in the position of the explorer to the scientific facts. In addition, the Ministry of Education provides the proper training for the students in the scientific research fields throughout thinking in order to enable them to use their minds as tools of the scientific thinking. As we present the book of Calculus for the third secondary to be a helping for lightening the minds of the students to trace the scientific thinking and to motivate them to search and explore.

In the light of what previously mentioned, the following details have been considered:

★ The book has been divided into related and integrated units. Each unit has an introduction illustrating the learning outcomes, the unit planning guide, and the related key terms. In addition, the unit is divided into lessons where each lesson shows the objective of learning it through the title You will learn. Each lesson starts with the main idea of the lesson content. It is taken into consideration to introduce the content gradually from easy to hard. The lesson includes some activities, which relate Math to other school subjects and the practical life. These activities suit the students' different abilities, consider the individual differences throughout Discover the error to correct some common mistakes of the students, confirm the principle of working together and integrate with the topic. Furthermore, this book contains some issues related to the surrounding environment and how to deal with.

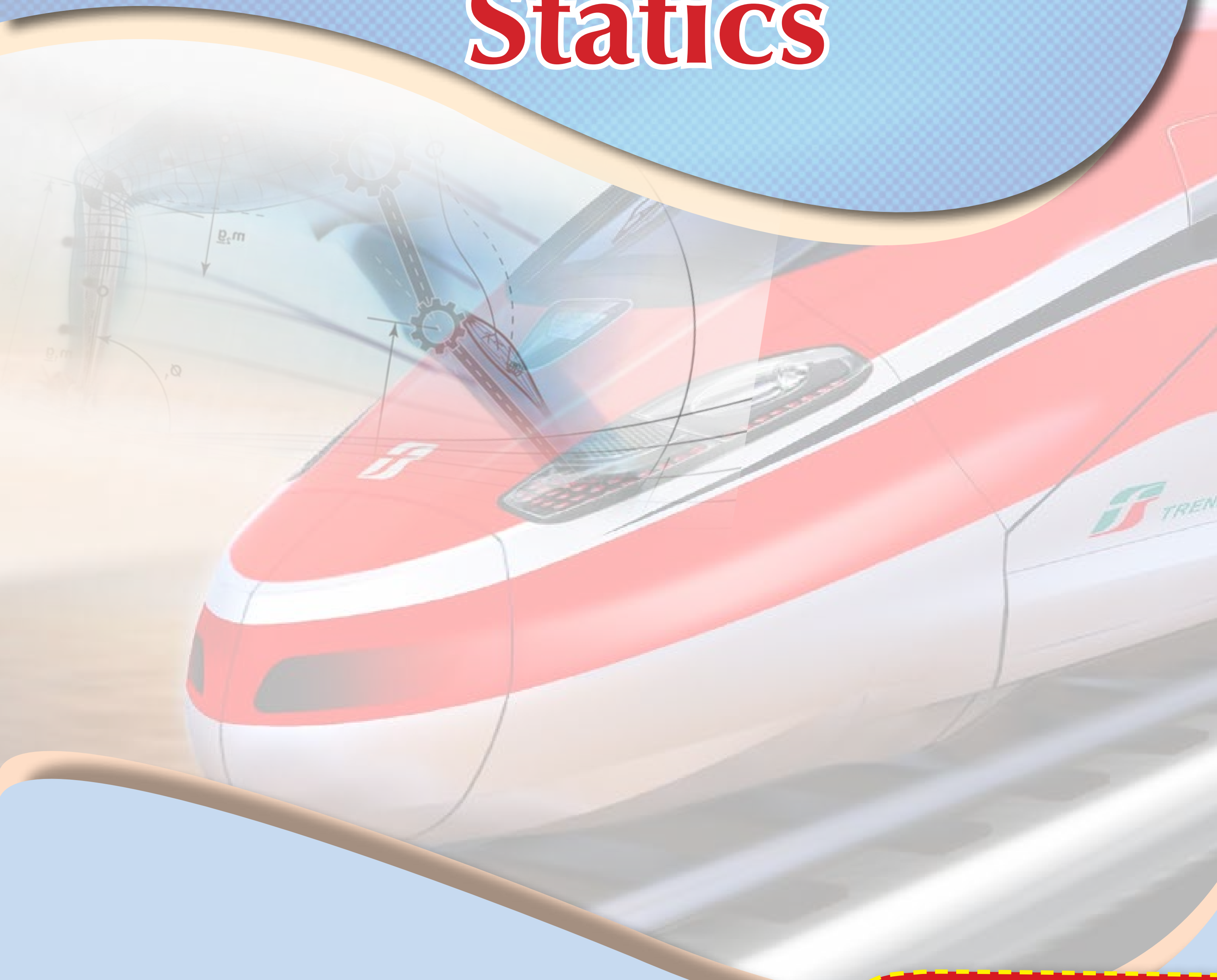
★ Each lesson contains examples starting gradually from easy to hard and containing various levels of thoughts accompanied with some exercises titled Try to solve. Each lesson ends in Exercises that contain various problems related to the concepts and skills that the students learned through the lesson.

Last but not least. We wish we had done our best to accomplish this work for the benefits of our dear youngsters and our dearest Egypt.

Contents

Pre Requiste for

Statics

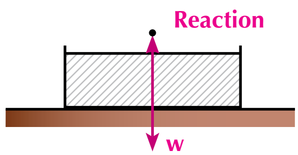


Third secondary

Not required for examination

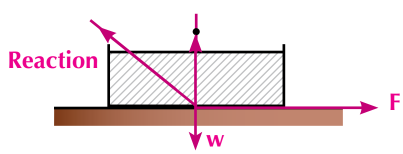
1 Smooth and Rough Surfaces

Scientists relate the friction forces among bodies in the presence of microscopic cavities and projections in the surfaces of the bodies whatever their smoothness is . The overlapping of such projections and cavities of the two surfaces in contact produce what is called the friction force. As a result, we can find resistance as we try to move one of the two surfaces against the other. The coefficient of the friction a good scale to measure the roughness degree of the surfaces. If the value of the coefficient of the friction increases, the roughness increases and vice versa. If the coefficient of the friction equals zero, the friction forces are not existed totally. The reaction between the two contact bodies depends upon the nature of the two bodies and upon the other forces acting on the body. In case of the smooth surfaces, the reaction is normal to the common tangent plane to the surfaces of the two contact bodies. On the contrary, when the two bodies are rough, the reaction would have a component in the direction of the tangent surface which is called the static friction. Besides the reaction has a normal component on the tangent surface which is called the normal reaction.



Reaction in case of a smooth surfaces

figure (1)



Reaction in case of a rough surfaces

figure (2)

2 The properties of the static friction force:

(1) The static friction force (F) acts in opposing the slide. It is in the opposite direction to the direction at which the body tends to slide.

(2) The static friction force (F) is only equal to the tangential force, which tends to move the body so that it can't be more than such a force and remains equal to the force as long as the body balanced.

(3) The static friction force (F) increases, whenever the tangential force which cause the motion increases until you arrive up to a certain limit which it doesnot exceed it . At such a limit, the body is about to slide in this case, the friction is called the limiting static friction and it is denoted by the symbol (F_s).

(4) The ratio between the limiting static friction and the normal reaction N is constant and this ratio depends on on the nature of the two bodies in contact but not up to their shape or masses. This ratio is called the coefficient of the static friction and is denoted by the symbol (μ_s) .

i.e. $\mu_s = \frac{F_s}{N}$ where F_s the limiting static friction.

3 Kinetic Friction Force

if a body moves upon a rough surface , it is subjected to the kinetic friction force (F_k) and its direction is opposite to the direction of its motion and its value is given by the relation: $F_k = \mu_k N$, where μ_k is the kinetic friction coefficient and R the normal reaction.

i.e.: The kinetic friction force equals the product of the kinetic friction coefficient multiplied by the normal reaction force.

Hence, the kinetic friction coefficient can be defined as the ratio between the kinetic friction force and the normal reaction force .

i.e. : $\mu_k = \frac{F_k}{N}$, where F_k is the kinetic friction force.

4 Resultant Reaction (R')

In case of the rough surfaces, the resultant reaction is inclined on the tangent surface since it expresses the resultant of the normal reaction and the static friction force. It is called the resultant reaction.

Definition

The resultant reaction ($\overrightarrow{R'}$) is the resultant of the normal reaction $\overrightarrow{N}$ and the static friction force $\overrightarrow{F}$

5 Angle of Friction

Note that the measure of the angle included between the normal reaction and the resultant reaction increases as the magnitude of the friction force increases [suppose that the normal reaction is constant] and this value is limiting as λ, when the friction becomes limiting and this angle in this case is called the angle of friction.

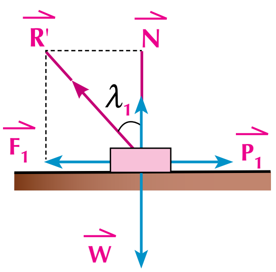


figure (3)

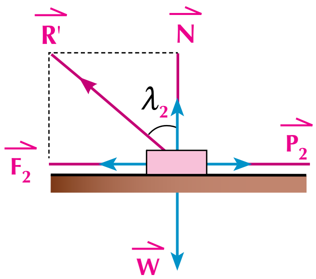


figure (4)

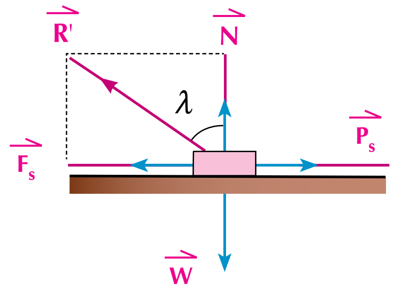


figure (5)

In fig (3) and fig (4,) we find: the vector resultant $\vec{R'}$ is the resultant of the normal reaction $\vec{N}$ and the friction force $\vec{F}$, it's magnitude is given by $R' = \sqrt{N^2 + F^2}$.

In fig (5) when the friction force is limiting we have :

$\therefore R' = \sqrt{N^2 + F_s^2}$

$\because F_s = \mu_s N$

$\therefore R' = \sqrt{N^2 + N^2 \mu_s^2}$

$\therefore R' = N\sqrt{1 + \mu_s^2}$

6 The Relation between Coefficient of Friction and Angle of Friction :

In case the friction is limiting as in shown fig (5) :

we find : $\tan\lambda = \frac{F_s}{N}$ where $\frac{F_s}{N} = \mu_s$

i.e. : $\mu_s = \tan \lambda$

i.e : In the case of limiting friction , the coefficient of friction is equal to the tangent of the angle of friction

Critical Thinking: Compare between the static and kinetic angle of friction.

7 Equilibrium of a Body on a Rough Horizantal Plane

If a body of weight (w) is in equilibrium on a horizontal rough plane and acted upon by a force p that inclined by an angle of measure θ with the horizontal fig (6) the body is equilibrium under the action of :

1) The weight $\vec{w}$ which is directed vertically downward
2) The resultant reaction $\vec{R'}$ and its magnitude is R'
3) The given force $\vec{P}$ with magnitude P.

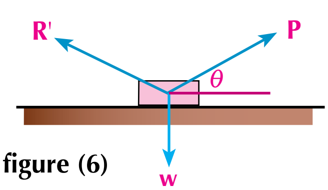


figure (6)

by resolve $\vec{P}$ into two components in the horizontal and vertical, direction then their magnitudes are $P\cos\theta$, $P\sin\theta$.

and by resolve $\vec{R'}$ into two perpendicular components which are the normal reaction $\vec{N}$ and its magnitude N, and the friction force $\vec{F}$ and its magnitude F as shown in fig (7).

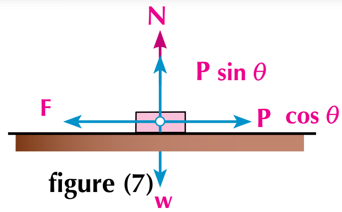


figure (7)

The equations for equilibrium are : $F = P\cos\theta$, $N + P\sin\theta = w$

Example The acting force on a body

(1) Karim pushes a box full of books towards his car on a horizontal road, if the weight of the box and books together 124 Newton and coefficient of friction between the road and the box 0.45 then find the magnitude of the horizontal force required by karim to push the box to make it about to move.

Solution

Suppose that $w = 124$ Newton , $\mu_s = 0.45$

From the conditions of equilibrium of the body in the horizontal plane :

$N = w$ i.e : $N = 124$ (1)

$F = \mu_s N$ and from (1) then : $F = 0.45 \times 124 = 55.8$ Newton

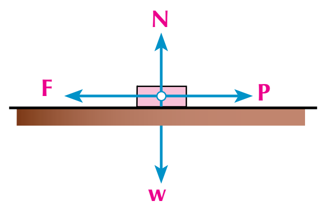


figure (8)

Try to solve

(1) A mass of weight 32 Newton is put on a horizontal rough plane and act on it a horizontal force magnitude P until the mass becomes about to move.

a) If $P = 8$ Newton, find the coefficient of the static friction between the mass and the plane

b) If $\mu_s = 0.4$ find P

Example Angle of friction

(2) A body of weight 12 kg.wt is placed on a horizontal rough plane, two forces act on the body of magnitudes 4 , 4 kg.wt and include at an angle of measure 60° where the two horizontal forces are on the same horizontal plane. If the body is about to move, find the coefficient of friction between the body and the plane also find angle of friction.

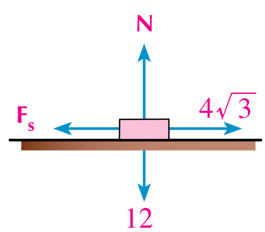


figure (9)

Solution

$\because$ **The body is about to move:**

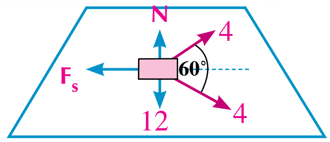


figure (10)

the body in equilibrium

$\therefore\ N = W$ $\qquad\qquad$ $\therefore\ N = 12$ kg.wt.

The resultant of the forces 4, 4 kg.wt = limiting friction forces

$\because F = \sqrt{F_1^2 + F_2^2 + 2F_1\ F_2 \cos \propto}$ the resultant of two forces law

$\therefore F = \sqrt{4^2 + 4^2 + 2 \times 4 \times 4 \times \frac{1}{2}} = 4\sqrt{3}$ kg.wt

$\mu_s N = F$ $\qquad\qquad$ $\therefore\ 12\mu_s = 4\sqrt{3}$

$\therefore \mu_s = \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3}$ $\quad \therefore\ \mu_s = \frac{1}{\sqrt{3}}$

$\because \mu_s = \tan\lambda$ $\qquad\qquad$ $\therefore\ \tan\lambda = \frac{\sqrt{3}}{3}$ $\qquad\qquad$ $\therefore\ \lambda = 30°$

8 Equilibrium of a body on an Inclined rough plane

We consider the body is in equilibrium on a rough horizontal inclined plane with an angle of measure θ.

The body become in equilibrium under the action of two forces:

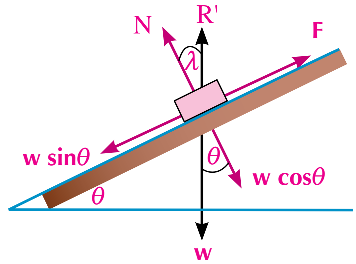


figure (11)

(1) The weight of the body $\vec{w}$ that acts vertically downwards and its magnitude is (w)

(2) The resultant reaction and its magnitude is (R')

And from the conditions of equilibrium we find that :

The resultant reaction force act vertically up.

and : R' = w .... (1)

we can designate two forces, the friction force and the normal reaction considering the resultant reaction resolved into two directions one of them is parallel to the plane and the other perpendicular to it as shown in the figure (11) .

Friction force .

$F = w \sin\theta$ .... (2)

and this force act on opposite direction of the expected motion, this means that is parallel to the line of the greatest slope upwards .

Normal Reaction .

$N = w \cos\theta$ ....(3)

The relation between the static friction angle and the measure of the angle of inclination of the plane to the horizontal.

If we put a body on a rough inclined plane and the body is about to slipping, then the measure of the static friction force equals the measure of the angle of inclination of the plane to the horizontal.

Proof :

$\because$ The friction is limiting

$\therefore$ The resultant reaction force makes an angle with the normal to the plane of measure equal the measure of the static friction force, and its measure is (λ) .

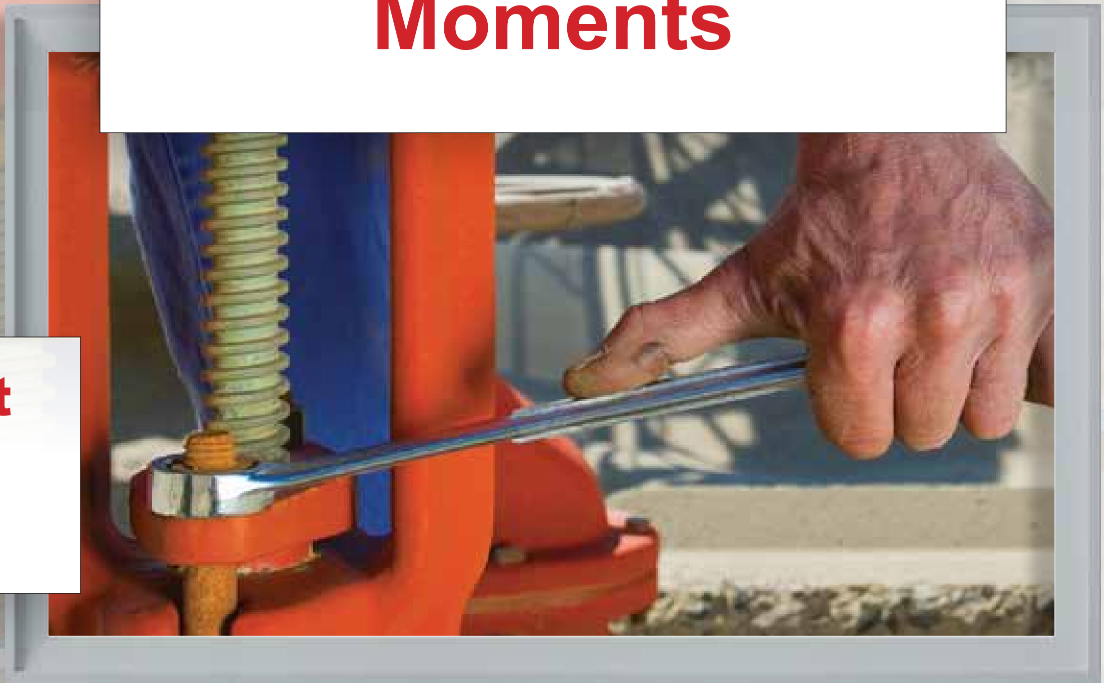
and from the previous figure, we find : $\theta = \lambda$

and we can put the equation in terms of coefficient of friction as follow :

$\tan \lambda = \mu_s$ or $\mu_s = \tan \theta$

Moments

Unit
1



Introduction

Since the early history, people had depended upon the idea of levers to enable them to carry and transfer their objects from one place to another. The human locomotor system is similar to the idea on which levers are based on . Bones are the materialistic solid bodies acted by the muscular power joint to the bones to turn around a fixed point (the center). This necessitate to understand the rational effect of the force (moment of a force) in this unit, we are going to shed the light on the concept of the moment of a force about a point in a 2-D coordinate system

Unit objectives

At the end of this unit and doing all the involved activities , the student should be able to:

- Find the norm and direction of the moment of a force about a point.
- Find the moments of coplanar forces about a point lying in their plane.
- Identify the general theorem of moments " if the system of forces acting on a rigid body has a resultant , then the algebraic sum of the moments of these forces about a certain point is equal to the moment of the resultant about this point ".
- Solve various applications on the moments.

Key terms

- Moment
- Moment Centre
- Moment Axis
- Moment Arm
- Rotation
- Resultant
- Moment Component
- Anti Clockwise
- Clockwise
- Algebraic Measure of the Moment
- Norm of the Moment

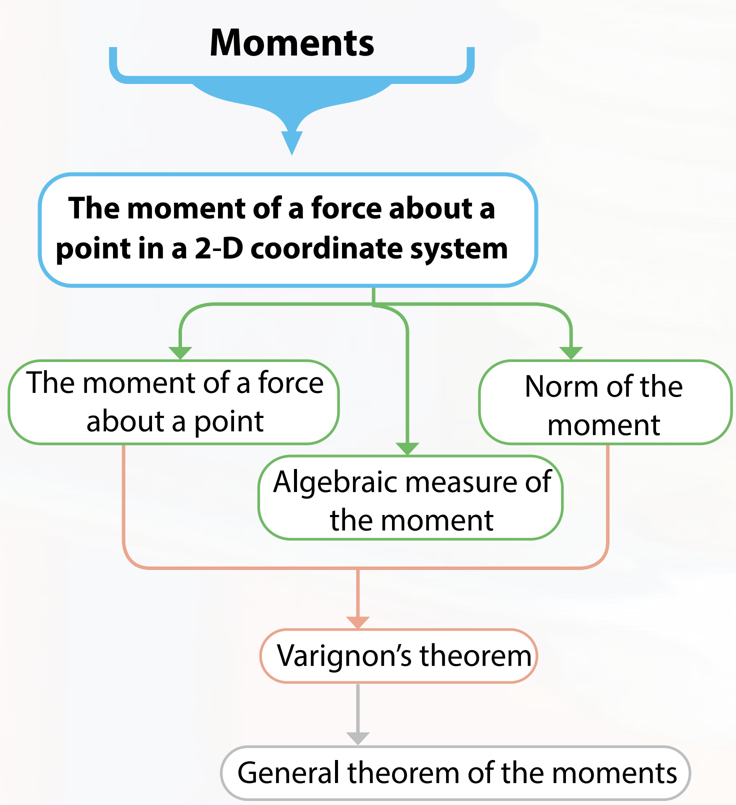
Lessons of the unit

Lesson (1 -1): The moment of a force about a point in a 2-D coordinate system

Materials

- Scientific calculator
- Computer graphics

Chart of the unit



Unit One

1 - 1

The moment of a force about a point in 2D-coordinate system

You will learn

- The moment of a force about a point
- The moments of the coplaner forces about a point in their plane

Key terms

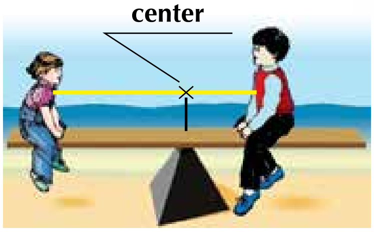
- Moment
- Moment centre
- Moment axis
- Moment arm

Materials

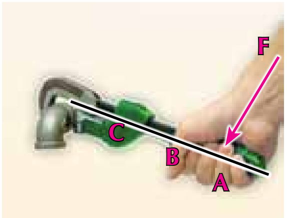
- Scientific calculator.

You have previously learned that the force is produced when a body acts on another body and this effect takes and any form (kinetic effect-formality effect). If a body moves from a place to another, the force acts on the body in a kinetic transitional way. If the body moves in a rotational motion about a point, the force acts on the body in a kinetic rotational way. Here, we say that the force can rotate the body about a point . It is called the moment of a force about a point . This rotational effect of the force (moment) depends up on the magnitude of the force and how distant the action line of this force from this point is.

Think and discuss



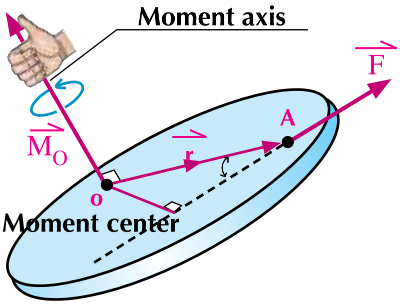
(1) The opposite figure shows two kids on a balanced swing in a horizontal position. which kid is (the heavier - the lighter) nearer to the rotation center? What does the heavier kid do if he/she wants to rotate the swing where the lighter kid rises up?



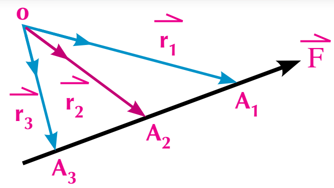
(2) The opposite figure shows a hand of a person trying to tie a pipe. The most proper position to the force (F) to tie the pipe perfectly is (A, B, C).

Learn

The moment of a force about a point in a 2D-coordinate system



The moment of a force $\overrightarrow{F}$ about the point O is known as the ability of the force to rotate the body about point O. The rotational effect can be calculated by the relation $\overrightarrow{M_O} = \overrightarrow{r} \times \overrightarrow{F}$ where $\overrightarrow{r}$ is the position vector of the of point A on the line of action the force about point O. The point O is called the moment center and the straight line passing through the point and perpendicular to the plane containing the force ($\overrightarrow{F}$) and point (O) is called the moment axis. We notice that the moment of a force is a vector quantity. According to the rule of the right hand of the vector multiplication, the direction of the moment of a force about a point and perpendicular to the plane containing the force $\overrightarrow{F}$.



Critical thinking: is the moment of a force $\vec{F}$ about point O based on the position of point A on the line of action of the force?

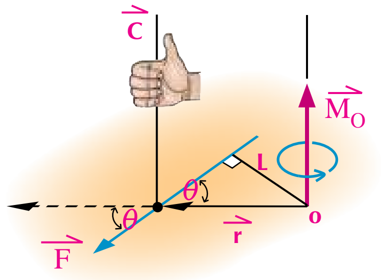
(1) The moment of a force about a point

From the definition of the vector multiplication of two vectors, then:

$\vec{M}_O = (\|\vec{r}\|\,\|\vec{F}\|\sin\theta)\,\vec{C}$ where $\vec{C}$ is a unit vector perpendicular to the plane of $\vec{F}$ and $\vec{r}$ where the rotation is from $\vec{r}$ to $\vec{F}$ in the direction of the vector $\vec{C}$ and θ is the measure of the angle between $\vec{r}$ and $\vec{F}$

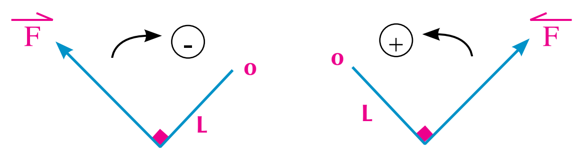
let $\|\vec{F}\| = F$ and $\|\vec{r}\| \sin\theta = L$

where L is the length of the perpendicular segment on the action line of force $\vec{F}$ (L is called the moment arm) then the moment of $\vec{F}$ about point O is $\vec{M}_O = (FL)\,\vec{C}$ (1)



(2) Algebraic measure of the moment

If the force $\vec{F}$ act on rotating about O in the anti clockwise direction, the algebraic measure of the moment vector is positive (the moment vector is in the direction of the vector $\vec{C}$). If the force $\vec{F}$ acts on rotating about O in the clockwise direction, the algebraic measure of the moment vector is negative (the moment vector is in the direction of the vector $-\vec{C}$)



(3) The norm of the moment: the norm of the moment is $\|\vec{M}_O\| = FL$ (2)

(4) The moment of a force about a point on its line of action = $\vec{0}$ zero

the action line of the force passes through point O

$\therefore\ M_O = Zero$

(5) The magnitude of the moment measuring unit:

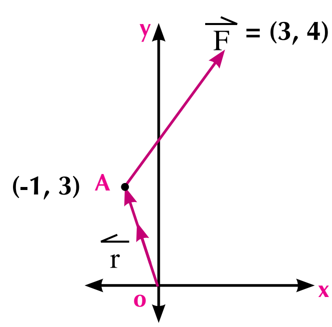
the magnitude of the moment measuring unit = the magnitude of force measuring unit × length measuring unit. Such as Newton. meter, Dyne. cm , kg.wt. meter ...

Example

1 If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are a right system of the unit vectors and the force $\vec{F} = 3\hat{i} + 4\hat{j}$ acts at the point A (-1, 3) of a body find:

a the moment of the force $\vec{F}$ about the origin point (0, 0)

b the length of the perpendicular segment from the point O and on the action line of the force $\vec{F}$



Solution

a $\vec{r} = \overrightarrow{OA} = \vec{A} - \vec{O}$

$= (-1, 3) - (0, 0) = (-1, 3)$

$\overrightarrow{M_O} = \overrightarrow{r} \times \overrightarrow{F}$

$= (-1, 3) \times (3, 4) = (-1 \times 4 - 3 \times 3)\ \hat{k}$

$= -13\ \hat{k}$

The norm of the moment = 13 moment unit and the algebraic measure of the moment vector = -13 moment unit

Interpreting the result: i.e the force $\overrightarrow{F}$ produces a rotation to the body about the point O and in the clockwise direction(the direction of the moment is in the direction of $-\hat{k}$)

b To find the length of the perpendicular drawn from O on the action line of the force $\overrightarrow{F}$

$\because \|\overrightarrow{M_O}\| = F\,L \qquad \therefore L = \frac{\|\overrightarrow{M_O}\|}{F} = \frac{13}{\sqrt{3^2 + 4^2}} = \frac{13}{5}$ length unit.

Try to solve

1. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are a right system of the unit vectors and the force $\overrightarrow{F} = \hat{i} - 2\hat{j}$ acts at the point A (2, 3) find:
   - **a** The moment of the force $\overrightarrow{F}$ about point B (2 , 1)
   - **b** The length of the perpendicular segment from point B on the line of action of the force .

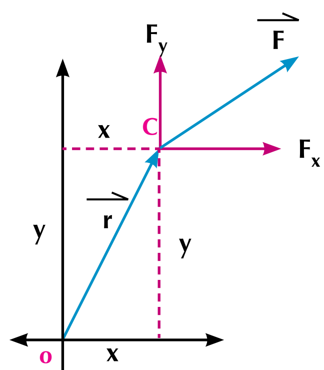
Critical thinking: what would it mean if the moment of the force about a point is vanished?

Learn

Principle of moments (Varignons theorm)

The moment of a force $\overrightarrow{F}$ about a point equals the sum of the moments of the components of this force about the same point.

Let the force $\overrightarrow{F} = F_x\,\hat{i} + F_y\,\hat{j}$ acts at the point A whose position vector with respect to the point O is $\overrightarrow{r} = (x, y)$ then



$\overrightarrow{M_O} = \overrightarrow{r} \times \overrightarrow{F}$

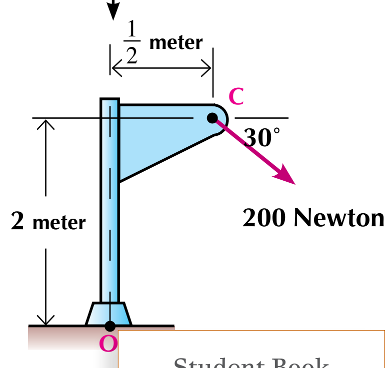
$= (x, y) \times (F_x, F_y)$

$= (x\,F_y)\ \hat{k} + (-y\,F_x)\ \hat{k}$

moment F_y about O + the moment of F_x about O

Example

2. **In the figure opposite:**
   Find the algebraic measure of the moment of the force about the point O.



First Solution:

We factorize the force 200 Newton into two components

$F_1 = 200 \cos 30 = 100\sqrt{3}$ Newton

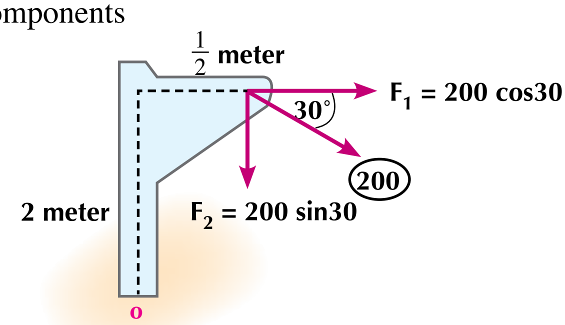
$F_2 = 200 \sin 30 = 100$ Newton

In regard to Varignon's theorem

$M_O = - F_1 \times 2 - F_2 \times \frac{1}{2}$

$= - 100\sqrt{3} \times 2 - 100 \times \frac{1}{2}$

$= (-200\sqrt{3} - 50)$ Newton . meter



Second Solution:

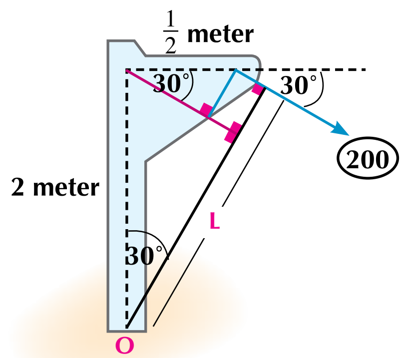
the length of the perpendicular segment from O on the line of action of the force = L

where $L = 2 \cos 30 + \frac{1}{2} \sin 30 = (\sqrt{3} + \frac{1}{4})$ meter

$\because$ the force is working to rotate about O in the clockwise direction

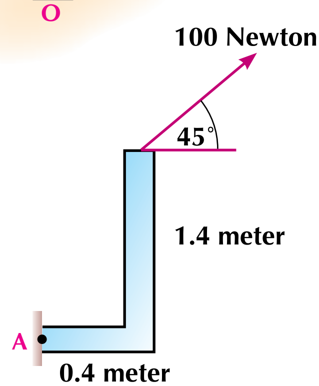
$\therefore$ the algebraic measure of the moment of a force is negative

$\therefore M_O = - 200 \times (\sqrt{3} + \frac{1}{4}) = (-200\sqrt{3} - 50)$ Newton.meter



Try to solve

2) **In the figure opposite:** calculate the algebraic measure for the moment of a force 100 Newton about point A .



Theorem

The algebraic sum of the moments of a system of forces acting at a point about any point in space is equal to the moment of the resultant of these forces about the same point

Proof

Let $\vec{F_1}, \vec{F_2}, ..., \vec{F_n}$ be a finite system of forces acting at point A

and let that is point (O) be the point required to find the moments about it

$\therefore \vec{r} = \vec{OA}$

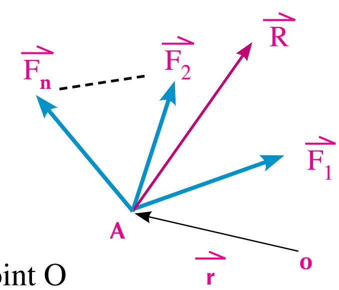
The sum of the moments of the forces about point O

$= \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2} + ... + \vec{r} \times \vec{F_n}$

$= \vec{r} \times (\vec{F_1} + \vec{F_2} + ... + \vec{F_n})$

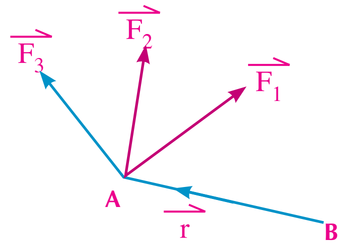
$= \vec{r} \times \vec{R}$

= The moment of the resultant of these forces about the same point O



Example (The moment of a system of forces acting at a point)

(3) The forces $\overrightarrow{F_1} = \hat{i} + 2\hat{j}$, $\overrightarrow{F_2} = (1, 3)$ and $\overrightarrow{F_3} = 4\hat{i} + 4\hat{j}$ act at the point A (-2, 1). Find the sum of the moments of these forces about point B (0, 2), then find the moment of the resultant of these forces about B. What do you notice?



Solution

$\overrightarrow{r} = \overrightarrow{BA} = \overrightarrow{A} - \overrightarrow{B} = (-2, -1)$

$\overrightarrow{M_1} = \overrightarrow{r} \times \overrightarrow{F_1}$

$= (-2, -1) \times (1, 2)$

$= (-4 + 1)\hat{k} = -3\hat{k}$

$\overrightarrow{M_2} = \overrightarrow{r} \times \overrightarrow{F_2} = (-2, -1) \times (1, 3) = (-6 + 1)\hat{k} = -5\hat{k}$

$\overrightarrow{M_3} = \overrightarrow{r} \times \overrightarrow{F_3} = (-2, -1) \times (4, 4) = (-8 + 4)\hat{k} = -4\hat{k}$

$\therefore$ The sum of the moment of the forces about point B

$= \overrightarrow{M_1} + \overrightarrow{M_2} + \overrightarrow{M_3}$

$= -3\hat{k} - 5\hat{k} - 4\hat{k} = -12\hat{k}$

Resultant of the force: $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = (1, 2) + (1, 3) + (4, 4) = (6, 9)$

$\therefore$ Moment of the resultant $= \overrightarrow{r} \times \overrightarrow{R}$

$= (-2, -1) \times (6, 9)$

$= (-18 + 6)\hat{k} = -12\hat{k}$

We notice that the sum of the moments of the forces about a point equals the moment of the resultant of the forces about the same point.

General theorem of moments

Theorem

The algebraic sum of the moments of forces about a point is equal to the moment of the resultant about this point.

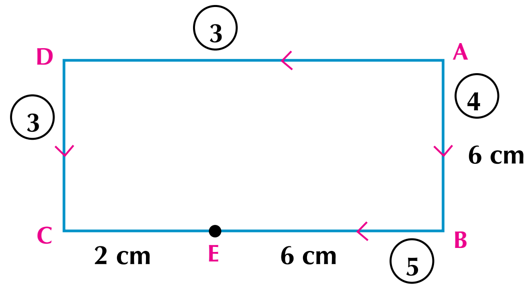
Try to solve

(3) The force $\overrightarrow{F_1} = 3\hat{i} - \hat{j}$, $\overrightarrow{F_2} = -2\hat{j} - 3\hat{j}$ act at point A (-1, 4). Find the sum of the moments of forces about point B (1, 1) hence find the moment of the resultant of these forces about point B.

Example

(4) ABCD is a rectangle in which AB = 6 cm and BC = 8 cm forces of magnitudes 4, 5, 3 and 3 Newtons act along the directions of $\overrightarrow{AB}$, $\overrightarrow{BE}$, $\overrightarrow{DC}$ and $\overrightarrow{AD}$ where E ∈ $\overline{BC}$, BE = 6cm. Prove that the resultant of these forces passes through point E.

Solution



The sum of the algebraic measures of the moments of forces about point E = -4 × 6 + 3 × 2 + 3 × 6 = zero

According to the theorem of moments, the moment of the resultant about point E = zero

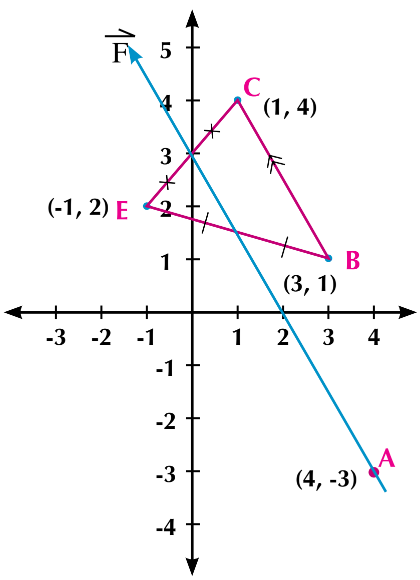
i.e. the resultant passes through point E

Try to solve

(4) A B C D is a square of side length is 6 cm and E ∈ $\overline{BC}$ where BE = 1 cm, forces of magnitudes 1, 2, 3, 4, and F Newtons act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and $\overrightarrow{AC}$ respectively. If the line of action of the resultant passes through point E, find the value of F.

Example

(5) A force $\overrightarrow{F} = -2\hat{i} + 3\hat{j}$ acts at the point A (4, -3). Find the moment of $\overrightarrow{F}$ about each of the points B(3, 1), C (1, 4) and D(-1, 2)



Solution

$\overrightarrow{r_1} = \overrightarrow{BA} = \overrightarrow{A} - \overrightarrow{B} = (1, -4)$

$\therefore \overrightarrow{M_B} = \overrightarrow{r_1} \times \overrightarrow{F} = (1, -4) \times (-2, 3) = (3 - 8)\hat{k} = -5\hat{k}$

$\overrightarrow{r_2} = \overrightarrow{CA} = \overrightarrow{A} - \overrightarrow{C} = (3, -7)$

$\therefore \overrightarrow{M_C} = \overrightarrow{r_2} \times \overrightarrow{F} = (3, -7) \times (-2, 3) = (9 - 14)\hat{k} = -5\hat{k}$

$\overrightarrow{r_3} = \overrightarrow{DA} = \overrightarrow{A} - \overrightarrow{D} = (5, -5)$

$\therefore \overrightarrow{M_D} = \overrightarrow{r_3} \times \overrightarrow{F} = (5, -5) \times (-2, 3) = (15 - 10)\hat{k} = 5\hat{k}$

From the previous example, we deduce that:

(1) If the moment of a force about point B = the moment of force about point C, then the line of action of this force // $\overleftrightarrow{BC}$

(2) If the moment of a force about point B = - the moment of this force about point D, then the line of action of this force bisects $\overline{BD}$

Try to solve

(5) A force $\overrightarrow{F}$ acts at point A (-3, 2). If the moment of $\overrightarrow{F}$ about each of the two points B (3, 1) and C (-1, 4) equals 28 $\hat{k}$, find $\overrightarrow{F}$.

Generalization of the previous conclusion

If a system of coplanar forces act on a body and A and B are two points at the same plane, then:

(1) If the sum of the moments of forces about A = the sum of the moments of the forces about B then the line of action of the resultant // $\overleftrightarrow{AB}$.

(2) If the sum of the moments of forces about A = - the sum of the moments of the forces about B, then the line of action of the resultants passes through the midpoint of $\overline{AB}$

Note: If the sum of the moments of the forces about a point- (say C) vanishes then either C lies on the line of action of the resultant or the resultant is the zero vector.

Example

(6) The force $\overrightarrow{F_1} = 2\hat{i} - \hat{j}$, $\overrightarrow{F_2} = 5\hat{i} + 2\hat{j}$, $\overrightarrow{F_3} = -3\hat{i} + 2\hat{j}$ act at point A(1, 1) prove using the moment that the line of action of the resultant is parallel to the straight line passing through the two points B(2, 1) and C (6, 4).

Solution

$\because \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 4\hat{i} + 3\hat{j}$

$\overrightarrow{r_1} = \overrightarrow{BA} = \overrightarrow{A} - \overrightarrow{B} = (-1, 0)$ $\qquad \overrightarrow{M_B} = \overrightarrow{r_1} \times \overrightarrow{R} = (-1, 0) \times (4, 3) = -3\overrightarrow{k}$

$\overrightarrow{r_2} = \overrightarrow{CA} = \overrightarrow{A} - \overrightarrow{C} = (-5, -3)$ $\qquad \overrightarrow{M_C} = \overrightarrow{r_2} \times \overrightarrow{R} = (-5, -3) \times (4, 3) = -3\overrightarrow{k}$

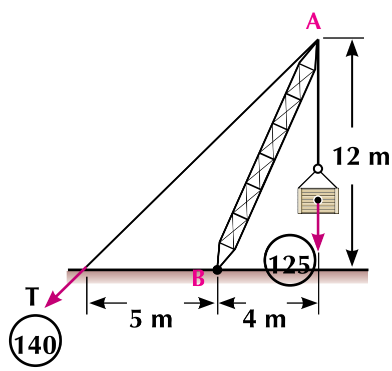
$\because \overrightarrow{M_B} = \overrightarrow{M_C}$, $\overrightarrow{R} \neq \overrightarrow{0}$ $\qquad \therefore$ line of action of $\overrightarrow{R}$ // $\overleftrightarrow{BC}$

Try to solve

(6) The forces $\overrightarrow{F_1} = \hat{i} + 2\hat{j}$, $\overrightarrow{F_2} = 3\hat{i} - \hat{j}$ act at point A (-2, 3) prove using the moments that the line of action of the resultant bisects the line segment drawn between the two points B(-1, 5) and C(1, 2).

Try to solve

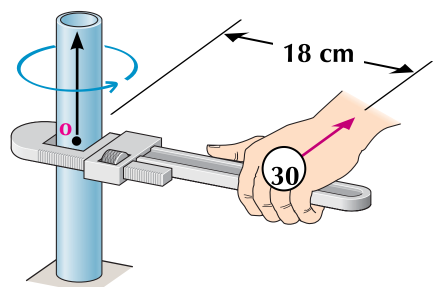
(7) **In the opposite figure:** $\overline{AB}$ represents a crane for lifting the goods. If the tension in the string is equal to 140 Newtons, and the weight of the box is 125 Newtons, find the sum of the two moments of the two forces about B.



Exercises 1 - 1

Complete the following

1) A force of a magnitude 50 Newtons and is 8 cm it is a line of action from point A, then the norm of the moment about point A equals .... Newtons . cm

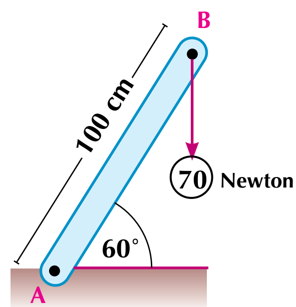


2) **In the opposite figure:** the norm of the moment of the force about the point (O) equals ....

3) A force of 4 $\hat{j}$ Newton acts at a point whose position vector with respect to the origin point O equals 5 $\hat{i}$ meters, then the moment of the force about the origin point equals ....

4) If the moment of a force about a point is equal to zero, this means ....

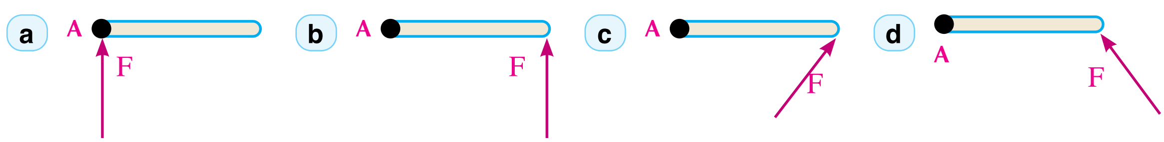
5) If the moment of the force about point is constant, then the magnitude of the force is inversely proportional to ....

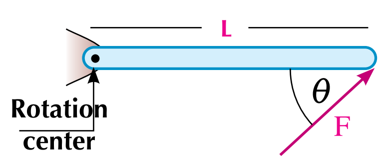


6) **The opposite figure:** shows a rod fixed by a hinge at A. If a vertical force of a magnitude 70 Newton acts on the end B downward, then the norm of the moment of the force about A is equal to .... Newtons. meter

Choose the correct answer:

7) **The opposite figure** presents a door attached with a hinge at A. If a force $\overrightarrow{F}$ acts on the door which of the following figures in which the force $\overrightarrow{F}$ has the greatest moment about A?





8) A rod of length L. It can rotate easily about a point. at one of its ends. A force of a magnitude F acts on the other end and inclines on the rode with an angle measured θ if $\overrightarrow{F}$ should be perpendicular to the rod, at which distance from the rotation center can F affect such that it has the same moment

a) $L \sin \theta$ b) $L \cos \theta$ c) L d) $L \tan \theta$

9) If the moment of a force $\overrightarrow{F}$ about point A is equal to its moment about point B then ....

a) $\overrightarrow{F} \perp \overline{AB}$ b) $\overrightarrow{F}$ bisects $\overline{AB}$

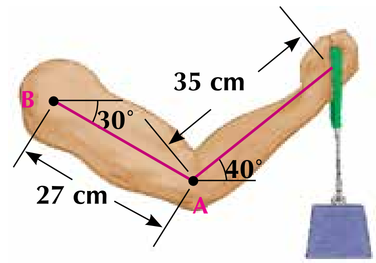
c) $\overrightarrow{F} \parallel \overline{AB}$ d) $\overleftrightarrow{AB}$ and action line of $\overrightarrow{F}$ are skew

Answer the following questions

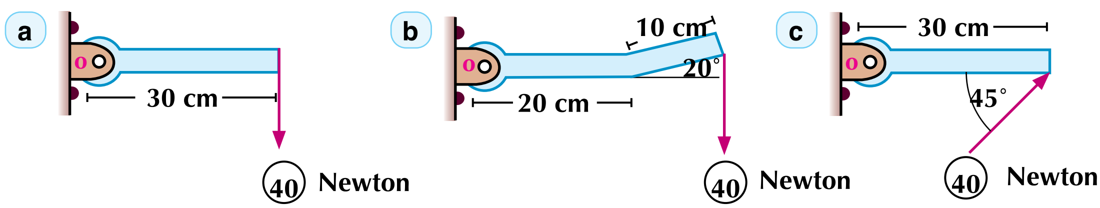
10 Two forces $\overrightarrow{F_1} = M\,\hat{i} + 2\,\hat{i}$ and $\overrightarrow{F_2} = L\,\hat{i} - \hat{j}$ act at the two points $A_1\,(1, 1), A_2\,(-1, -2)$ respectively. Determine the value of the two constants M and L such that the sum of the two moments of those two forces about the origin point and about point B $(2, 3)$ vanishes.

11 The forces $\overrightarrow{F_1} = 2\,\hat{i} - \hat{j}$, $\overrightarrow{F_2} = 5\,\hat{i} + 2\,\hat{j}$, $\overrightarrow{F_3} = -3\,\hat{i} + 2\,\hat{j}$ act at point A $(1, 1)$. Prove using the moments that the line of action of the resultant is parallel to the straight line passing through the two points $(2, 1)$ and $(6, 4)$

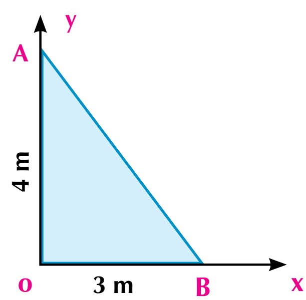
12 **The opposite figure** represents a person carrying a weight in his hand. If the norm of the moment of the weight about point A is equal to 80 Newtons meter, find the moment of the weight about point B



13 Find the algebraic measure of the moment of the force about point O in each of the following figures



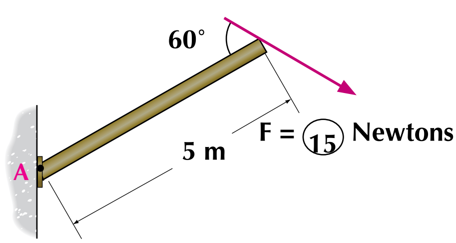
14 A force $\overrightarrow{F}$ at the xy-plane acts on the triangle A O B. If the algebraic measure of the moment of $\overrightarrow{F}$ about point O is equal to 84 Newton . m , the algebraic measure of its moment about point A is equal to - 100 Newton . m , and the algebraic measure of its moment at point B is equal to zero, determine $\overrightarrow{F}$



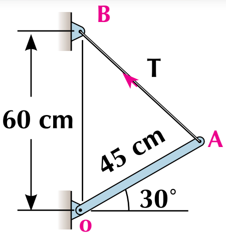
15 AB C D is a square of side length 10 cm. Forces of a magnitudes 3, 5, 8, $5\sqrt{2}$ kg.wt act along the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{AC}$ respectively. Find the algebraic measure the sum of the moments of the forces:

- **a** about point A
- **b** about point B
- **C** about the center of the square

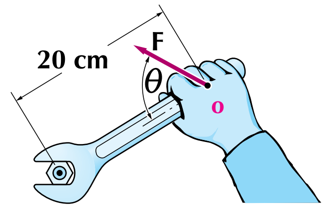
16 **The opposite figure** represents the effect of a force of 15 Newton on an arm fixed in a hinge at A. Find the algebraic measure of the moment of the force about point A.



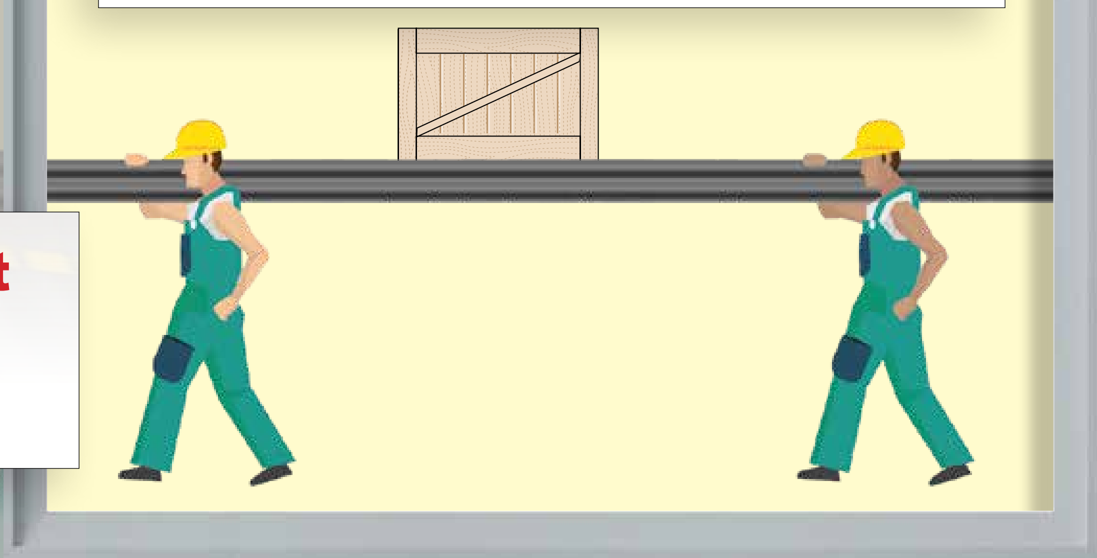
17 **In the opposite figure** the magnitude of the tension in the thread $\overline{AB}$ is 150 Newton. Find the algebraic measure of the moment of the tension force about point O.



18 If the moment needed to rotate a nail is equal to 400 Newton. cm, find the least value of the force F and the value of θ which satisfies the nail rotation.



Coplanar forces



Unit 2

Introduction

In our previous study of the system of coplanar forces acting on a materialistic point, the lines of action of the resultant of these forces meet at one materialistic point. As a result, the line of action ot the resultant of these forces passes through one point which is the common point of intersection to this system of forces. In this unit, we are going to learn a system of forces acting on a rigid body since the lines of action of these forces do not necessarily meet at a point

Unit objectives

At the end of this unit and by doing all the involved activities , the student should be able to:

- Identify the parallel coplanar forces.
- Identify the line of action of the resultant of two like and unlike forces.
- Identify one of the two parallel forces if the other force and the resultant are known.
- Find the moments of a system of parallel coplanar forces about a point
- Find the resultant of a system of parallel coplanar forces.
- Deduce that the sum of the moments of a system of parallel forces about a point is equal to the moment of the resultant about the same point.
- Deduce that the sum of the moments of a system of parallel forces about a point is equal to zero if their resultant passes through such a point.
- Deduce that the sum of the moments of a system of parallel forces about a point is equal to zero if the resultant of these forces vanishes.
- Specify the general comditions of Equilibrium of a body under the action of a set of coplanar forces
- Equilibrium of a set of parallel forces
- Equilibrium of a set of forces that are not parallel and not meet at a point (not concurrent)
- Solve applications on equilibrium of a ladder or a rod on rough horizontal ground and smooth vertical wall or support.

Key Terms

- Parallel forces
- Resultant
- Magnitude
- Norm
- Point of action
- Reaction
- Weight
- Parallel
- Support
- Beam
- Tension
- Pulley
- General Equilibrium
- Vertical reaction
- Horizontal component
- Vertical component
- Equilibrium of original body
- Triangle of force
- Friction

Unit Lessons

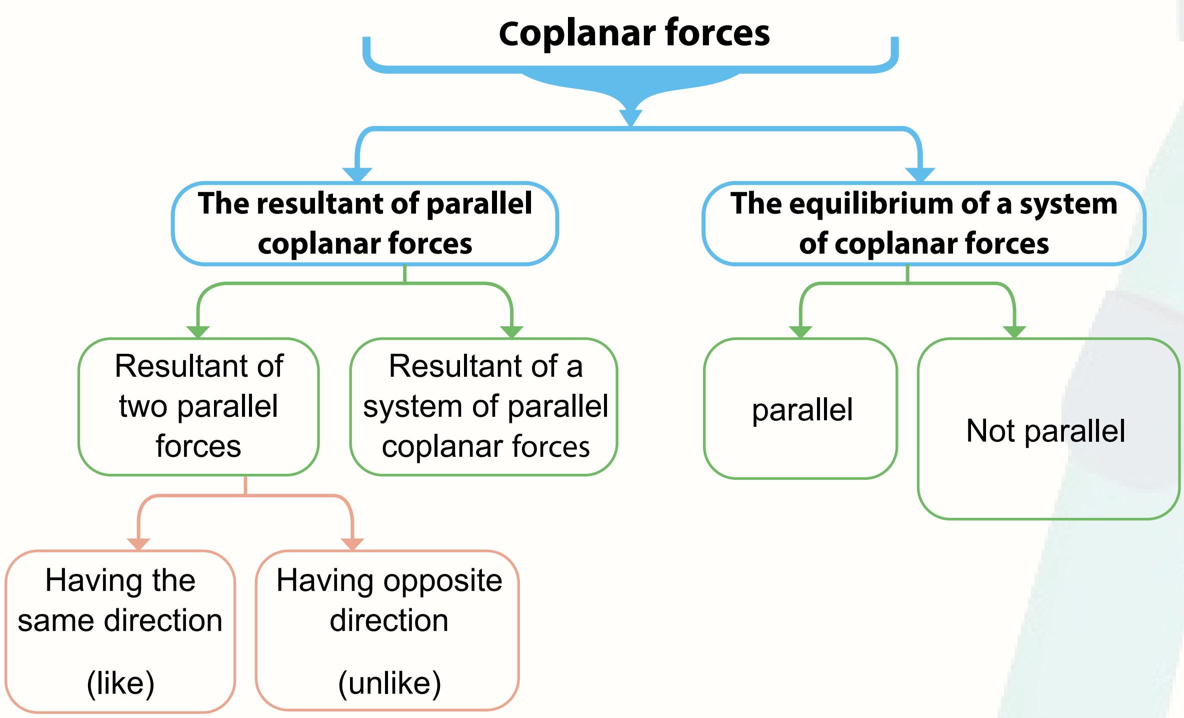
Lesson (2 -1): The resultant of parallel coplanar forces.

Lesson (2 -2): The equilibrium of a system of parallel coplanar forces.

Materials

- Scientific calculator
- Computer graphics

Unit planning guide



Unit Two

2 - 1 Resultant of a parallel coplanar forces

You will learn

- The resultant of two like forces.
- The resultant of two unlike forces .
- The resultant of a system of parallel and coplanar forces.

Key terms

- Parallel forces
- Resultant
- Magnitude
- Norm
- Point of action

Materials

- Scientific calculator

Cooperative work

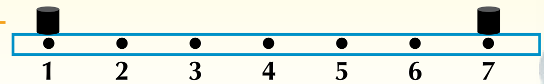


Figure (1)

Figure (1) shows a graduated wood ruler from 1 to 7 and two identical stones are placed at the ends of the ruler.

(1) Identify the position of a point on the ruler from which the ruler can be suspended in a horizontally equilibrium way.

(2) Does the position of the suspension point change if two weight are placed at one end as in figure (2)?

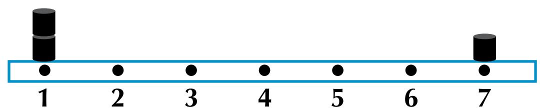


Figure (2)

Identify the new suspension point if the position changes.

Resultant of two parallel forces having the same direction (like forces)

You have learned that the resultant of a system of coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, ..., $\overrightarrow{F_n}$ are meet at a point is the force $\overrightarrow{R}$ where $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + ... + \overrightarrow{F_n}$ and passes through the same point. In this lesson, you are going to find the resultant of a system of parallel coplanar forces.

You start to find the resultant of two parallel coplanar forces having the same direction (like forces).

Let $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ be two like forces forces, act at a rigid body at two points A and B, then the resultant of these two forces is $\overrightarrow{R}$ where:

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$.

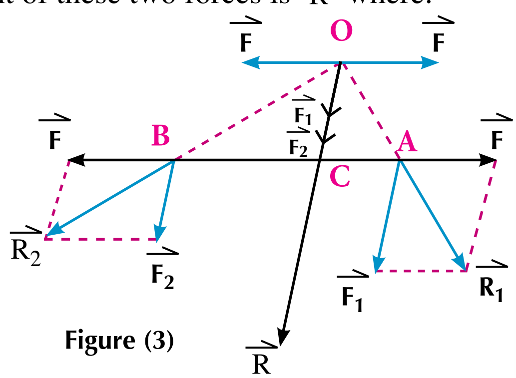


Figure (3)

To identify the position of the point of action of the resultant, let two forces of equal magnitude and opposite direction act at A and B and this does not change the action of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$.

Let R_1 be the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F}$ at A which represents a diagonal in the parallelogram and $\overrightarrow{R_2}$ be the resultants of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ at B.

and let the two lines of action of the two resultants $\vec{R_1}$ and $\vec{R_2}$ are intersect at O.

The force $\vec{R_1}$ can be replaced by their two original components $\vec{F_1}$ and $\vec{F}$ and the force $\vec{R_2}$ can be also replaced by their two original components $\vec{F_2}$ and $\vec{F}$.
The force acting at point (O) are: $\vec{F_1}$ and $\vec{F_2}$ They act in the direction of $\vec{OC}$ (parallel to the line of action of the two original forces) the two forces $\vec{F}$, $\vec{F}$ which and act at two opposite directions where they can be removed without any change to the effect of the two forces $\vec{F}$ and $\vec{F}$ at point (O). The two forces $\vec{F_1}$ and $\vec{F_2}$ acting at point (O). act in the direction of OC and have the same effect of the two forces $\vec{F_1}$ and $\vec{F_2}$ acting at A and B , then their resultant is

$$\vec{R} = \vec{F_1} + \vec{F_2} \text{ and act in the direction of } \vec{OC}$$

since the forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{R}$ are parallel, then

$$\frac{F}{F_1} = \frac{AC}{OC} \quad (1) \qquad \text{and} \quad \frac{F}{F_2} = \frac{BC}{OC} \quad (2)$$

By dividing (2) by (1) then : $\frac{F}{F_2} \times \frac{F_1}{F} = \frac{BC}{OC} \times \frac{OC}{AC}$ i.e $\frac{F_1}{F_2} = \frac{BC}{AC}$

Thus : $F_1 \times AC = F_2 \times BC$

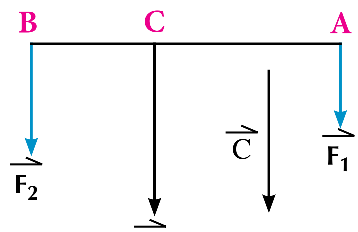


Figure (4)

In figure (4) by taking the unit vector $\vec{C}$ in the direction of the two forces, then :

$\vec{F_1} = F_1 \vec{C}$, $\vec{F_2} = F_2 \vec{C}$

$\therefore \vec{R} = (F_1 + F_2)\vec{C}$ this means that the resultant is in the direction of the two forces and its magnitude is equal to the sum of the magnitude of the two forces i.e:

The resultant of two like forces is a force working in the same direction as the two forces, its magnitude is equal to the sum of the magnitudes of the two forces and its line of action divides the distance between the lines of action of the two forces internally in an inverse ratio to their magnitudes.

Example Identify the resultant of two like forces

1 Two like forces of magnitudes 5 and 7 Newtons act at the two points A and B where AB = 36 cm find the resultant of the two forces

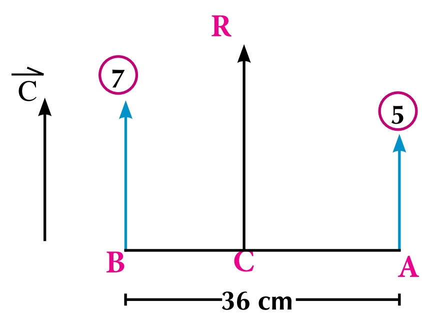


Figure (5)

Solution

Let $\vec{C}$ be the unit vector in the direction of the two forces

$\therefore \vec{F_1} = 5\vec{C}$ and $\vec{F_2} = 7\vec{C}$

the magnitude and direction of the resultant:

$\vec{R} = \vec{F_1} + \vec{F_2} = 5\vec{C} + 7\vec{C} = 12\vec{C}$

Identifying the point of action of the resultant

Let the resultant acts at point $C \in \overline{AB}$ $\therefore \frac{AC}{CB} = \frac{7}{5}$ **i.e** $\frac{AC}{36 - AC} = \frac{7}{5}$

$\therefore 5\,AC = 252 - 7\,AC$ **i.e** $AC = 21cm$

i.e the magnitude of the resultant is equal to 12 Newtons, it acts in the same direction of the two forces and it acts at a point which is 21 cm distant from A.

Try to solve

1. Two like forces of magnitudes 4 and 6 Newtons act at the two points A and B where AB = 25cm. Find the resultant of the two forces

Critical thinking: Where does the point of action of the resultant lie if the two forces are equal.

Learn

Resultant of two unlike forces

Similarly in figure (6), if F_1 and F_2 are two parallel, unequal forces in the opposite directions act at the two points A and B in a rigid body and their resultant is $\vec{R}$ then: $\vec{R} = \vec{F_1} + \vec{F_2}$ and acts at point C which divides $\overline{AB}$ externally in an inverse ration of the magnitude of the two forces.

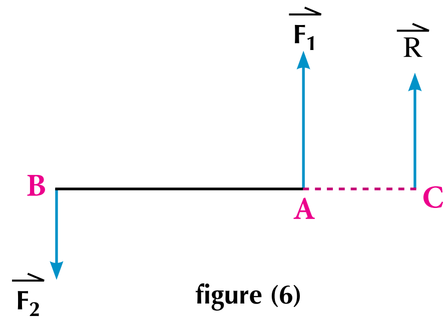


figure (6)

If $F_1 > F_2$ **then** $\frac{AC}{BC} = \frac{F_2}{F_1}$ i.e: $F_1 \times AC = F_2 \times BC$

i.e: the resultant of two unequal and unlike forces is a force working in the direction of the force of the greater magnitude, its magnitude is equal to the difference between their magnitudes and its line of action divides the distance between the lines of action of the two forces externally from the side of the force of the greater magnitude in an inverse ratio to their magnitude.

Example Identifying the resultant of two unlike forces

2. Two unlike forces of magnitudes 40 and 100 Newton and the distance between their two lines of action is 240 cm. Find their resultant.

Solution:

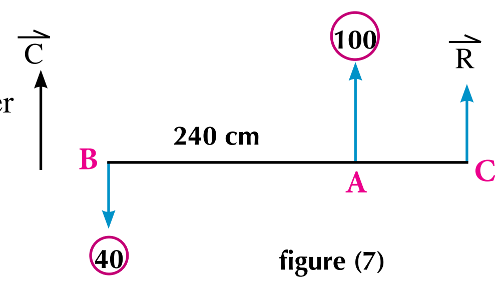


figure (7)

Let $\vec{C}$ be a unit vector in the direction of the greater force

$\therefore \vec{F_1} = 100\,\vec{C}$, $\vec{F_2} = -40\,\vec{C}$

the magnitude and direction of the resultant

$\therefore \vec{R} = \vec{F_1} + \vec{F_2} = 100\,\vec{C} - 40\,\vec{C} = 60\,\vec{C}$

Identifying the point of action of the resultant let the resultant acts at point $C \in \overrightarrow{BA}$ where

$\frac{CA}{CB} = \frac{40}{100}$

$\therefore \dfrac{CA}{240 + CA} = \dfrac{2}{5} \quad \therefore 5\,AC = 480 + 2\,AC \quad \therefore AC = 160$ cm

i.e the magnitude of the resultant is equal to 60 Newtons, its direction is the same direction of the force 100 Newtons, it acts at a point $\in \overrightarrow{BA}$ and $\notin \overline{AB}$ and it is 160 cm a part of A

Try to solve

2) Find the resultant of two unlike forces of magnitudes 7 and 12 Newtons act at A and B where AB = 20 cm

Critical thinking: What would you say about the resultant of two equal and unlike forces?

Theorem

The sum of the moments of a finite number of parallel coplanar forces about any point in its plane is equal to the moment of the resultant of these forces about the same point

Proof (is not required)

We start to prove this theorem in a special case when the system is made up of two forces only.

(1) If the two forces are like

Let a point such as (O) lie in the plane of the two forces. From point O, draw a common perpendicular on the two lines of action of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ to intersect them at the two point A and B respectively and to intersect the line of action of the resultant at point C.

Then, the algebraic sum of the moments of the forces about point O

$= - F_1 \times AO - F_2 \times OB = - F_1 (OC - AC) - F_2 (OC + CB)$

$= - F_1 \times OC + F_1 \times AC - F_2 \times OC - F_2 \times CB$ (1)

but: $\dfrac{F_1}{F_2} = \dfrac{BC}{AC}$ = i.e $F_1 \times AC = F_2 \times BC$

by substituting in (1) $\quad \therefore M_O = - F_1 \times OC - F_2 \times OC$

$= - (F_1 + F_2) \times OC$

$= - R \times OC$ = the moment of the resultant about point O

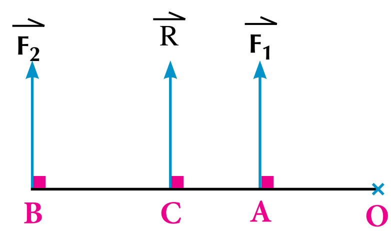


figure (8)

(2) If the two forces are unlike

Let $F_1 > F_2$ then , then algebraic sum of the moments of the forces about point O

$= F_1 \times OA - F_2 \times OB$

$= F_1 (OC + CA) - F_2 (OC + CB)$

$= F_1 \times OC + F_1 \times CA - F_2 \times OC - F_2 \times CB$ (2)

but: $\dfrac{F_1}{F_2} = \dfrac{CB}{CA}$ i.e $F_1 \times CA = F_2 \times CB$

by substituting in (2)

$\therefore M_O = F_1 \times OC - F_2 \times OC = (F_1 - F_2) \times OC$

$= R \times OC$ = the moment of the resultant about point O

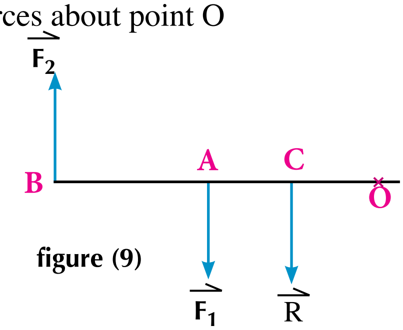


figure (9)

(3) If the system is made up of any definite number of forces (more than two forces) whose resultant does not vanish, then the theorem can be proved by getting the resultant of any two forces whose resultant does not vanish and applying the theorem in pairs and so on until we get the resultant of the system.

Example **Identifying one of two parallel forces if the other and the resultant are given**

(3) Two parallel forces of magnitudes 20 and F Newton act at the two points A and B. The magnitude of their resultants is 35 Newtons and the distance between the lines of action of the known force and the resultant is equal to 15 cm, Find $\overrightarrow{F}$ in each of the following two cases :

(a) The known force and the resultant are in the same direction.

(b) The known force and the resultant are in the opposite directions.

Solution:

(a) Let $\overrightarrow{C}$ be a unit vector in the direction of the resultant

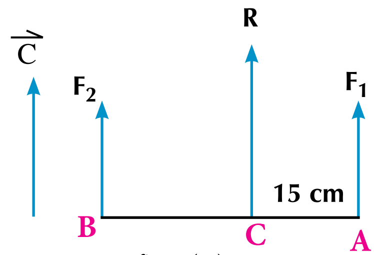


figure (10)

$\therefore \overrightarrow{R} = 35\ \overrightarrow{C}\ ,\ \overrightarrow{F_1} = 20\ \overrightarrow{C}$

$\because \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$ i.e $35\ \overrightarrow{C} = 20\ \overrightarrow{C} + \overrightarrow{F_2}$

$\therefore \overrightarrow{F_2} = 15\ \overrightarrow{C}$

i.e the force $\overrightarrow{F_2}$ of a magnitude 15 Newtons , and in the same direction of the known force and the resultant

$\because$ The sum of the moments about point C is equal to the moment of the resultant about point C = zero

$\therefore 20 \times 15 - 15 \times BC = \text{zero}$

$\therefore BC = 20$ cm i.e the force F_2 acts at point B at a distance of 35 cm from A

(b) Let $\overrightarrow{C}$ be the unit vector in the direction of the resultant

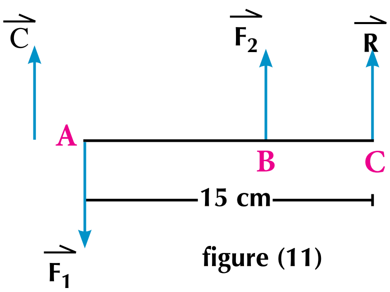


figure (11)

$\therefore \overrightarrow{R} = 35\ \overrightarrow{C}\ ,\ \overrightarrow{F_1} = -20\ \overrightarrow{C}$

$\therefore \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$ i.e $35\ \overrightarrow{C} = -20\ \overrightarrow{C} + \overrightarrow{F_2}$

$\therefore \overrightarrow{F_2} = 55\ \overrightarrow{C}$

i.e the force $\overrightarrow{F}$ of magnitude 55 Newtons and its direction is in the same direction of the resultant force

$\because$ the sum of the moments of forces about point C is equal to the moment of the resultant about C = zero

$\therefore 20 \times 15 - 55 \times BC = \text{zero}$ i.e $BC = \frac{60}{11}$ cm

i.e force F_2 acts at point B at a distance $\frac{105}{11}$ cm from A

Try to solve

(3) Two parallel forces, the magnitude of their resultants is 350 Newtons and the magnitude of one of the two forces is 500 Newtons acting at a distance of 51 cm from the resultant. Find

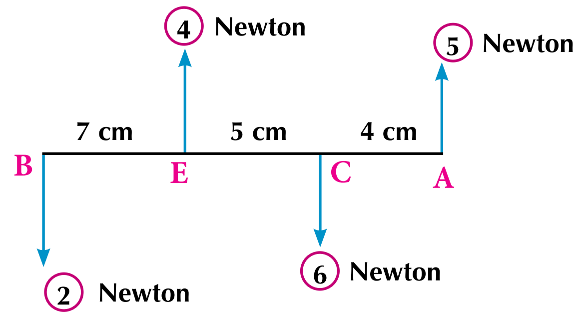
the second force and the distance between the two lines of action of the two forces if the known force and the resultant have

First: the same direction **Second:** the opposite directions

Example The moments of a system of parallel coplanar forces about a point

(4) The opposite figure represents a system of the parallel forces perpendicular to $\overline{AB}$. Find the algebraic measure of the sum of the moments of these forces about:

(a) point A (b) point C



Solution

(a) The force 5 Newtons acts at point A, then its moment about A is equal to zero. By considering the direction of revolving the forces about A (clockwise or anti clockwise direction), then the algebraic measure of the sum of the moments of forces about point A

$= 6 \times 4 - 4 \times 9 + 2 \times 16 = 20$ Newton. cm

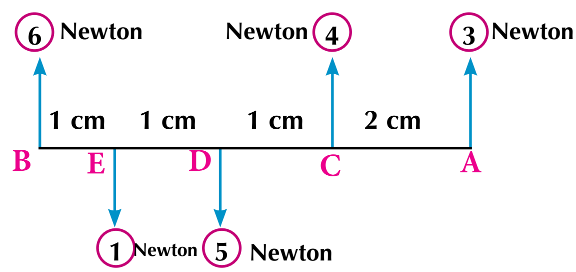
(b) The force 6 Newtons acts at point C, then its moment about C is equal to zero. The algebraic measure of the sum of the moments of forces about point C

$= 5 \times 4 - 4 \times 5 + 2 \times 12 = 24$ Newton. cm

Try to solve

(4) The opposite figure represents a system of parallel forces perpendicular to $\overline{AB}$ Find the algebraic measure of the sum of the moments of forces about

(a) Point A (b) Midpoint of $\overline{AB}$



Example The resultant of a system of parallel coplanar forces

(5) A , B , C , D and E are points lying on a straight line where: AB : BC : CD : DE = 2 : 3 : 4 : 7. Five parallel forces having the same direction whose magnitudes are 30, 50, 20, 70 and 40 Newtons act at the points A , B , C , D and E respectively. Find the resultants of these forces

Solution:

Let AB = 2x , BC = 3x

CD = 4x , DE = 7x

and let $\vec{C}$ be the unit vector in the direction of the forces

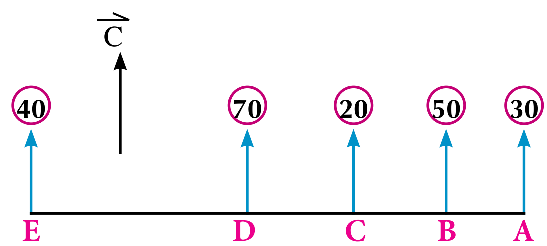


figure (12)

$\therefore \vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5}$

$= 30\vec{C} + 50\vec{C} + 20\vec{C} + 70\vec{C} + 40\vec{C} = 210\vec{C}$ Newtons i.e the magnitude of the resultant is 210 Newton in the same direction of the forces

to find the point of action of the resultant, let the resultant act at point $O \in \overline{AE}$

$\because$ the sum of the moments of the forces about A is equal to the moment of the resultant about A

$\therefore -50 \times 2x - 20 \times 5x - 70 \times 9x - 40 \times 16x = -210 \times AO$

$\therefore AO = \frac{1470x}{210} = 7x$ cm

$\frac{AO}{AE} = \frac{7x}{16x} = \frac{7}{16}$ i.e the resultant acts at point (O) which divides $\overline{AE}$ internally by a ratio 7 : 16 from the direction of A

Try to solve

5) If C , D and E $\in \overline{AB}$ such that AC : CD : DE : EB = 1 : 3 : 5 : 7. Parallel forces in the same directions and equal in the magnitudes act at points A, C, D, E and B. Prove that the resultant divides $\overline{AB}$ by a ratio 3 : 5

Example the resultant of a system of parallel forces

6) In the opposite figure (fig.13) A, B, C, D and E are five points lying on a straight line. Two forces of magnitudes 20 and 30 Newtons act vertically upwards at the two points B and D and two forces of magnitudes 40 and 60 Newtons act vertically downwards at the points A and C. Find the magnitude, direction and the point of action of the resultant.

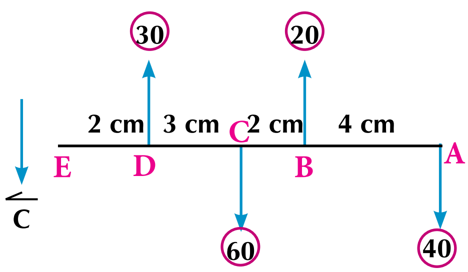


Figure (13)

Solution

Let $\overleftarrow{C}$ the unit vector down as shown in figure (13)

$\therefore \vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

$= -30\overleftarrow{C} + 60\overleftarrow{C} - 20\overleftarrow{C} + 40\overleftarrow{C} = 50\overleftarrow{C}$

Let the resultant acts at a point on $\overleftrightarrow{AE}$ and is distant x cm from A

$\because$ the sum of the moments of forces about A = the moment of the resultant about A

$-30 \times 9 + 60 \times 6 - 20 \times 4 = 50 \times X$

$\therefore X = 0.2$

i.e the resultant acts at a point on $\overleftrightarrow{AE}$ and at a distance of 0.2 cm from A

Try to solve

6) The opposite figure shows a light rod $\overline{AB}$. The parallel forces shown in the figure act on it. If the magnitude of the resultant is 300 Newtons and it acts up at a point on the rod and is 4 meters from A. Find F and K

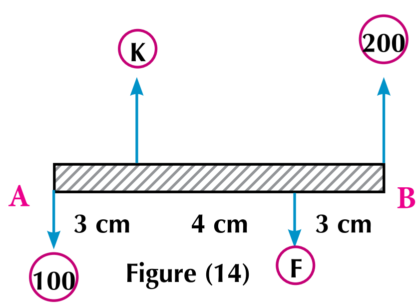


Figure (14)

Example — The theoritical proof

7 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two like forces which act at the two points A and B and their resultant is $\overrightarrow{R}$. If the force $\overrightarrow{F_2}$ moves parallel to itself in the direction of $\overrightarrow{AB}$ a distance x cm, then prove that the resultant of the two forces moves in the direction of $\overrightarrow{AB}$ a distance $(\frac{F_2}{F_1 + F_2})$ x

Solution

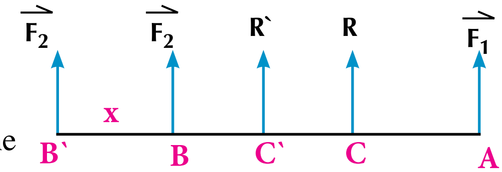


Figure (15)

In the first case:

Let the resultant act at point C

$\because$ the moment of the resultant at A = the sum of the moments of the forces at A

$\therefore$ R × AC = F_2 × AB (1)

in the second case:

if force $\overrightarrow{F_2}$ parallel to itself moves in the direction of $\overrightarrow{AB}$ a distance of x cm.

let the resultant act at C`

$\because$ the moment of the resultant at A = the sum of the moments of forces at A

$\therefore$ R × AC` = $F_2$ × AB` (2)

By subtracting (1) from (2)

$\therefore$ R (AC` - AC) = $F_2$ (AB` - AB) $\therefore$ R × CC` = F_2 × x

$\therefore$ CC` $= \frac{F_2}{R} \times x = (\frac{F_2}{F_1 + F_2})x$

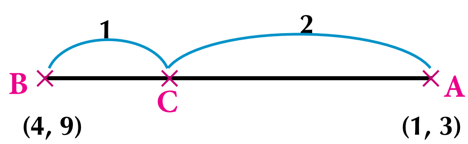
Try to solve

7 Two like forces of magnitudes F and 2F act at the two points A and B. If the force $2\overrightarrow{F}$ moves parallel to itself in the direction of $\overrightarrow{AB}$ a distance x cm, prove that the resultant of the two forces moves in the same direction a distance $\frac{2}{3}$ x

Example

8 Two forces $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$ and $\overrightarrow{F_2} = 4\hat{i} - 6\hat{j}$ act at the two points A(1, 3) and B(4, 9) respectively. Find the resultant of the two forces and its point intersection of it's line of action with $\overleftrightarrow{AB}$.

Solution



$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} = 6\hat{i} - 9\hat{j}$

we notice that $\overrightarrow{F_2} = 2\overrightarrow{F_1}$ i.e the two forces are parallel and in the same direction.

Let the resultant act at point C ∈ $\overline{AB}$ where $\frac{AC}{CB} = \frac{2}{1}$

From the rule of the dividing a line segment internally point C = $(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2})$

$\therefore$ C = $(\frac{2 \times 4 + 1 \times 1}{2 + 1}, \frac{2 \times 9 + 1 \times 3}{2 + 1}) = (3, 7)$

Try to solve

8. Two forces $\overrightarrow{F_1} = 3\hat{i} - \hat{j}$ and $\overrightarrow{F_2} = -9\hat{i} + 3\hat{j}$ act at the two points A(-1, 0) and B (1, 2) respectively. Find the resultant of the two forces and its point of intersection of it's line of action $\overleftrightarrow{AB}$.

Exercises 2 - 1

Choose the correct answer :

1. Two unlike forces of magnitudes 7 and 12 Newtons, then their resultant is equal to:
   - a 19 Newton
   - b 12 Newton
   - c 7 Newton
   - d 5 Newton
2. Two like forces of magnitudes 7 and 10 Newtons act at the two points A and B where AB = 51 cm. If their resultant acts at point C then AC =
   - a 30 cm
   - b 27 cm
   - c 21 cm
   - d 12 cm
3. Two like forces of magnitudes 5 and 7 Newtons then their resultants is equal to
   - a 12
   - b 6
   - c 2
   - d 1

Answer the following questions:

In the exercises 4 - 6, two parallel forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ act at the two points A and B. If their resultant $\overrightarrow{R}$ acts at point $C \in \overleftrightarrow{AB}$

4. Find the magnitude and the direction of the resultant and the length of $\overline{AC}$ in each of the following (the two forces are in the same direction)
   - a $F_1 = 9$ Newton , $F_2 = 17$ Newton , AB = 13 cm
   - b $F_1 = 23$ Newton , $F_2 = 15$ Newton , AB = 57 cm
   - c $F_1 = 16$ Newton , $F_2 = 10$ Newton , BC = 30 cm
5. If $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are in the same direction , answer the following:
   - a $F_1 = 8$ Newton , R = 13 Newton , AC = 10 cm find F_2 , A B
   - b $F_2 = 6$ Newton , AC = 24 cm , AB = 56 cm find F_1 , R
   - c $F_1 = 6$ Newton , AC = 9 cm , CB = 8 cm find F_2 , R
6. If $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are in opposite directions, answer the following:
   - a $F_1 = 15$ Newton , R = 20 Newton , AC = 70 cm find F_2 , AB
   - b $F_2 = 6$ Newton , AC = 24 cm , $C \notin \overline{AB}$, AB = 56 cm find F_1 , R
   - c $F_1 = 6$ Newton , AC = 9 cm , $C \notin \overline{AB}$, CB = 8 cm find F_2 , R
7. In each of the following, find the magnitude and the direction of the resultant and the distance of its point of action from point A

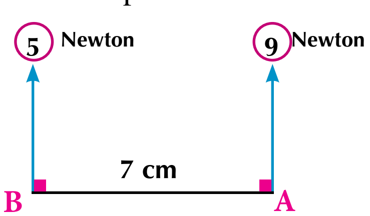


figure (16)

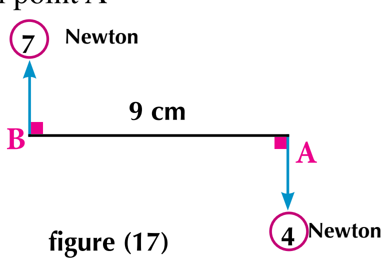


figure (17)

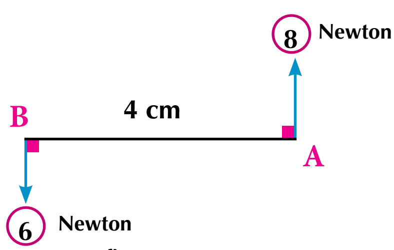


figure (18)

8. Two unlike forces of magnitudes 4 and 9 Newtons act at the two points A and B where AB = 15 cm. Find their resultants.

9. If the resultant of the two parallel forces $7\vec{C}$ and $5\vec{C}$ Newtons act at point $2\frac{1}{3}$ meters distant from the line of action of the smaller force, then find the distance between the two lines of action of the two forces.

10. Two parallel forces, the smaller one is 30 Newtons and acts at the end A of a light rod $\overline{AB}$ and the greater acts at the end B. If the magnitude of their resultant is 10 Newtons and is 90 cm distant from end B. How long is the rod?

11. A, B, C, D and E are points on one straight line such that AB = 4 cm, BC = 6 cm, CD = 8 cm and DE = 10 cm. Five forces of magnitudes 60, 30, 50, 80 and 40 kg.wt act at the points A, C, D, B and E respectively and in a perpendicular direction to $\overleftrightarrow{AE}$ such that the first three forces are in the same direction and the other two forces are in opposite directions. Identify the resultant of the system.

12. In figure (19) four weights of magnitudes 1, 7, 5 and 3 kg.wt have been placed on a light rod as in the figure. Identify the point of suspending the rod such that the rod remains horizontal.

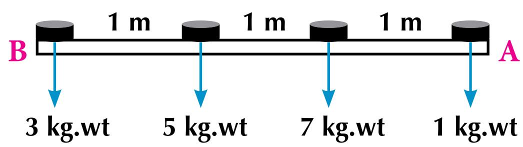


figure (19)

13. Two like forces of magnitudes 5 and 8 Newton act at the two points A and B where AB = 39 cm. If another force of a magnitude F in the same direction is added to the first force, then the resultant will move 8 units. Find F.

14. A, B and C are three points on one straight line where AB = 1 meter, AC = 3 meter $B \in \overline{AC}$. The forces of magnitudes 2 and $\frac{1}{2}$ Newton act vertically downwards at the two points A and B respectively and a force of a magnitude 4 Newtons acts vertically upwards at point B. Find the magnitude and the direction of the resultant and the distance of its point of action from point A.

Unit Two

2 - 2

Equilibrium of a system of coplanar forces

You will learn

- The equilibrium of a rigid body under the action of a system of parallel coplanar forces.

Key terms

- Reaction
- Weight
- Parallel
- Support
- Beam
- Tension
- Pulley
- Rotate

Materials

- Scientific calculator
- Mechanics lab

Rule

The body under the action of a set of coplanar forces in a static equilibrium state if the following two conditions are satisfied:

1- **The resultant vector of a set forces ($\vec{R} = \vec{o}$) vanishes**

2- **The moments of the forces about a point ($\vec{M} = \vec{o}$) vanishes**

These conditions are necessary and sufficient of the equilibrium of a set of coplanar forces. figure (20) shows a set of the orthogonal unit vectors ($\hat{i}$, $\hat{j}$, $\hat{k}$) where $\hat{i}$ and $\hat{j}$ lie on the plane of the forces and $\hat{k}$ is perpendicular to this plane. Thus, the resultant vector $\vec{R}$ can be resolved in the two directions of $\hat{i}$ and $\hat{j}$, while the moment vector $\vec{M}$ is parallel to the unit vector $\hat{k}$, then:

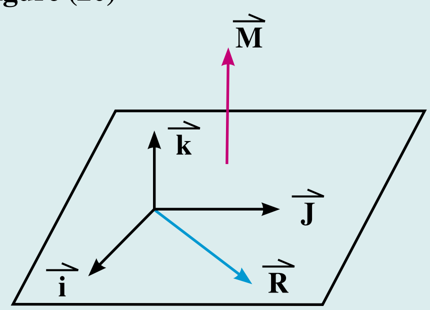


figure (20)

$\vec{R} = x\,\hat{i} + y\,\hat{i}$ **and** $\vec{M} = M\,\hat{k}$

where : x = The sum of the algebraic components of the forces of the system in directions of $\hat{i}$, y = the sum of algebric components of the forces of the system in direction of $\hat{j}$

$\vec{M}$ = The sum of the algebraic measures of the moment for the forces of the system in directions of $\hat{k}$ then the condition are sufficient and necessary for the equilibrium of a set of coplanar forces are x = 0 , y = 0 and M = 0.

Example

Equilibrium of a rigid body under the action of a system of coplanar parallel forces

1 The opposite figure shows a wood board of mass 30 kg for each meter of its length. If it rests horizontally on two supports A and B and carries a box of mass 240 kg, find the pressure exerted on each support.

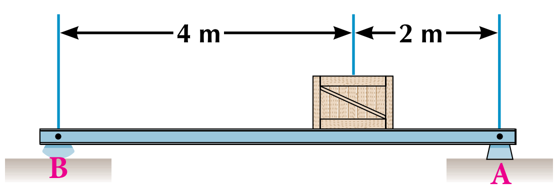


Figure (21)

Solution

Since the board is uniform, then its weight acts at its midpoint

Mass of the board $= 30 \times 6 = 180$ kg

$\therefore$ the weight of the board $= 180$ kg.wt

the reaction at each support is equal to the pressure exerted on it

The sum of the algebraic measures of the forces in the perpendicular direction to the board $= 0$

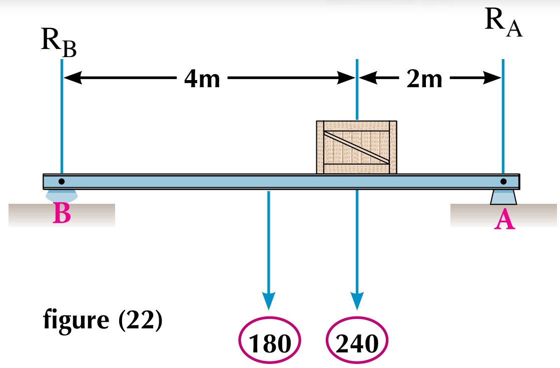


figure (22)

$\therefore R_A + R_B = 240 + 180 \qquad R_A + R_B = 420 \quad (1)$

the sum of the algebraic measures of the moments of the forces about point B $=$ zero

$-180 \times 3 - 240 \times 4 + R_A \times 6 =$ zero $\qquad$ i.e $R_A = 250$ kg.wt

$\therefore$ By substituting in (1) then $R_B = 170$ kg.wt

Critical thinking: What would happen to the reaction at each of A and B as the box gets closer from point A?

Try to solve

1. Two men A and B carry a wood board of length 2 meters and weight 16 kg.wt acts at its midpoint and the board carries a box of weight 24 kg.wt as shown in figure (23). Find the pressure exerted on the shoulder of each man, then identify which point of the board the shoulder of man B should be to equilibrate the two pressures.

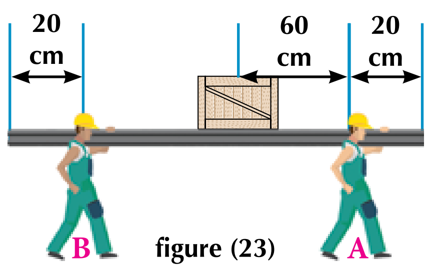


figure (23)

Example

Equilibrium of a system of coplanar parallel forces

2. $\overline{AB}$ is a uniform rod of length 90 cm and weight 60 Newton suspended horizontally by two vertical strings at its two ends A and B. Where should a weight of a magnitude 150 Newton be suspended in order that the tension magnitude at A is twice its magnitude at B.

Solution

Let the weight of 150 Newton suspended from a point distant x cm of A and the tension at B $= T$, and tension at A $= 2T$

$\therefore$ the sum of algebraic measures of forces $=$ zero

$\therefore 2T + T - 150 - 60 =$ zero, then $T = 70$ Newton

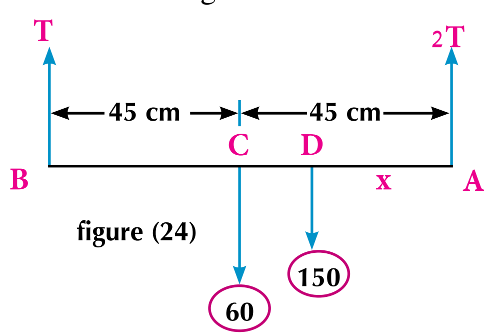


figure (24)

$\because$ The sum of algebraic measures of the moments of forces about A is equal to zero $\quad \therefore 150 \times x + 60 \times 45 - T \times 90 =$ zero

By substituting $T = 70$

$\therefore 150\,x = 3600$

$\therefore x = 24$ cm

Try to solve

(2) $\overline{AB}$ is a uniform wooden board of mass 10 kg and length 4 meters. If it rests horizontally on two supports; one of them at A and the other at a point distant 1 meter from B, then show at which distance a 50 kg.wt child can stand on the board in order for the two reactions on the two supports get equal.

Example

(3) $\overline{AB}$ is a non-uniform wooden board of length 4 meters resting horizontally on two supports at C and D such that AC = 1 meter and BD = $1\frac{1}{2}$ meter. If the maximum distance a 780 Newton man can move on the board from C to B without getting the board imbalanced is 3 meters from point A and the maximum distance the same man can move from C to A is $3\frac{1}{2}$ meters from point B. Find the weight of the board and its point of action.

Solution

Let the weight of the board be equal to (w) Newton and act at a point distant x meter from the end A.

First case:

When the man travels the maximum distance 3 meters from A to B the board is about to rotate about D.

i.e the reaction of the support at C vanishes.

$\because$ The sum of moments of forces about D = zero

$780 \times \frac{1}{2} - w\,(2\frac{1}{2} - x) = \text{zero}$

$\therefore\ w\,(2\frac{1}{2} - x) = 390$ **(1)**

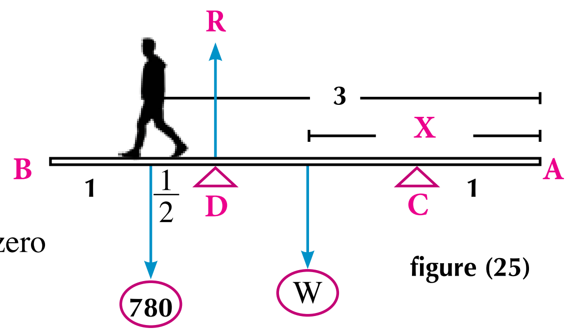


figure (25)

Second case:

When the man travels the maximum distance $3\frac{1}{2}$ meter from B to A the board is about to rotate about C. i.e the reaction of the support at D = zero

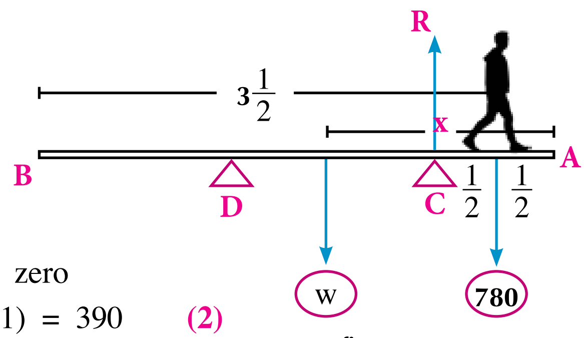


figure (26)

$\because$ The sum of moments of forces about C = zero

$\therefore\ w\,(x - 1) - 780 \times \frac{1}{2} = 0 \qquad \therefore\ w\,(x - 1) = 390$ **(2)**

from **(1)** , **(2)**

$\therefore\ x - 1 = 2\frac{1}{2} - x$ then $x = 1.75$ meters

By substituting in **(2),** we find that w = 520 Newton

i.e the weight of the board is equal to 520 Newton and acts at a point distant 1.75 meters from the end A.

Try to solve

3) $\overline{AB}$ is a rod of length 90 cm and weight 50 Newton acting at its midpoint rests horizontally on two supports. One of them at end A and the other is distant 30 cm from B carries a 20-Newton weight at a point distant 15 cm from B. Find the value of pressure exerted on each support and find the magnitude of the weight which should be suspended from end B such that the rod is about to rotate. What is the value of the pressure exerted on C hence?

Example

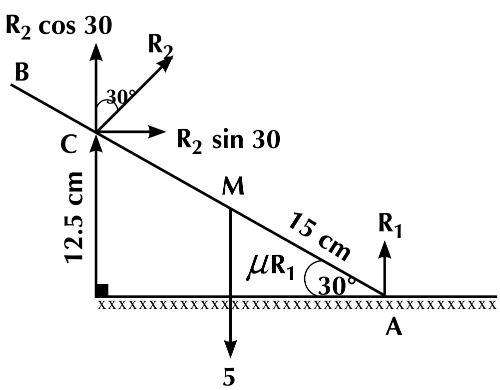
Equilibrium of a bar on a rough horizontal plane and a smooth wedge

4) A uniform bar $\overline{AB}$ of weight 5 kg.wt and length 30 cm rests with its end A on rough horizontal ground and at one of its point C on a smooth wedge which is 12.5 cm up on the ground surface. If the bar is about to slip when it inclines at 30° to the horizontal ground and lies in a vertical plane, find:

First : the reaction of the wedge.

Second : the coefficient of friction between the end A and ground.

Solution



We notice that $AC = 25$ cm

The bar is in equilibrium under the action of the forces:

The weight of the bar 5 kg.wt and acts vertically downwards.

The reaction at end A of the ground and its two orthogonal components R_1 and μR_1 .

The reaction of the wedge at the bar R_2, and it is perpendicular to the bar at the point of tangency C .

By applying the conditions of the equilibrium which are : $x = 0$, $y = 0$, $M_A = 0$

$\because M_A = 0 \quad \therefore 5 \times 15 \cos 30° - R_2 \times 25 = 0 \qquad \therefore R_2 = \frac{3\sqrt{3}}{2}$ .... (1)

From the two equations of the equilibrium : $x = 0$, $y = 0$

$\therefore R_2 \sin 30° - \mu R_1 = 0 \qquad \therefore R_2 = 2\mu R_1$ By substituting from (1)

$\therefore 2\mu R_1 = \frac{3\sqrt{3}}{2} \qquad \therefore \mu R_1 = \frac{3\sqrt{3}}{4}$ .... (2)

, $R_1 + R_2 \cos 30° = 5 \qquad \therefore R_1 + \frac{\sqrt{3}}{2} R_2 = 5$ By substituting from (1)

$\therefore \quad R_1 + \frac{\sqrt{3}}{2} \times \frac{3\sqrt{3}}{2} = 5 \qquad \therefore R_1 = 5 - \frac{9}{4} = \frac{11}{4}$ kg.wt .

By substituting the value of R_1 in equation (2) to find the value m .

$\therefore \mu \times \frac{11}{4} = \frac{3\sqrt{3}}{4} \qquad \therefore \mu = \frac{3\sqrt{3}}{11}$

Example

Equilibrium of a ladder on two orthogonal planes one of them is rough

5 A uniform ladder of weight 20 kg.wt rests in a rough horizontal ground with one of its ends and the other end on a smooth vertical wall and the ladder is being kept in a vertical plane and inclined at 60° to the horizontal. If it is known that the coefficient of friction between the ladder and the ground is equal to $\frac{1}{2\sqrt{3}}$. Prove that the maximum distance a girl of weight 60 kg.wt can ascend the ladder is equal to half the length of the ladder.

Solution

The ladder is in equilibrium under the action of the forces:

The weight of the ladder 20 kg.wt and acts vertically downwards at its midpoint.

The weight of the girl 60 kg.wt and acts vertically downwards at a distance x from the ladder base.

The reaction of the rough ground at end A whose two components the vertical R_1 and the horizontal μR_1 .

The reaction of the smooth wall R_2 and its orthogonal on the wall

By applying the conditions of the equilibrium which are :

$x = 0$, $y = 0$, $M_a = 0$ let the ladder length = ℓ ,

and the maximum distance the girl can ascend = x then the rod is about to move

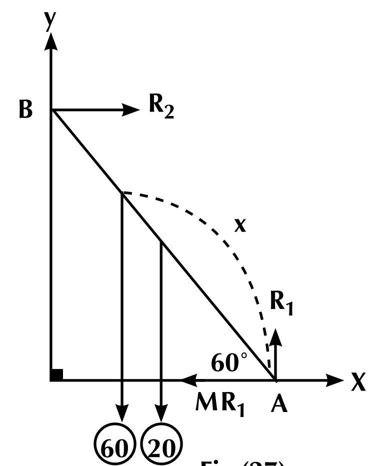


Fig (27)

$\because R_1 = 20 + 60 = 80$ kg.wt , $R_2 = \frac{1}{2\sqrt{3}} R_1$

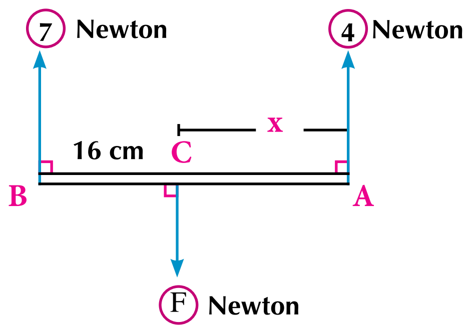
$\therefore R_2 = \frac{1}{2\sqrt{3}} \times 80 = \frac{40}{\sqrt{3}}$ kg.wt (1) $\because M_A = 0$

Try to solve

4 A uniform ladder AB of weight 30 kg.wt and length 4 meters rests with its end A on a smooth horizontal plane and its other end B against a smooth vertical wall. The ladder is being kept in a vertical plane and inclinded at 45° by a horizontal rope joining the end A with a point of the horizontal plane lying vertically under B exactly. If a man of weight 80 kg.wt ascends the ladder, prove that the tension of the rope increases whenever the man ascends. If the rope does not stand tension more than 67 kg.wt, find the length of the maximum distance the man can ascend without cutting the rope.

Exercises 2 - 2

In each of the following figures, a light rod is equilibrium horizontally. Find the magnitude (norm) each of the forces F and K

1


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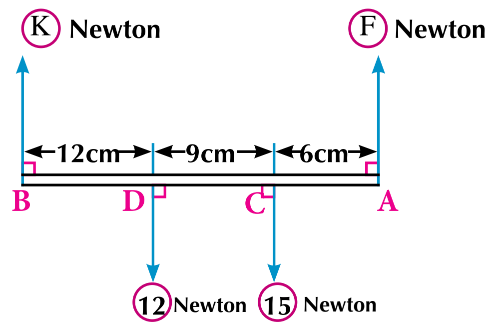

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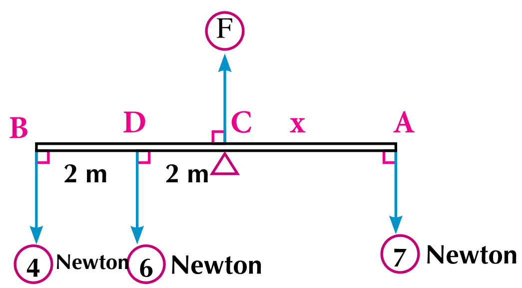
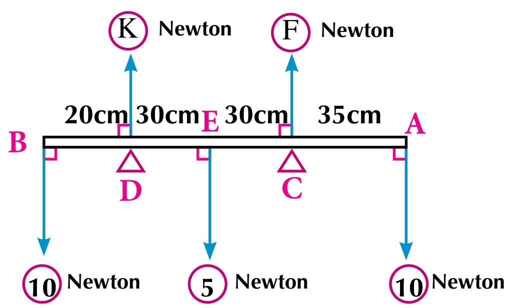


4


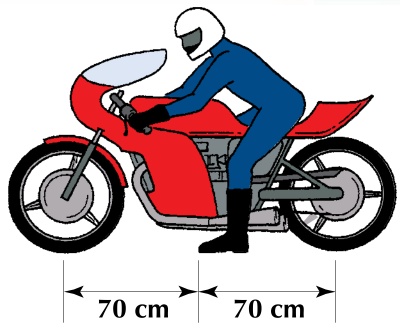
Answer the following :

5. A uniform rod of length 2 meters and mass 75 kg rests horizontally on two supports at its ends. A weight of magnitude 15 kg.wt is suspended from a point on the rod distant 50 cm from one end. Find the reaction at each support.

6. A uniform rod of length 3 meters and mass 4 kg carries two bodies of masses 5 kg and 1.5 kg at its two ends. Find the position of the suspension point on the rod in order for the rod to equilibrate horizontally.

7. AB is a non-uniform rod of length 120 cm. If a weight of a magnitude 1 Newton is fixed at its end B and a weight of a magnitude 16 Newton is suspended from end A. If the rod gets equilibrated in this case at a point distant 30 cm from A and if the weight existed at A is decreased to be 8 Newton, then the rod gets equilibrated at a point distant 40 cm from A. Find the weight of the rod and the distance from its point of action to A.

8) Figure (38) shows a motorcycle of mass 200 kg and its weight acts at the vertical line passing through the midpoint between the two wheels, and the mass of the motorcyclist is 84 kg and its weight acts the vertical line distant 1 meter behind the front wheel. Find the reaction of the ground on each of the two wheels for each of the following cases :



a) The motorcycle is without a motorcyclist.

B) The motorcycle and the motorcyclist.

9) $\overline{AB}$ is a rod of length 60 cm and weight 400 gm.wt acts at its midpoint on a wedge distant 20 cm from A to keep the rod horizontally in an equilibrium state by a light vertical string attached by its end B. Find:

a) The magnitude of the string tension and the reaction of the wedge.

B) The magnitude of the weight required to be suspended from A to make the tension in the rod about to vanish.

10) AB is a uniform rod of length 60cm and weight 10 gm.wt acting at its midpoint. It is suspended horizontally by vertical strings. One of them is attached at point A and the other is attached at point C where AC = x cm. A weight of a magnitude 12 gm.wt is suspended at point D where AD = 25 cm. If the maximum tension each string can stand is 15 gm.wt, find the values in which x lies and find the maximum and minimum values of the tension in each of the two strings.

11) Light ruler $\overline{AB}$ measured by cm rests horizontally on two supports at C, D where C $\in \overline{AD}$, 2A C = 2 BD = CD, suspended weight of a magnitude (w) Newton from the point (m) on the ruler it is about to rotate if a weight of magnitude 10 Newton is suspended from A or a weight of magnitude 6 Newton is suspended from B. Find the magnitude (w) and prove that $\frac{AM}{MB} = \frac{9}{7}$.

12) Two men A and B carry a body of mass 90 kg suspended from a strong light iron rod. If the distance between the two men is 60 cm and the suspension point of the body is distant 20 cm from A. What is the magnitude that each man can carry of this weight? If man B cannot carry more than 50 kg.wt, identify the maximum distance from A the weight can be suspended at until man B can keep carrying the rod.

13) A uniform ladder of weight 64 kg.wt rests in a smooth vertical wall with one of its ends and the other end on a smooth horizontal plane. The ladder is being kept in a position inclinded at 45° to the horizontal by a rope fixed at the ladder base and at a point of the plane lying vertically downwards the top of the ladder. A man of weight equal to the ladder weight stands at a position of the ladder distant $\frac{3}{4}$ of the ladder length from the direction of the base. Find the tension in the rope and the reactions at the wall and plane as well.

14. A uniform ladder of weight 10 kg.wt rests on a smooth horizontal plane with its end A and on a smooth vertical wall with its end B. The ladder is being kept in a vertical plane in a position inclined at 45° to the horizontal by a horizontal rope attaches end A to a point on the horizontal plane vertically under B. If a man of weight 80 kg.wt ascends this ladder, find:
First : the tension in the rope when the man has ascended $\frac{3}{4}$ of the ladder length.
Second : the maximum value of tension which the rope can stand known that it is about to be cut as the man reaches the ladder top.

15. A uniform rode of weight 40 Newton rests in rough horizontal ground with its end A and on a smooth vertical wall with its end B such that the rode is in a vertical plane perpendicular to the wall and inclindes at 45° to the horizontal ground. Find the minimum horizontal force acting at end A of the rod to make it about to slip away from the wall known that the coefficient of friction between the rod and ground is 0.75

16. A uniform rod rists in a vertical plane with its upper end on a smooth vertical wall and on a rough horizontal plane with its lower end such that it makes an angle whose tangent $\frac{3}{2}$ to the horizontal . Find the coefficient of friction between the rod and the horizontal plane as it is about to slip.

Couples

Unit 3

Introduction

In the previous unit, you learned to obtain two parallel forces in opposite direction by replacing them into two forces meeting at a point (concurrent). You noticed that it can be possible as long as the two forces are equal. But if the two parallel forces are equal in the magnitude, they cannot be replaced by two non-parallel forces. We always obtain two parall, forces equal in magnitude and of opposite direction. Hence, such two forces cannot be obtained at the same time.

In regard to what is mentioned above, we believe that the system made up of two parallel forces of equal magnitudes and of opposite direction has a new name in statistics which is called "couple" in this unit, we are going to demonstrate the concept and definition of a couple and, calculate its moment , the equilibrium of a rigid body under the action of two coplanar couples, and the moment of the resultant couple. This unit ends up with learning the sum of any finite number of couples.

At the end of this unit and by doing all the activities involved , the student should be able to:

- Identify the concept of a couple.
- Find the moment of a couple.
- Deduce that the moment of a couple is a constant vector.
- Identify equivalent of two couples and equilibrium of two couples.
- Identify the concept of the equilibrium of a body under the action of two coplanar couples.
- Find the resultant of several couples
- Prove that a system of forces is equivalent to a couple (the resultant = zero and the moments about any point $\neq$ zero) or (the sum of moments of the forces about three non-collinear points = a constant $\neq$ zero.)
- Prove that a system of forces is equivalent to a couple using the definition.
- Recognise the theorem stating "If many coplanar forces act on a rigid body and are completely represented by the sides of a polygon، taken the same cyclic order، then the system is equivalent to a couple.
- Solve various applications on the couples.

Key Terms

- Couple
- Line of action
- Equilibrium
- Rigid body
- Equivalence

Unit Lessons

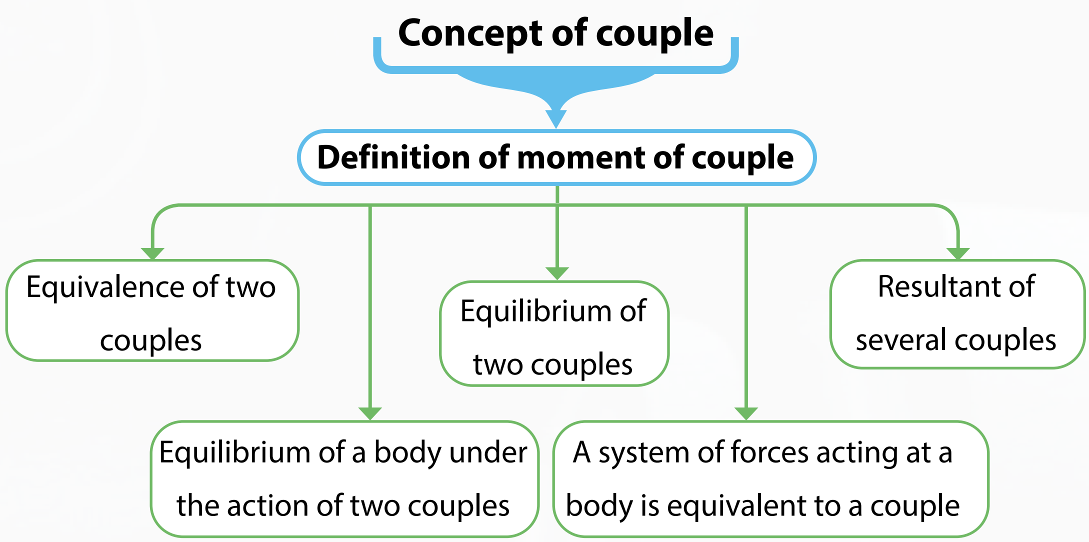
Lesson (5-1): Couples

Lesson (5-2): Resultant couple

Materials

Scientific calculator

Unit planing guide



Unit Five

3 - 1

COUPLES

You will learn

- Couple and moment of couple
- Equivalence of two couples
- Equilibrium of a body under the action of two or more couples

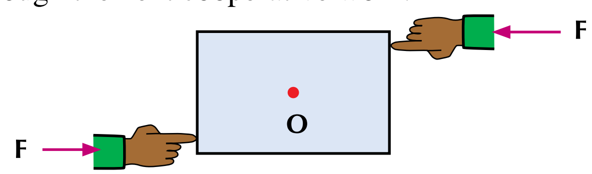
Key terms

- Couple
- Line of action
- Equilibrium
- Rigid body
- Equivalence

Materials

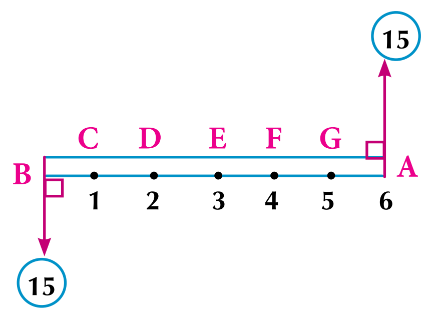
- Scientific calculator
- Mechanical Lab

Introduction: Some people may believe that if the resultant of the forces acting on a body is equal to zero, then this body is kept at rest (static). If you look at the opposite figure, you will find two forces of equal magnitudes and opposite direction (their resultant is equal to zero). You see that this body will move a rotational motion about (0). The velocity of rotation depends upon several things which the students can discover through the next cooperative work:



Cooperative work

The opposite figure represents a graduated ruler two parallel forces in opposite direction of magnitude 15 Newton of each act at its ends. With your classmates. Find the sum of the moments of the two forces about each of the points A, B, C, D, E, F and G and place your results in the next table:



The points	A	B	C	D	E	F	G
Sum of moments of two forces							

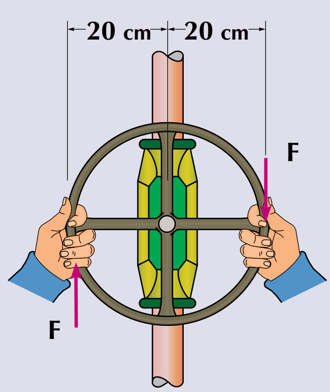
What do you notice from the results?

Learn

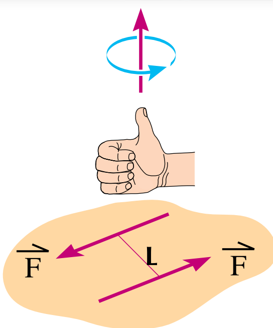
Couple

Definition

Couple: a system of two forces of equal magnitudes and opposite directions and acting in different lines of action.



Moment of a couple



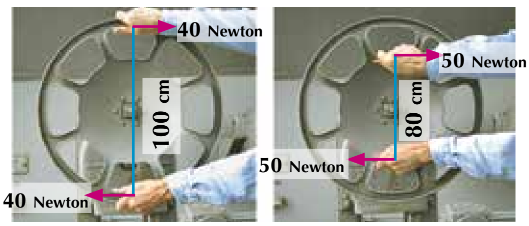
The moment of a couple is known as the sum of the moments of the two forces of the couple about a point in space, and its magnitude is equal to the product of the magnitude of one of the two forces by the normal distance between them and is denoted by $M = \|\vec{M}\|$

$\therefore \|\vec{M}\| = F \times L$ where $F = \|\vec{F}\|$ and L is called the arm of the couple

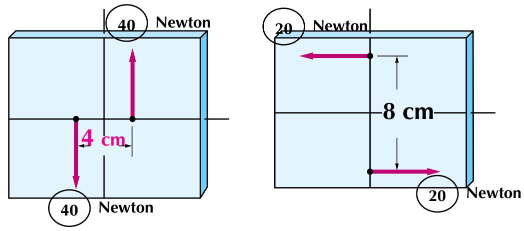
Example

1 Find the algebraic measure of the moment of the couple in each of the following figures:

a



b



Solution

a The algebraic measure of both the couples in figure (A) is equal to - 4000 Newton. cm

b The algebraic measure of both the couples in figure (B) is equal 160 Newton. cm. Notice the increase of the normal distance between the two forces, the decrease of the magnitudes of the two forces, and the constancy of the magnitude of the moment.

Try to solve

1 Find the algebraic measure of the moment of the couple in the following figure:

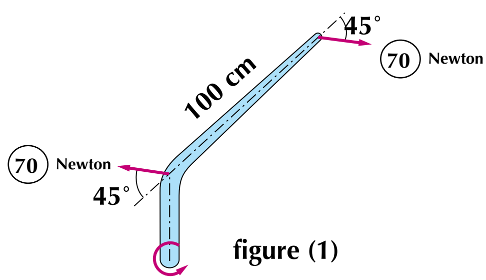
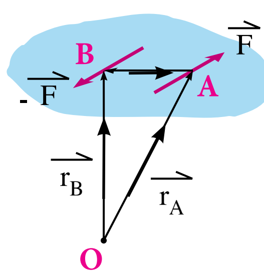


figure (1)

Theorem

The moment of a couple is a constant vector independent of the point about which we take the moments of the two forces.

The proof (is not required)



Let the two forces $\vec{F}$ and $-\vec{F}$ act at the two points A and B respectively and let point O be a general point in space.

We find the sum of the moments of the forces about point O

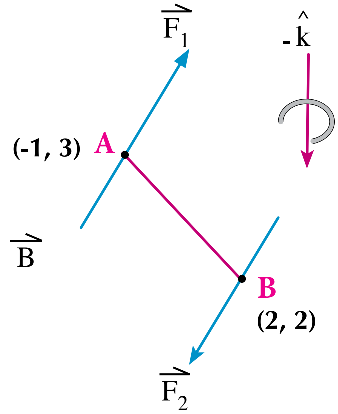
$$\vec{M} = \vec{OA} \times \vec{F} + \vec{OB} \times -\vec{F} = (\vec{OA} - \vec{OB}) \times \vec{F}$$

$$\because \vec{OA} - \vec{OB} = \vec{BA} \qquad \therefore \vec{M} = \vec{BA} \times \vec{F}$$

The last form of the moment shows that the moment of the couple independent of the position of point O about which we take the moments of the two forces.

Example

(2) If the two forces $\vec{F_1} = 2\hat{i} + b\hat{j}$ and $\vec{F_2} = a\hat{i} - 5\hat{j}$ form a couple and act at the two points A (-1 , 3) and B (2 , 2) respectively, find the value of a and b, and the moment of the couple.



Solution

∵ The two forces form a couple ∴ $\vec{F_1} = -\vec{F_2}$

∴ a = -2 , b = 5

The moment of the couple = moment of $\vec{F_1}$ about B

$= \vec{BA} \times \vec{F_1}$ where $\vec{BA} = \vec{A} - \vec{B}$

$= (-3 , 1) \times (2 , 5)$

$= (-15 - 2)\hat{k} = -17\hat{k}$

Try to solve

(2) If $\vec{F_1}$ and $\vec{F_2}$ are two forces of a couple where $\vec{F_1} = -3\hat{i} + 2\hat{j}$ acting at point A (1 , 1) , $\vec{F_2}$ acting at point B (-1 , 2), find $\vec{F_2}$, moment of the couple, and the length of the perpendicular drawn from A on the line of action of $\vec{F_2}$.

Equilibrium of a rigid body under the action of two or more coplanar couples

Definition

A rigid body is said to be in equilibrium under the action of two coplanar couples if the sum of their moments is the zero vector

If $\vec{M_1}$ and $\vec{M_2}$ are the two moments of two couples, then the condition of the equilibrium of a body under the action of the two couples is $\vec{M_1} + \vec{M_2} = \vec{O}$

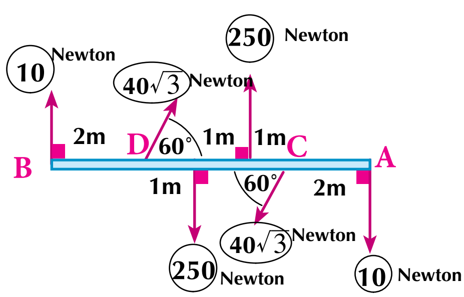
In general: If the body is acted on by several coplanar couples of moments $\vec{M_1}$, $\vec{M_2}$, ..., $\vec{M_n}$, then the condition of the equilibrium of the body under the action of such couples is $\vec{M_1} + \vec{M_2} + ... + \vec{M_n} = \vec{O}$

Corllary

A rigid body is said to be in equilibrium under the action of two or more coplanar couples if the sum of the algebraic measures of the moments of the couple vanishes

Example

(3) The forces shown in the figure act at the light rod AB. prove that the rod is in equilibrium.



Solution

The two forces 10, 10 from a couple.

The algebraic measure of its moment is $M_1 = -10 \times 7 = -70$ Newton. meter

The two forces $40\sqrt{3}$ and $40\sqrt{3}$ form a couple. The algebraic measure of its moment

$M_2 = -40\sqrt{3} \times 3 \sin 60 = -180$ Newton. meter

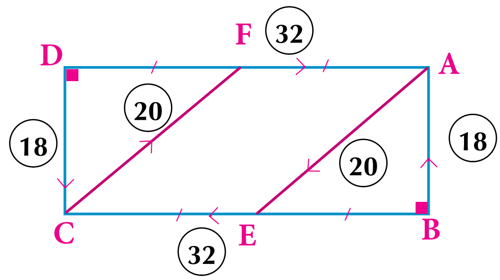
the two forces 250 and 250 form the couple of the algebraic measure of its moment

$M_3 = 250 \times 1 = 250$ Newton. meter

$\because M_1 + M_2 + M_3 = -70 - 180 + 250 =$ zero $\quad \therefore$ the rod is in equilibrium.

Try to solve

3 **In the opposite figure:** ABCD is a rectangle. E and F are the midpoints of $\overline{BC}$ and $\overline{AD}$ respectively, AB = 6cm and BC = 16cm. If the forces acting in Newton and their magnitudes and directions are shown in the figure, prove that the system is in equilibrium.



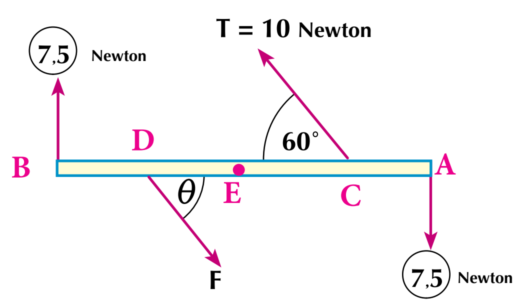
Example

4 AB is a rod of negligible weight is horizontally suspended by a pin at its midpoint, two forces each of a magnitudes 7.5 Newton act at its two ends that at one end is vertically upwards and at the other end it is vertically down wards. It is also pulled by a string in a direction making an angle of measure 60° with the rod from point C on it. Find the magnitude and direction and the point of action of the force which if acts with the previous forces on the rod will keep it in equilibrium in a horizontal position, given that the tension in the string is of a magnitude 10 Newton and the rod length is 30 cm.

Solution

The two forces 7.5 and 7.5 Newton at A and B form a couple. The algebraic measure of its moment is $M_1 = -7.5 \times 30 = -225$ Newton. cm

Since the rod is to be in equilibrium, the force of tension in the string and the other force should be form a couple. The algebraic measure of its moment is 225 Newton. cm



$\therefore$ the other force $F = T = 10$ Newton , $\theta = 60°$ and $10 \times CD \sin 60 = 225$

$\therefore CD = 15\sqrt{3}$ cm i.e. point D is distant $15\sqrt{3}$ cm from point C on the rod.

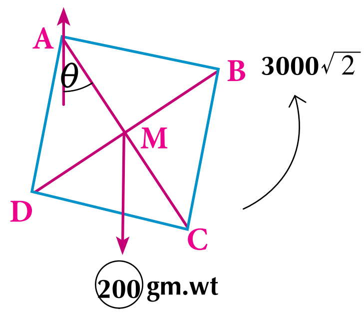
Try to solve

4 ABCDEF is a regular hexagon, The forces 3 , 9 , F_1 , 3 , 9 , F_2 gm.wt act along the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ and $\overrightarrow{AF}$ respectively. Find the value for each of F_1 and F_2 so that the system is in equilibrium.

Example

(5) ABCD is a fine lamina in the form of a square whose side length is 60 cm and of weight 200 gm.wt acts at the point at which the diagonals meet. The lamina is suspended by a pin from a small hole near the vertex A such that its plane is vertical and a couple acts on its plane of a magnitude $3000\sqrt{2}$ gm.wt. Find, in the position of equilibrium, the measure of the angle of inclination of $\overline{AC}$ to the vertical.

Solution



In the position of equilibrium, the lamina is under the action of two forces which are; the weight of the lamina and the reaction of the pin at A, in addition to the external couple. Let the external couple act in the anticlockwise direction (as in the figure). Since the couple is in equilibrium, only with a similar couple, then the reaction at point A and the weight form a couple whose algebraic measure of its moment is

$M_2 = -200 \times AM \sin\theta$ where $AM = 30\sqrt{2}$

$M_1 + M_2 = \text{zero}$ $\quad 3000\sqrt{2} - 200 \times 30\sqrt{2}\sin\theta = \text{zero}$

where $\sin\theta = \frac{1}{2}$ $\quad \theta = 30^\circ$ or 150°

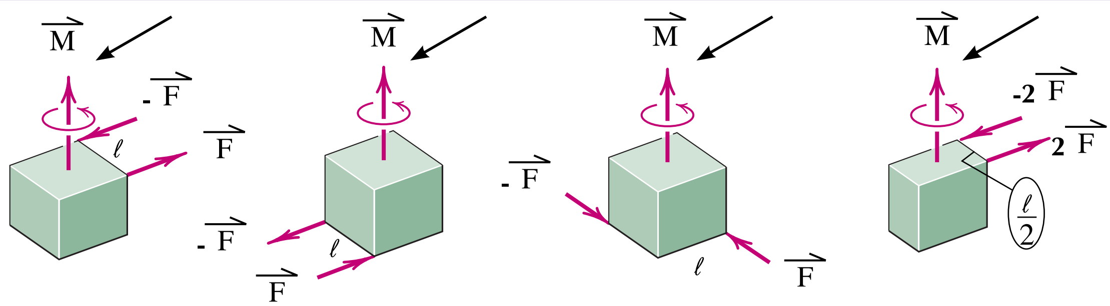
Try to solve

(5) A rod of length 40 cm and weight 2.4 kg.wt act at its midpoint. It can easily rotate at a vertical plane about a fixed hinge at its end. A couple of moment 24 kg.wt.cm and of orthogonal direction to the vertical plane at which the rod can rotate in acts on the rod. Identify the magnitude and the direction of the reaction of the hinge and the angle of inclination of the rod to the vertical in the equilibrium position.

Equivalence of two couples

Definition

Two couples in the same plane are equivalent if the algebraic measures of their moments are equal

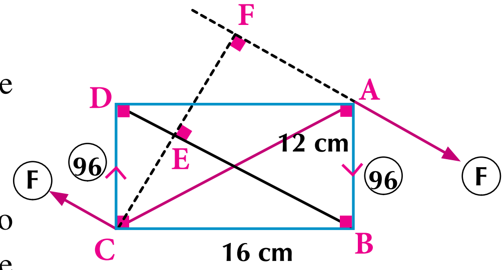


Example

(6) ABCD is a rectangle in which, AB= 12 cm and BC = 16 cm. Two equal forces each of a magnitude 96 Newton in the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ act on it. Find the magnitudes of

two equal forces acting at A and C in a direction parallel to $\overleftrightarrow{BD}$ such that the couple formed from the first two forces is equivalent to the couple formed from the second two forces.

Solution



The two forces 96 and 96 Newton form a couple whose algebraic measure of its moment is

$M_1 = -96 \times 16 = -1536$ Newton. cm

since the two couples are equivalent, then the two forces at A and C act on the rotation in the clockwise direction (as in the figure).

$\therefore M_2 = -F \times CF$

$\therefore M_2 = -F \times 19.2$

$\because$ two couples are equivalent $\therefore M_1 = M_2$

$\therefore -F \times 19.2 = -1536$ $\therefore F = 80$ Newton

from Euclidean theorem

$CE = \dfrac{AB \times AD}{BD}$

$CE = \dfrac{16 \times 12}{20} = 9.6$cm

$CF = 2\,CE$

$CF = 19.2$cm

Try to solve

6. AB is a light rod of length 50 cm, two forces each of a magnitude 30 Newton act at A and B in two opposite directions. Two other forces, each of a magnitude 100 Newton, act in two opposite directions at points C and D of the rod where CD = 30 cm such that they form a couple equivalent to the couple formed by the first two forces. Find the measure of the angle of inclination of the other two forces on the rod.

Exercises 3 - 1

Choose the correct answer:

1. The couple is:
   - a) Two parallel forces of equal magnitudes and in the same direction.
   - b) Two perpendicular forces of equal magnitudes.
   - c) Two parallel forces of equal magnitudes and on one line of action.
   - d) Two parallel forces of equal magnitudes, in opposite directions and are not on one line of action.
2. Which of the following conditions does not change the effect of the couple on the body?
   - a) Translating the couple into a new position in its plane.
   - b) Translating the couple into another plane parallel to its plane
   - c) Rotating the couple in its same plane.
   - d) All what previously mentioned.
3. The two forces acting on the steering wheel of a car and producing the rotation of the steering wheel form:
   - a) Friction.
   - b) Couple.
   - c) Perpendicular force on the steering wheel.
   - d) Non-zero resultant.

4 To form a couple of two forces, the two forces should be :

a Equal in magnitudes. b Opposite in directions.

c Not on one line of action. d All what previously mentioned.

5 If M_1 and M_2 are the algebraic measures of the moments of two couples and $M_1 + M_2$ = zero then:

a The two couples are equivalent b The two couples are not in equilibrium

c The two couples are equilibrium d The two couples are equivalent to a force

6 The product of the magnitude of a force of the two forces of a couple and the arm of the couple is called:

a The resultant of the couple. b The moment of the couple.

c The moment of one force of the two forces of the couple.

d Nothing of the previous.

7 If $\overrightarrow{F_1} = 3\hat{i} - b\hat{j}$ and $\overrightarrow{F_2} = a\hat{i} - 5\hat{j}$ form a couple, then (a, b) =

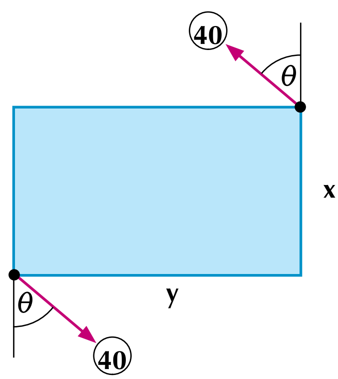
a (3, -4) b (3, 5) c (-3, 5) d (-3, -5)

8 If the norm of the moment of a couple is 350 Newton. m and the magnitude of one of its two forces is 70 Newton, then the arm length of the couple is equal to:

a 50 meter b 5 meters c 5 cm. d 24500 cm.

Answer the following questions :

9 **The opposite figure** shows two forces each of a magnitude 40 Newton act on the two edges of a lamina in the form of a triangle of dimensions X and Y cm. Find the moments of the couple of the two forces in each of the following cases:



a $x = 3cm$, $y = 4$ cm , θ = zero

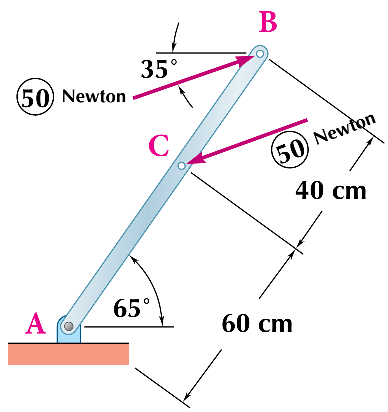
b $x = y = 6cm$, $\theta = \frac{\pi}{4}$

c $x = 0$, $y = 5cm$, $\theta = 30^\circ$

d $x = 6cm$, $y = 0$, $\theta = 60^\circ$

e $x = 5cm$, $y = 12$ cm , $\tan\theta = \frac{5}{12}$

10 **The opposite figure** shows two forces each of a magnitude 50 Newton, act on a lever AB find the algebraic measure of the moment of the couple in two methods:



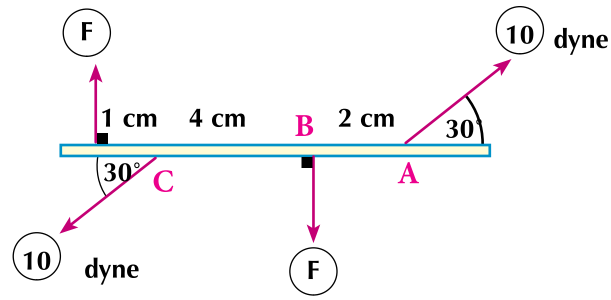
a Using the perpendicular distance between two forces.

b By finding the sum of the moments of the two forces about point A

(11) Two forces $(3\hat{i} - 5\hat{j})$ and $(-3\hat{i} + 5\hat{j})$ Newton act at the two points A and B respectively whose position vectors are, $(6\hat{i} + \hat{j})$ and $(4\hat{i} + \hat{j})$ meters. Prove that the system is equivalent to a couple and find its moment.

(12) Two forces $(a\hat{i} + b\hat{j})$ and $(5\hat{i} - 2\hat{j})$ Newton act at the two points C and D respectively where C(-2, 1) and D(3, 1). If the two forces form a couple, find the value for each a and b, then find the moment of the couple and the perpendicular distance between the two lines of action of the forces.

(13) **The opposite figure** represents an equilibrium rod under the action of four forces. Find the value of F.



(14) ABCD is a rectangle in which AB = 8cm, BC = 6 cm, x, y, z and L are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively, the forces of magnitudes F, F, F, F, 6 and 6 Newton act in the direction of $\overrightarrow{AX}$, $\overrightarrow{CZ}$, $\overrightarrow{YX}$, $\overrightarrow{LZ}$, $\overrightarrow{CY}$ and $\overrightarrow{AL}$ respectively. If the rectangle is in equilibrium, find the value of F.

(15) AB is a rod of length 60 cm and weight 18 Newton, acts at its midpoint. The rod can rotate easily about a fixed vertical pin passing through a small hole in the rod at point C which is distant 15 cm from A. If the rod rests with its end B on a smooth horizontal table and the end A is pulled horizontally by a string until the reaction of the table is equal to the weight of the rod, find the tension in the string and the reaction of the pin known that the rod is in equilibrium as it inclines at 60° to the horizontal.

(16) ABCD is a fine lamina in the form of a rectangle in which AB = 18 cm, BC = 24 cm and its weight equals 20 Newton acting at the intersection point of the two diagonals. The lamina is suspended by a pin in a small hole near the vertex D such that its plane is vertical. If the lamina is acted by a couple whose moment is 150 Newton and in a perpendicular direction of the plane of the lamina , find the angle of inclination of $\overline{DB}$ to the vertical in the equilibrium position.

(17) ABCD is a square of side length 10 cm, Two forces each of magnitude 60 Newton act in the directions of $\overrightarrow{BA}$ and $\overrightarrow{DC}$. Find two forces equal in the magnitude, acting at A and C, parallel to $\overrightarrow{BD}$ and forming a couple equivalent to the couple formed by the first two forces.

Unit Five

3 - 2

Resultant couple

You will learn

- The sum of coplanar couples (resultant couple)
- The condition of a system of coplanar forces equivalent to a couple

Key terms

- Resultant couple
- Equivalent

Materials

- Scientific calculator.

Think and discuss

1) What is the action occurring on a body if this body is under the action of a couple?

2) Does the body under the action of a couple move in a linear motion or circular motion?

3) If the resultant of a system of concurrent coplanar forces is equal to zero; can these forces represent a couple?

4) If the resultant of a system of non-concurrent forces is equal to zero, can these forces represent a couple?

Learn

The system of the coplanar forces are equivalent to a couple

A system of coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, ... , $\overrightarrow{F_n}$ is said to be equivalent to a couple if the following two conditions are satisfied together :

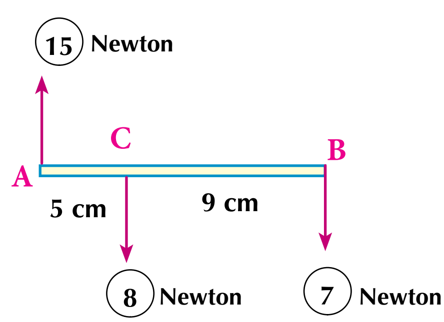
1) Vanishing the resultant of the forces (or the sum of the algebraic components of the forces in any direction = zero)

2) The sum of the moments of the forces about any point does not vanish.

Notice: satisfying one of the two conditions only is not enough to prove that the system is equivalent to a couple. If the resultant of the concurrent forces, vanishes, then the system of the forces is in equilibrium but is not equivalent to a couple.

Example

1 AB is a light rod acted on by the forces shown in the figure. Prove that the system of forces is equivalent to a couple and find the algebraic measure of its moment.



Solution

Let $\overrightarrow{C}$ be the unit vector in the direction of the force 15 Newton

$\therefore \overrightarrow{R} = 15\overrightarrow{C} - 8\overrightarrow{C} - 7\overrightarrow{C} = \overrightarrow{0}$

i.e. the resultant vanishes

$\therefore$ Either the system is in equilibrium or it is equivalent to a couple.

As a result, we find the sum of the moments of the forces about a point (say A)

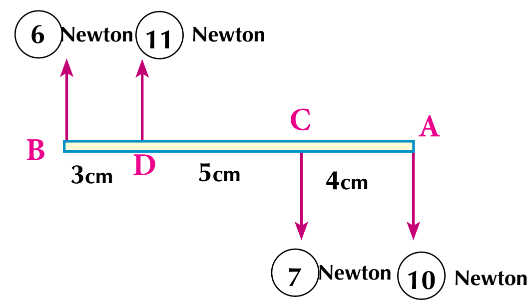
$M_A = -8 \times 5 - 7 \times 14 = -138$ Newton.c

$\therefore$ the system is equivalent to a couple and the algebraic measure of its moment is equal to - 138 Newton. cm

Critical thinking: Find the sum of the moments of the forces about each of B, C. What do you notice?

Try to solve

1) **In the opposite figure,** prove that the system is equivalent to a couple and find the algebraic measure to its moment.



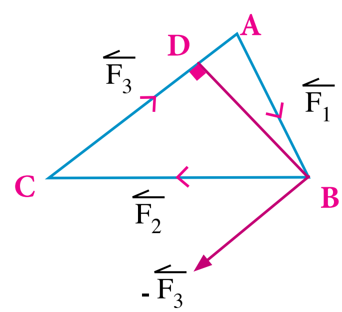
Rule

If three coplanar forces act on a rigid body and are completely represented by the sides of a triangle, taken in the same direction, then this system is equivalent to a couple, the magnitude of its moment is equal to twice the area of the triangle multiplied by the magnitude of the force represented the unit length.

Proof (is not required)

The directed straight segments $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ represent the three forces completely.

i.e. in the magnitude, direction and line of action, let m represent the magnitude of the force of the unit length



i.e. $m = \frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{AC}$

$\because \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{O}$

$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \overrightarrow{O}$

$\therefore \overrightarrow{F_1} + \overrightarrow{F_2} = -\overrightarrow{F_3}$

i.e the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ is the force $(-\overrightarrow{F_3})$ acting at point B - thus the system is equivalent to the two forces $\overrightarrow{F_3}$ acting at C and $(-\overrightarrow{F_3})$ acting at point B, it is equivalent to a couple.

To identify the magnitude of the moment of such a couple, draw a perpendicular from B to $\overline{AC}$ to meet it at point D.

The magnitude of the moment of the couple $= \| \overrightarrow{F_3} \| \times BD$

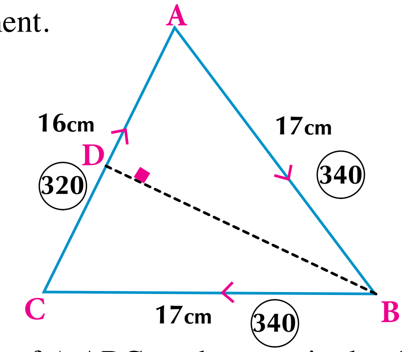
But $\| \overrightarrow{F_3} \| = AC \times m$

The magnitude of the norm of the couple $= AC \times m \times BD$

$= (AC \times BD) \times m = m \times$ twice the area of triangle ABC

Example

(2) ABC is a triangle, in which AB = BC = 17 cm and AC = 16 cm. Forces of magnitudes 340 , 340 and 320 Newton act at $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.



Solution

Since $\frac{340}{17} = \frac{340}{17} = \frac{320}{16} = 20$

$\therefore$ The magnitude of the force representing the unit length is equal to 20 Newton and since the forces are taken in one cyclic order (taken in the same direction)

$\therefore$ the system is equivalent to a couple

the magnitude of the moment of the couple = twice the area of $\triangle$ ABC $\times$ the magnitude of the force representing the unit couple

To find the area of $\triangle$ we draw $\overline{BD} \perp \overline{AC}$ to bisect it

$\therefore$ BD = $\sqrt{17^2 - 8^2}$ = 15 cm

$\therefore$ the magnitude of the moment of the couple = $2 \times \frac{1}{2} \times 16 \times 15 \times 20 =$ 4800 Newton. cm

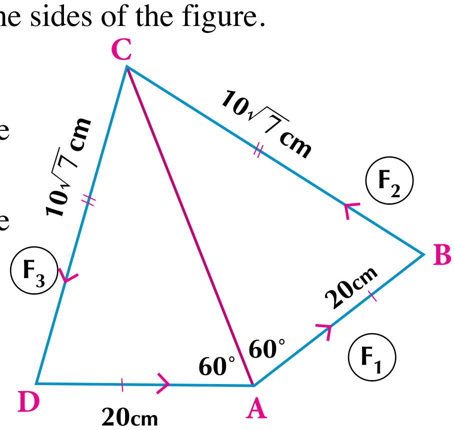
Try to solve

(2) ABC is a right-angled triangle at B in which AB = 30 cm and B C = 40 cm. Forces of magnitudes 6, 8 and 10 Newton act at $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ respectively, Prove that the system is equivalent to a couple and find the magnitude of its moment.

Generalization: If a system of coplanar forces act on a rigid body and are completely represented by the sides of a closed polygon taken the same cyclic order, then this system is equivalent to a couple. The magnitude of its moment is equal to the product of twice the surface area of the polygon by the magnitude of the force representing the unit length.

Example

(3) AB CD is a quadrilateral in which AB = AD = 20 cm, BC = CD = 10 $\sqrt{7}$ cm , m($\angle$A) = 120°. forces represented by the directed straight segments $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ act on. If the system ends to a couple the magnitude of its moment is 180 $\sqrt{3}$ Newton. cm in the direction of AB CD Find the magnitude of the forces acting on the sides of the figure.



Solution

$\because$ The forces act on the sides of the polygon and take the same way round

$\therefore$ the magnitude of the moment = twice the area of the figure $\times$ m

Twice the area of the figure $\times$ m = 180 $\sqrt{3}$ (1)

from the geometry of the figure $\triangle$ ABC $\equiv$ $\triangle$ ADC

from the cosine rule in $\triangle$ ABC

$(BC)^2 = (AB)^2 + (AC)^2 - 2\,AB \times AC \times \cos(B\hat{A}C)$

$\therefore (10\sqrt{7})^2 = 20^2 + (AC)^2 - 2 \times 20 \times AC \times \cos 60$

$\therefore \quad 700 = 400 + (AC)^2 - 20\,AC$

$\therefore (AC)^2 - 20\,AC - 300 = \text{zero}$

$\therefore (AC + 10)(AC - 30) = 0$ then $AC = 30$

Area of figure ABCD $= 2 \times$ area $\triangle ABC$

$= 2 \times \frac{1}{2} \times AB \times AC \times \sin 60 = 20 \times 30 \times \sin 60 = 300\sqrt{3}\,cm^2$

By substitution in (1)

$\therefore 2 \times 300\sqrt{3} \times m = 180\sqrt{3}$ then $m = \frac{3}{10}$

$\because \frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{CD} = \frac{F_4}{DA} = m$

$\therefore \frac{F_1}{20} = \frac{F_2}{10\sqrt{7}} = \frac{F_3}{10\sqrt{7}} = \frac{F_4}{20} = \frac{3}{10}$

then $F_1 = 6$ Newton , $F_2 = 3\sqrt{7}$ Newton , $F_3 = 3\sqrt{7}$ Newton , $F_4 = 6$ Newton

Try to solve

3) ABCD is a trapezium in which $\overline{AD} \,//\, \overline{BC}$, $\overline{AB} \perp \overline{BC}$, AB = 6 cm, BC = 9 cm and AD = 3 cm. The forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$ completely represented by the directed straight segments act at $\overrightarrow{DA}$, $\overrightarrow{CD}$, $\overrightarrow{BC}$ and $\overrightarrow{AB}$ respectively. If the system is equivalent to a couple the magnitude of its moment is 360 Newton.cm in the direction of ABCD. Find the magnitude for each of $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$.

Rule

If the sum of the algebraic measures of the moments of a system of coplanar forces with respect to three non collinear points in its plane is equal to a non zero constant, then the system is equivalent to a couple. The algebraic measure of its moment is equal to the value of the constant.

Proof (is not required)

For any such system of forces reduces to either a single force $\vec{F}$ or a single couple or it is in equilibrium.

It is clear that the forces are not in equilibrium since the sum of the algebraic measures of the moments of the forces about a point does not vanish . Let the system be equivalent to one force of a magnitude F, the three points be A , B and C and their distances to the line of action of the force be L_1 , L_2 and L_3 respectively.

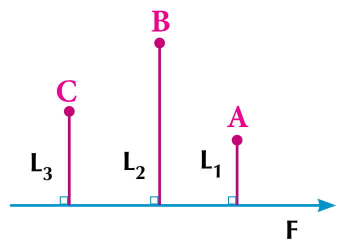
$\therefore F \times L_1 = F \times L_2 = F \times L_3$ = a constant magnitude

By dividing by F where $F \neq$ zero $\quad \therefore L_1 = L_2 = L_3$

i.e. points A , B and C lie on one straight line parallel to the line of action of F and this does not match with the hypothesis.

$\therefore$ The system forces is not equivalent to a force

$\therefore$ The system is equivalent to a couple. The algebraic measure of its moment is equal to the value of the constant.

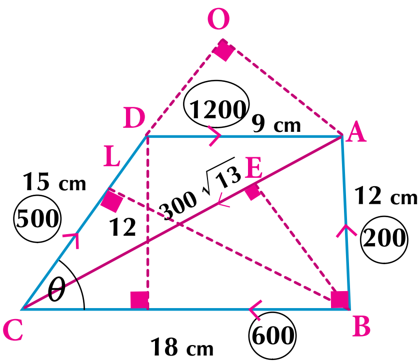


Example

(4) ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, m($\angle$B) = 90°, AB = 12 cm, BC = 18 cm and AD = 9 cm. Forces of magnitudes 200 , 600 , 500 , 1200 and 300 $\sqrt{13}$ kg.wt act at $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ $\overrightarrow{DA}$ and $\overrightarrow{AC}$ respectively. Prove that the system is equivalent to a couple and find its moment.

Solution

Calculate the sum of the algebraic measures of the moments of the forces with respect to three non-colinear points. Let them be A, B and C.



$M_A = -600 \times 12 - 500 \times AO$

where $AO = 9 \sin\theta = 9 \times \frac{12}{15} = 7.2$

$\therefore M_A = -600 \times 12 - 500 \times 7{,}2 = -10800$ kg.wt. cm

$M_B = -1200 \times 12 - 500 \times BL + 300\sqrt{13} \times BE$

where $BL = 18 \sin = 18 \times \frac{12}{15} = 14{,}4$

, $BE = \frac{12 \times 18}{6\sqrt{13}} = \frac{36}{\sqrt{13}}$

$\therefore M_B = -1200 \times 12 - 500 \times 14{,}4 + 300\sqrt{13} \times \frac{36}{\sqrt{13}}$

$= -10800$ kg.wt cm

$\therefore M_c = 200 \times 18 - 1200 \times 12$

$= -10800$ kg.wt.cm

$\therefore$ The system is equivalent to a couple acting on rotation in the clockwise direction. The magnitude of its moment is 10800 kg.wt. cm

Try to solve

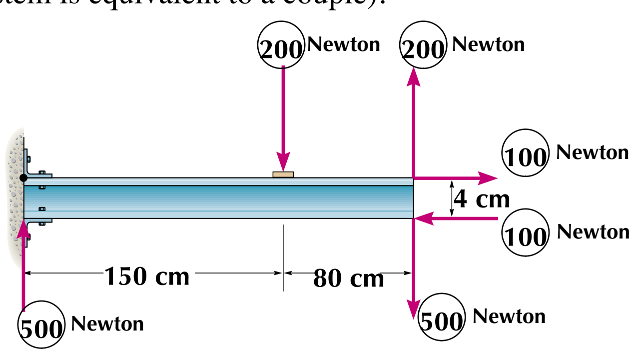
(4) ABCD is a square of side length 10 cm, E $\in \overrightarrow{CB}$, F $\in \overrightarrow{CD}$, such that C E = C F = 30 cm. Forces of magnitudes 40, 10, 20, 30 and 20 $\sqrt{2}$ kg.wt. cm act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and $\overrightarrow{EF}$ respectively. Prove that the system is equivalent to a couple and find its moment.

Resultant couple

The sum of two coplanar couples is known as the couple whose moment is equal to the sum of the two moments of those two couples $\overrightarrow{M} = \overrightarrow{M_1} + \overrightarrow{M_2}$ and the sum of the two coplanar couples is called the resultant couple (the system is equivalent to a couple).

Example

(5) **In the opposite figure** find the algebraic measure of the resultant couple.



Solution

The two forces 200 and 200 Newton form a couple the algebraic measure of its moment

$M_1 = 200 \times 0.8 = 160$ Newton .meter

The two forces 500 and 500 Newton form a couple the algebraic measure of its moment

$\overrightarrow{M_2} = -500 \times 2.3 = -1150$ Newton. meter

The two forces 100 and 100 Newton form a couple the algebraic measure of its moment

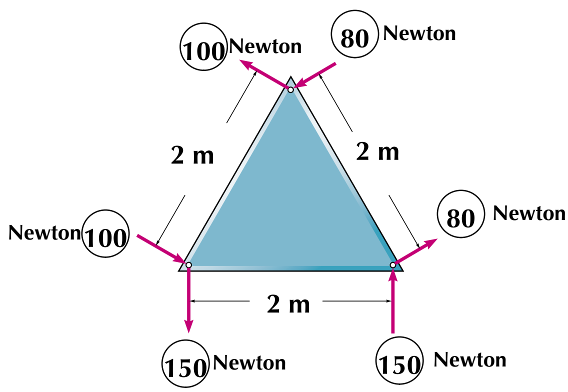
$M_3 = -100 \times 0.04 = -4$ Newton. meter

The resultant couple $= M_1 + M_2 + M_3$

$= 160 + (-1150) + (-4) = -994$ Newton. meter

Try to solve

5 **The opposite figure** represents a uniform lamina in the form of an equilateral triangle. If a forces act on the lamina as shown in the figure, find the algebraic measure of the moment of the resultant couple.



Example

6 ABCD is a square of side length 10 cm. Two forces, each of a magnitude 40 kg.wt act at $\overrightarrow{AD}$ and $\overrightarrow{CB}$ and two forces each of a magnitude 70 kg.wt at $\overrightarrow{AB}$, $\overrightarrow{CD}$. Find the algebraic measure of the moment of the resultant couple.

Solution

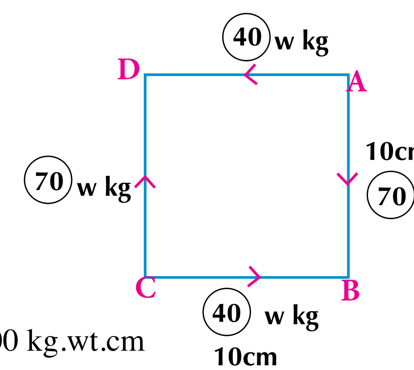
The two forces 40 and 40 form a couple the algebraic measure of its moment

$M_1 = 40 \times 10 = 400$ kg.wt.cm

the two forces 70 and 70 form a couple the algebraic measure of its moment

$M_2 = -70 \times 10 = -700$ kg.wt.cm

the resultant couple $= M_1 + M_2 = 400 + (-700) = -300$ kg.wt.cm

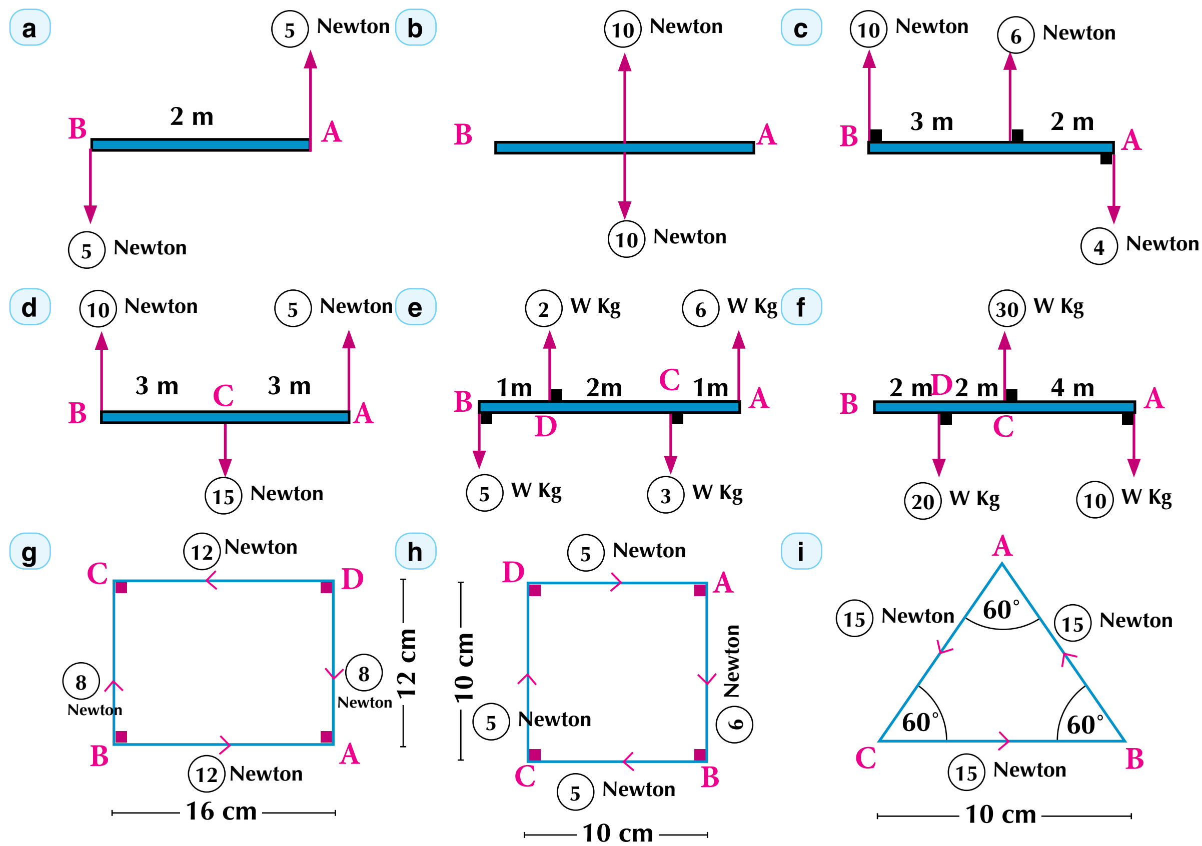


Try to solve

6 ABCD is a rectangle in which AB = 60 cm , BC = 160 cm , X and Y midpoints of $\overline{BC}$ and $\overline{AD}$ respectively. The forces of magnitudes 200 , 200 , 400 , 400 , F and F Newton act in the direction $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{CB}$, $\overrightarrow{AD}$, $\overrightarrow{XA}$ and $\overrightarrow{YC}$, respectively. If the algebraic measure of the moment of the resultant couple is equal to 6400 Newton.cm, find the value of: F.

Exercises 3 - 2

1. Show which of the following system of forces is equivalent to a couple and find the algebraic measure of its moment:



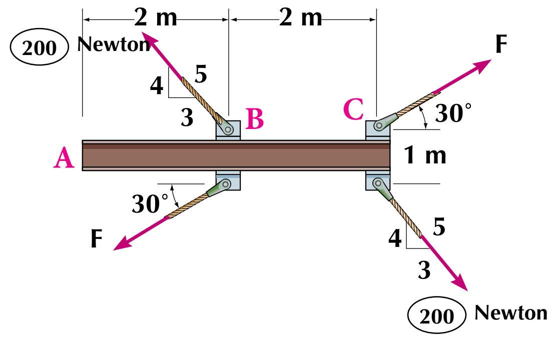
2. ABCD is a square of side length 3 meters. The forces of magnitudes 5, 2, 5 and 2 Newton act in the directions of $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$ and $\overrightarrow{DA}$, respectively. Show that the system is equivalent to a couple and find the magnitude of its moment.

3. ABCD is a rectangle in which AB = 6cm , B C = 8 cm forces of magnitudes 7 kg.wt act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ respectively prove that the system is equivalent to a couple and find the magnitude of its moment.

4. ABCD is a rectangle in which AB = 30 cm, B C = 40 cm forces of magnitudes 15, 30, 15 and 30 gm.wt act at $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$ and $\overrightarrow{DA}$ respectively. Prove that this system is equivalent to a couple and find its moment , then find the two forces acting at A and C perpendicular to $\overline{AC}$ such that the system is in equilibrium.

5. ABCD is a rhombus of side length 10 cm, m(∠ BAC) = 120°. Forces of magnitudes 20, 15, 20 and 15 kg.wt act at $\overrightarrow{AB}$ $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment. Find the two forces acting at B and D perpendicular to $\overline{BD}$ such that the system is in equilibrium.

6. **The opposite figure** represents an arch. The shown force in the figure acts on it. If the algebraic measures of the moment of the resultant couple is equal to $200 - 200\sqrt{3}$ Newton. meter, find F.



7. ABCD is an isosceles trapezium in which $\overline{AD} \,//\, \overline{BC}$, AD = 9 cm AB = DC = 15 cm and BC = 33 cm. The forces of magnitudes 45, 99 , 45 and 27 act in the directions of $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

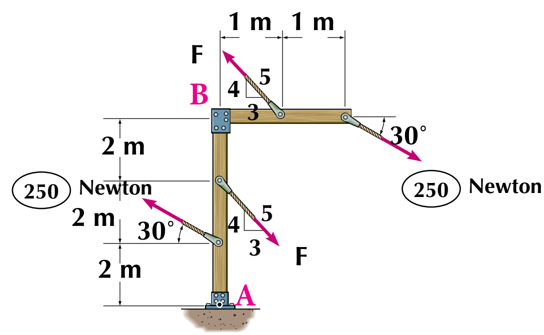
8. ABCDEF is a regular hexagon of side length 15 cm. Forces of magnitudes 40, 50, 30, 40, 50 and 30 Newton act at $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{OE}$ and $\overrightarrow{FA}$ respectively. Find the moment of the resultant couple.

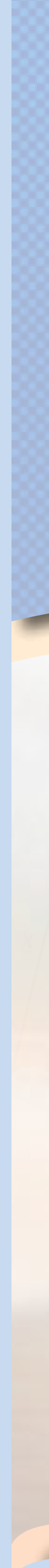
9. ABCDE is a regular pentagon of side length 15 cm. Forces of magnitudes 10w kg act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$ and $\overrightarrow{EA}$ respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

10. ABC is a triangle in which AB = BC = 6cm, m($\angle$ ABC) = 120°. Forces of magnitudes 18, 18 and $18\sqrt{3}$ act at $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ respectively prove that the system is equivalent to a couple and find the magnitude of its moment.

11. ABCD is a square of side length 60 cm. Forces of magnitudes 10, 20, 80 and 50 Newton act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ respectively and two forces of magnitude $50\sqrt{2}$ and $20\sqrt{2}$ Newton act in $\overrightarrow{AC}$ and $\overrightarrow{DB}$ respectively. Prove that the system is equivalent to a couple the magnitude of its moment is 4800 Newton. cm

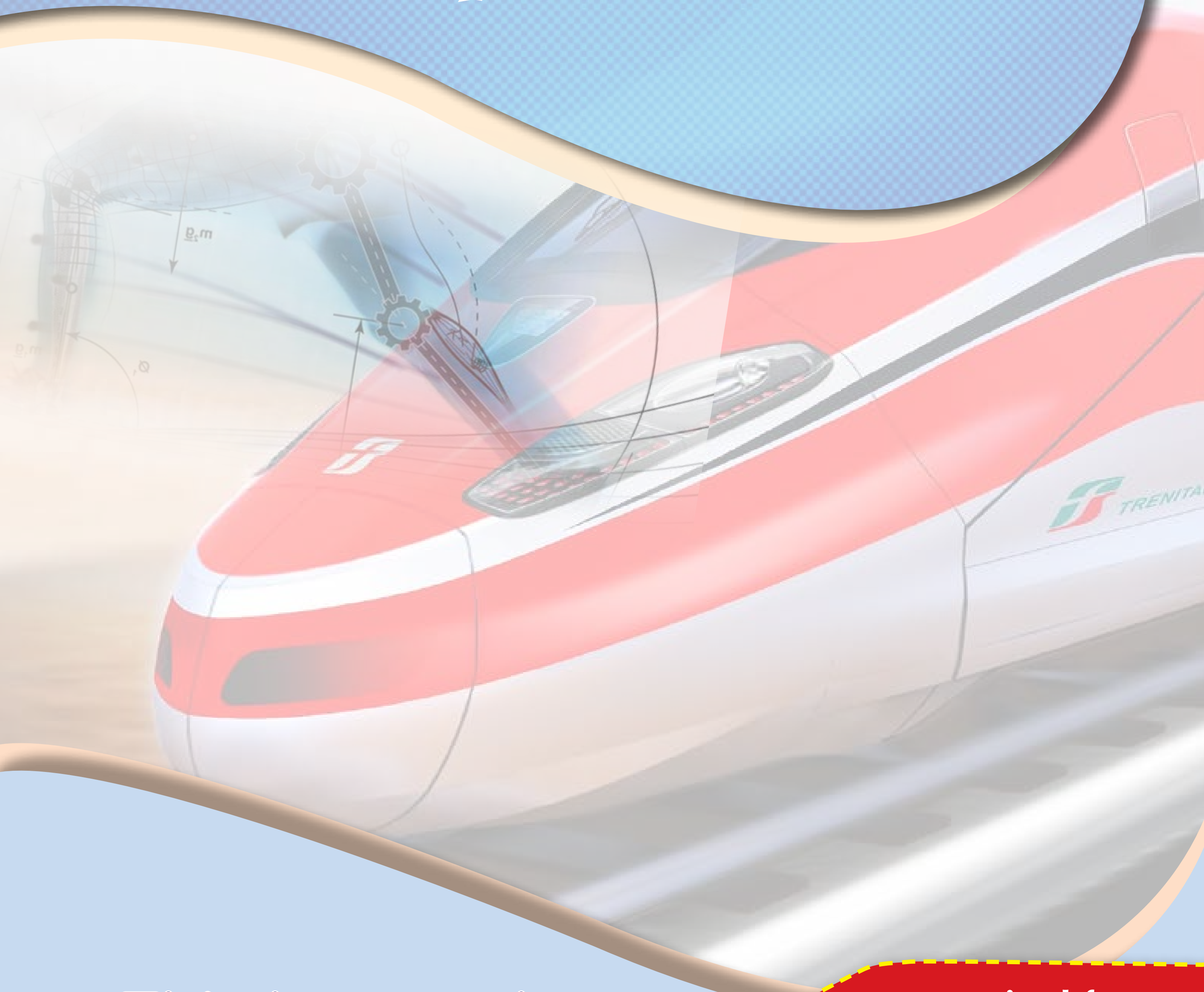
12. **In the figure opposite,** find F which makes the algebraic measure of the moment of the resultant couple equal to $150 - 500\sqrt{3}$





Pre-Requisites for

Dynamics



Third secondary

Not required for examination

1 Momentum

The momentum of a moving body is a vector quantity of the same direction of the velocity of this body and its magnitude at a moment is estimated by the product of the mass of this body by its velocity at this moment. The momentum vector is denoted by the symbol $\vec{H}$.

$$\vec{H} = m\vec{V}$$

In case of the rectilinear motion, each of $\vec{H}$ and $\vec{V}$ are parallel to the straight line on which the motion occurs. Each of $\vec{H}$ and $\vec{V}$ can be expressed in terms of the algebraic measure for each of them:

$$H = mV$$

Where: H and V are the two algebraic measures for the momentum vector and velocity respectively.

2 Measuring units of momentum

The magnitude unit of momentum = mass unit × velocity unit

In the international system of units, the magnitude of momentum is measured in kg. m/sec

i.e.: H (kg. m / sec) = m(kg) × V(m/sec).

Notice that: At the constancy of the mass of the body, H is proportional to V and the relation between them is linear. As a result the momentum in this case is called the linear momentum.

Example Definition of momentum

1. Calculate the momentum of a bike whose mass is 35 kg and moves in a uniform speed of a magnitude 12 m/sec towards East.

Figure (1)

Solution

$\because$ H = mV $\therefore$ H = 35 × 12 = 420 kg. m/sec

The momentum of the bike = 420 kg. m/sec towards East.

Try to solve

1. Calculate the momentum of a train whose mass is 40 tons moving towards the North with a uniform velocity of a magnitude 72 km/h.
2. Calculate the momentum of a car whose mass is 800 kg moving towards the Southwest with a uniform velocity of a magnitude 126km/h.

Example Using the vectors

2. A car of mass 2 tons moves in a straight line such that $\vec{r} = (3t^2 - 4t + 1)\vec{C}$ where $\vec{C}$ is the unit vector in the direction of the motion of the car. If x is measured in meter, find the magnitude of the momentum of the car when it starts to move, then after 3 seconds of its motion.

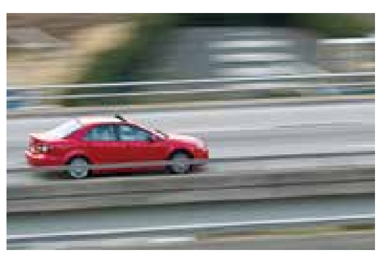


Figure (2)

Solution

$\because \overrightarrow{r} = (3t^2 - 4t + 1)\overrightarrow{C}$

$\therefore \overrightarrow{V} = \frac{d\overrightarrow{r}}{dt} = (6t - 4)\overrightarrow{C}$

(1) At the beginning of the motion $t = 0$, $\overrightarrow{V} = -4\overrightarrow{C}$

$\because \overrightarrow{H} = m\overrightarrow{V}$

$\therefore \overrightarrow{H} = 2000(-4\overrightarrow{C}) = -8000\overrightarrow{C}$

the magnitude of momentum = 8000 kg. m/sec

(2) when $t = 3$sec , **then** $\overrightarrow{V} = (6 \times 3 - 4)\overrightarrow{C} = 14\overrightarrow{C}$

$\because \overrightarrow{H} = m\overrightarrow{V}$ $\therefore \overrightarrow{H} = 2000(14\overrightarrow{C}) = 28000\overrightarrow{C}$

the magnitude of momentum = 28000 kg. m/sec.

Try to solve

3 A car of mass 1200 kg moves in a straight line such that $S = t^3 - 12t^2$ where S is measured in meter, Find the momentum of the car after 4 sec from the beginning of the motion.

3 The change of momentum

If the two velocity vectors of a moving body at two successive moments t_1 and t_2 respectively are $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ then the change of momentum of the body is determined by the relation:

$$\triangle\overrightarrow{H} = m\triangle\overrightarrow{V}$$

where m is the mass of the moving body, $\triangle\overrightarrow{V}$ is the change occurring in the value of its velocity

$\therefore$ The change of momentum of a body $\triangle\overrightarrow{H} = m(\overrightarrow{v_2} - \overrightarrow{v_1})$

Example The change of momentum

3 A rubber ball of mass 200 gm is let to fall on a horizontal surface from a height of 90 cm to rebound up to a height of 40 cm. Calculate using kg.m/sec unit the magnitude of the change of momentum of the ball as a result of impact.

Solution

Let $\overrightarrow{C}$ be the unit vector directed vertically upwards.

Studying the motion of the ball in case of falling.

$\because v^2 = V_\circ^2 + 2gS$

$\therefore V_1^2 = 0 + 2 \times 980 \times 90$

$v_1 = 420$ cm / sec

$\therefore \overrightarrow{v_1} = 420\overrightarrow{C}$

Studying the motion of the ball in case of rebounding.

$\because v^2 = V^2 + 2\,g\,S$

$\therefore 0 = V_2^2 - 2 \times 980 \times 40$

$v_2 = 280$ cm / sec

$\therefore \vec{v_2} = -280\,\vec{C}$

The change of momentum

$\Delta \vec{H} = m(\vec{v_2} - \vec{v_1})$

$= \frac{200}{1000}(-2.8 - 4.2)\,\vec{C} = -1.4\,\vec{C}$

$\therefore$ **the magnitude of the change of momentum** = 1.4 kg. m/sec

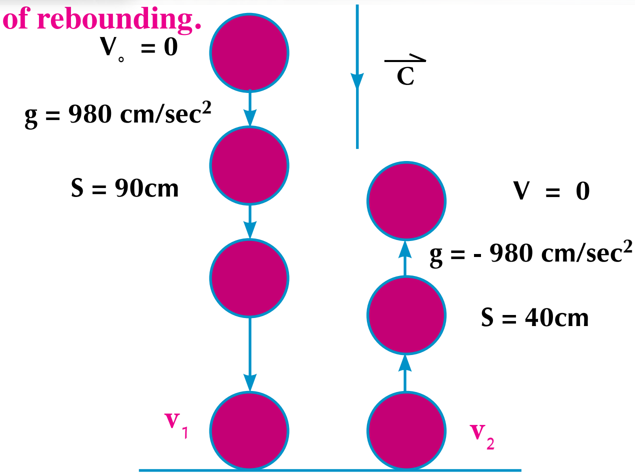


Figure (3)

4 Newton's First Law

Within this law, Isaac Newton stated what happens to a body when the vanish of the forces acting on it.

Every body preserves in its state of rest or of moving uniformly unless acted upon by an unbalanced external force by an external effect

From Newton's first law, we notice that:

(1) The body at rest preserves at rest unless acted upon by a force to move it and the body in a uniform motion preserves at motion unless acted upon by a force changes its motion.

(2) In the formulation of the law, the expression "force" means the resultant of all the forces acting upon the body. The force is measured by the Newton unit as in the international system of units Newton.

(3) The law considers the two states of rest and uniform motion in a straight line in an equivalent position since both states represent the "natural state" of the body when the resultant of the forces acting on the body is equal to zero.

(4) The law shows that the body which is at rest or moving uniformly in a straight line (i.e. when the body is in its normal state) cannot change its state and that's why Newton's first law is named " law of inertia ".

5 Inertia

From Newton's first law, we can deduce that the bodies naturally endeavor to preserve their states of rest or of moving uniformly in a straight line and this reluctance or resistance to the change is known as inertia.

Inertia principle:

Every body, as much as in its state, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

6 Force

Newton's first law includes a definition of the force as it is the effect which changes or try change the state of the body whether of rest or of moving uniformly forward in a straight line.

Example (Body at rest)

(4) The opposite figure shows a body moving horizontally in the direction shown with a uniform velocity of magnitude 8m/sec. Find F_1 and F_2.

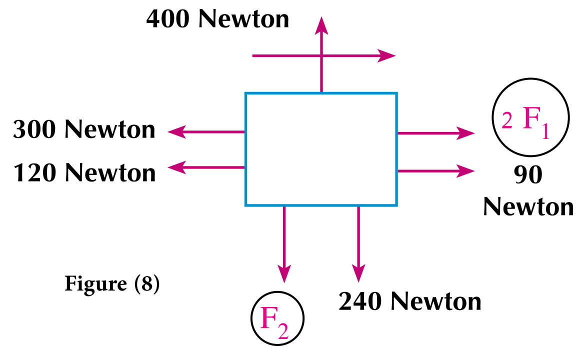


Figure (4)

Solution

$\because$ The body is in a uniform motion state

$\therefore$ The horizontal forces are in equilibrium

$\therefore 2F_1 + 90 = 300 + 120$

$\therefore F_1 = 165$ Newton

$\because$ The vertical forces are in equilibrium

$\therefore 240 + F_2 = 400$

$\therefore F_2 = 160$ Newton

Try to solve

(4) The opposite Figure shows a body moving vertically upwards with a uniform velocity and a system of forces acting on it. Find F_1 and F_2.

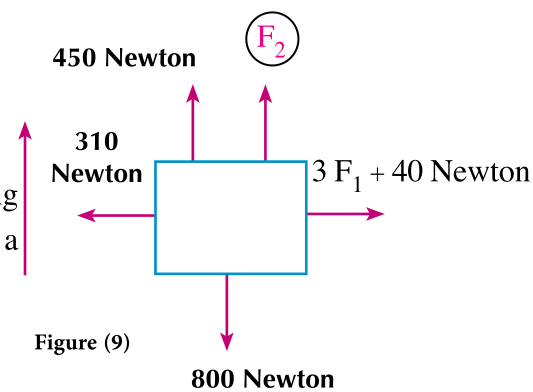


Figure (5)

Example

5 A train of mass 200 tons moves under the action of a resistance proportional to the square of its velocity. If this resistance is 9.6 kg.wt per ton of the mass of the train when the velocity of the train is 72 km/h, find the maximum velocity of the train if the engine of the train can entrain (pull) it with a uniform force of magnitude 4.32 ton.wt.

Figure (6)

Solution

Let the resistance $= R_1$ when the velocity of the train is v_1.

The resistance $= R_2$ when the velocity of the train is v_2.

$\because$ The resistance is proportional to the square of its velocity.

$$\therefore \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2}$$

Given data
$R_1 = 9.6 \times 200$ $= 1920$ kg.wt $v_1 = 72$ km / h

The train reaches the maximum velocity when the resistance is completely equal to the force of pulling the train.

If v_2 is the maximum velocity for the train, then $R_2 = 4{,}32$ tons.wt $\therefore R_2 = 4320$ kg.wt

$$\therefore \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2} \qquad \frac{1920}{4320} = \frac{72 \times 72}{V_2^2} \qquad \therefore v_2 = 108 \text{ km/h.}$$

7 Newton's second law

The rate of change of momentum with respect to the time is proportional to the acting force and takes place in the direction in which the force is acting

$$\frac{d}{dt}(m\vec{V}) \propto \vec{F} \quad \text{i.e.} \quad \frac{d}{dt}(m\vec{V}) = K\vec{F}$$

(where K is the proportionality constant)

As the mass of body is constant during motion, then:

$$m\frac{d\vec{V}}{dt} = K\vec{F}$$ (where K is the proportionality constant)

$$m\vec{a} = K\vec{F}$$

If we define the unit of forces as the force if it acts on a body of mass a unit of masses, it acquires the body the unit of acceleration. By substituting in the equation above, we find that:

$1 = K \times 1 \qquad \therefore K = 1$

and the equation above takes the form $m\vec{a} = \vec{F}$

This equation is called **the equation of motion of a constant mass**, it is considered the basic equation in dynamics. It can be applied on all the moving bodies of a constant mass.

From the equation of motion above, we find that $\vec{F}$ and $\vec{a}$ are in the same direction. If $\vec{a}$ is measured in a certain direction, it is necessary to measure $\vec{F}$ in the same direction so, it is better to write down the equation of motion in the form:

$$m\,\vec{a} = \vec{F}$$

to determine the direction of the acceleration first

If a and F express the algebraic measure of each of $\vec{a}$ and $\vec{F}$ respectively, then the equation of motion of a constant mass body is written in the form:

$$m\,a = F$$

where m is the mass of the moving body, a is the acceleration of motion and F expresses the algebraic measure of the resultant of the system of forces acting on the body. i.e.:

$$m\,a = \Sigma F$$

8 Units of Force and units of Mass

As we deduce the equation of motion of a moving body, we choose certain units for each of the force, mass and acceleration until the constant of proportionality is equal to 1 (unity). The equation of motion takes the form m a = F. As a result, when we use the equation of motion, we use the absolute units of force such as Newton and dyne.

$m \times a = F$
$1\text{kg} \times 1\text{ m/sec}^2 = 1$ Newton
$1\text{gm} \times 1\text{cm/sec}^2 = 1$ Dyne

Remember

1 kg.wt = 9.8 Newton
1 gm.wt = 980 dyne

9 Weight and Mass

The weight of a body is the Earth's attraction force to the body. If we have a body of mass 1 kg, then its weight according to the equation of motion is equal to 1 kg.wt.

$\because\ m\,a = F$

$\therefore\ 1 \times 9.8 = F$

$F = 9.8$ Newton $= 1$ kg.wt

Example

6 A force of magnitude 10 Newtons acts on a body at rest of mass 8 kg to move it in its direction with a uniform acceleration. Calculate the distance traveled after 12 sec and its velocity.

Solution

$F = 10$ Newton $\quad v_0 = 0$

$m = 8$ kg $\quad t = 12$ sec

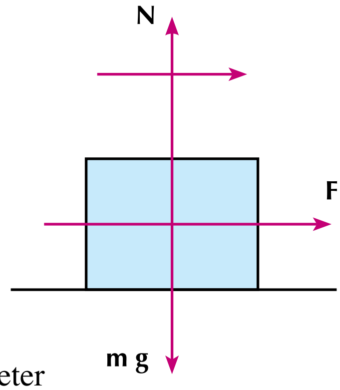


Figure (7)

The equation of motion of the body

$m\,a = F \quad \therefore 8\,a = 10$

$a = \frac{5}{4}$ m/sec^2

$\because V = v_0 + a\,t \quad \therefore V = 0 + \frac{5}{4} \times 12 = 15$ m/sec

$\because S = v_0\,t + \frac{1}{2}\,a\,t^2 \quad \therefore S = 0 + \frac{1}{2} \times \frac{5}{4} \times 144 = 90$ meter

Example

7 A body of mass 3 kg is let to fall from a height of 10 meters on a sandy ground to sink (embed) for a distance of 5 cm. Find the sand resistance to the body in kg.wt in position constant given that the body moves with a uniform acceleration within the sand.

Solution

The phase of free falling

$v^2 = V_\cdot^2 + 2\,g\,S$

$v^2 = 0 + 2 \times 9.8 \times 10$

$V = 14$ m/sec

The phase of sinking (embedding) in sand

$v^2 = v_\cdot^2 + 2a\,S$

$0 = (14)^2 + 2a \times 0.05$

$a = -1960$ m/sec ^{2}

Equation of motion

$m\,a = mg - R$

$3 \times -1960 = 3 \times 9.8 - R$

$\therefore R = 3 \times 9{,}8 + 3 \times 1960$

$R = 5909.4$ Newton

$R = 603$ kg.wt

10 Newtons third law

To every action, there is a reaction equal in magnitude and opposite in direction.

Example

(8) A body of mass 12 kg is placed on a smooth plane inclines at 30° to the horizontal. A force of magnitude 88.8 Newton acts in the direction of the line of the greatest slope upwards the plane. Find the velocity of this body after 14 seconds from the beginning of the motion. If the force acting on the body is ceased at this moment, find the distance which the body moves on the plane after that until it is at rest.

Solution

$\because$ F $= 88.8$ Newton

$\because$ mg sin $\theta = 12 \times 9.8 \times \frac{1}{2}$

$= 58.8$ Newton

F $>$ mg sin θ

$\therefore$ The body moves upwards the plane with a uniform acceleration a

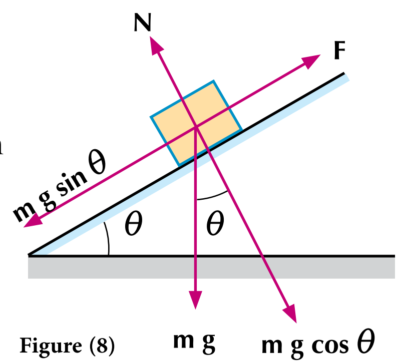


Figure (8)

Equation of motion:

m a $=$ F - mg sin θ

12 a $= 88.8 - 58.8$

a $= 2.5$ m/sec^2

$\because$ V $= V_\circ + a\,t = 0 + 2.5 \times 14 = 35$ m/sec

After leasing the action of the force, the body moves in the same previous direction with a uniform retardation a`

Equation of motion:

m a` $=$ -mg sin θ

a` $= -9.8 \times \frac{1}{2} = -4.9$ m/sec^2

the body travels a distance S until it reaches the instantaneous rest where

$v^2 = v_\circ^2 + 2$ a` S

$0 = (35)^2 - 2 \times 4.9$ S

S $= 125$ meters

Example

9 A rough inclined plane of length 250 cm and height 150 cm, another body at rest is placed on it to slide downwards the plane and the acceleration of motion is equal to 196 cm/sec^2. Find the coefficient of the kinetic friction, then find the velocity of the body after it travels (cuts) 200 cm on the plane.

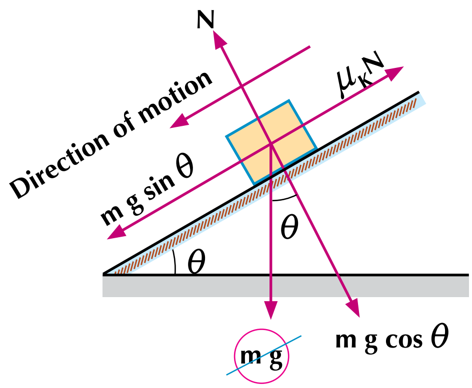
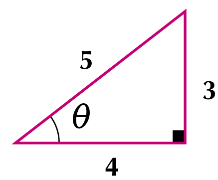


Figure (9)



Solution

$N = m\,g\cos\theta = \frac{4}{5}mg$

$\because$ The body moves downwards with a uniform acceleration

$m\,a = mg\sin\theta - \mu_K N$

$196\,m = \frac{3}{5}mg - \mu_K \times \frac{4}{5}mg$

$196 = \frac{3}{5} \times 980 - \mu_K \times \frac{4}{5} \times 980$

$\therefore \mu_K = \frac{1}{2}$

$\because V^2 = V_0^2 + 2aS$

$V^2 = 0 + 2 \times 196 \times 200$

$\therefore V = 280$ cm/sec

Example

10 A body of mass 12 kg. is placed on a rough horizontal plane. If the coefficient of the static friction between the body and the plane is equal to $\frac{\sqrt{3}}{3}$, whereas the coefficient of the kinetic friction is equal to $\frac{\sqrt{3}}{4}$, then calculate the force which make the body about to move, also find the force which makes the body move with an acceleration of magnitude $\frac{49\sqrt{3}}{20}$ m/sec^2 if the force inclines at 30° to the horizontal.

Solution

First: The force makes the body about to move

$N + F\sin 30^\circ = W$

$N = (12 - \frac{1}{2}F)$ kg.wt

$\because F\cos 30^\circ = \mu_S N$

$\therefore \frac{\sqrt{3}}{2}F = \frac{\sqrt{3}}{3}(12 - \frac{1}{2}F)$

$3F = 24 - F$

$4F = 24$

$F = 6$ kg.wt

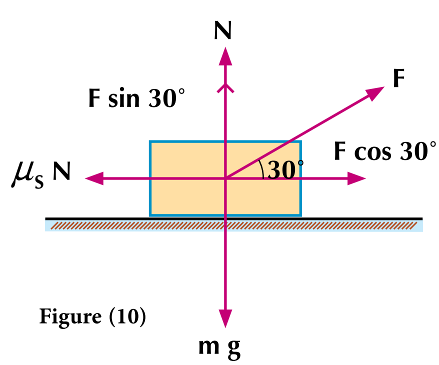


Figure (10)

Second: The force moves the body with an acceleration of magnitude $\frac{49\sqrt{3}}{20}$ m/sec^2

$\because$ N =mg - F sin 30 i.e. N = (12 × 9.8 - $\frac{1}{2}$ F) Newton

$\because$ m a = F cos 30 - μ_k N

$12 \times \frac{49\sqrt{3}}{20} = F \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}(12 \times 9.8 - \frac{1}{2} F)$

$12 \times \frac{49\sqrt{3}}{20} = \frac{5\sqrt{3}}{8} F - 3\sqrt{3} \times 9.8$

F = 94.08 Newton

Second Dynamics

Rectilinear Motion

Unit 1

Introduction

In this unit, you are going to learn the rectilinear motion of a moving particle, analyzing this motion, and studying the vectors of the position displacement, velocity, and the acceleration of the motion of a particle. The motion will be identified at any moment during the motion of the particle in a rectilinear motion whether this motion is uniform or non-uniform using the methods of differential and integral calculus to deduce the elements of study. The rectilinear motion will be analyzed graphically through the curves of the motion and using it to solve different problems. The study will not only be for the particle in motion but other different bodies such as cars, trains, and planes will be taken into consideration.

Unit objectives

By the end of this unit and by doing all the activities involved, the student should be able to:

- Use the related time to express the velocity if the displacement is a function of time ($v = \frac{ds}{dt}$)
- Use the related time to express the acceleration if the velocity is a function of time ($a = \frac{dv}{dt}$)
- Express the acceleration as a function of the displacement if the velocity is a function of the displacement ($a = v\frac{dv}{ds}$)
- If each of s , v. and a are functions in time , then:
- $v = \frac{ds}{dt} \Leftrightarrow \int v dt = \int ds \qquad \therefore s = \int v dt$
- $a = \frac{dv}{dt} \Leftrightarrow \int a dt = \int dv \qquad \therefore v = \int a dt$
- If a is a function of displacement then:
  - $a = V\frac{dv}{ds} \Leftrightarrow \int a ds = \int v dv$

Key terms

- Rectilinear Motion
- Position
- Displacement
- Distance
- Average Velocity
- Average Speed
- Velocity
- Speed
- Average Acceleration
- Acceleration

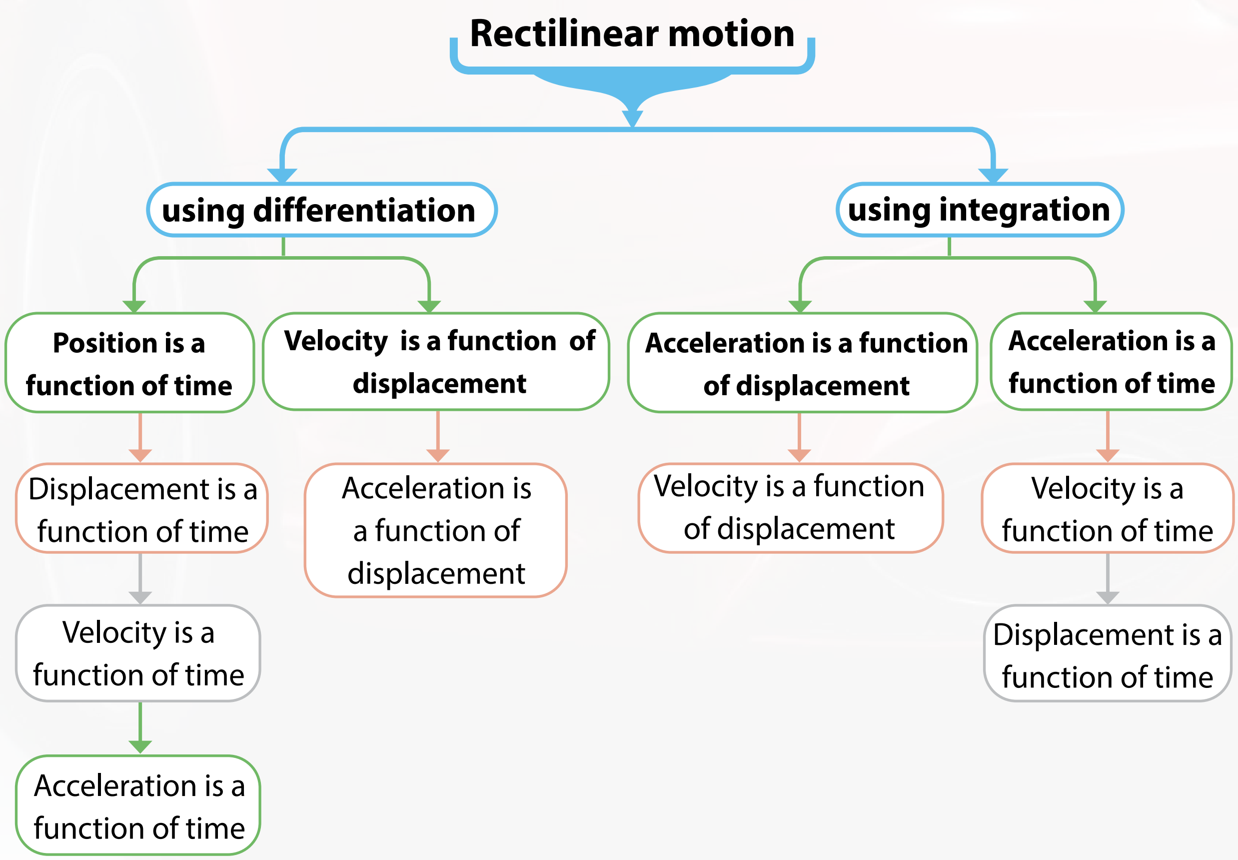
Lessons of the unit

(1 -1): Differentiation of the vector functions

Materials

- Scientific calculator
- Computer graphics

Unit planning guide



Unit One

1 - 1

Differentiation of vector functions

You will learn

- If $\vec{s}$ is a function of time then: $\vec{v} = \frac{d\vec{s}}{dt}$
- If $\vec{v}$ is a function of time, then: $\vec{a} = \frac{d\vec{v}}{dt}$
- If $\vec{v}$ is a function of displacement s then $\vec{a} = \vec{v}\frac{dv}{ds}$

Key terms

- Rectilinear motion
- Position of the porticle
- Displacement
- Distance
- Speed
- Velocity
- Average velocity
- Instantaneous velocity
- Average acceleration
- Acceleration

Materials

- Scientific calculator.
- Computer graphics

Definite integration:

$$_a\int^b f`(x) \bullet dx = f(b) - f(a)$$

i.e: $_1\int^4 (x^2 + 2x - 1)\, dx$

$= [\frac{x^3}{3} + \frac{2x^2}{2} - x]_1^4$

$= [\frac{x^3}{3} + x^2 - x]_1^4 = [\frac{64}{3} + 16 - 4] - [\frac{1}{3} + 1 - 1] = 33$

The definite integration will be learned in the integral.

Learn

Rectilinear motion

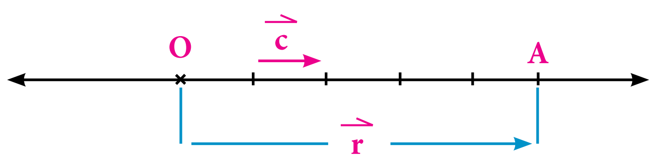
If a particle moves in a straight line, it is said that it moves in a linear motion.

Position of the particle

When a particle moves in a linear motion, at this moment it takes up a certain position on the straight line. To identify the position of $\vec{r}$ for a moving particle at any moment t, we choose a constant point "O" on the straight line as an origin point and determine the positive direction along the line.

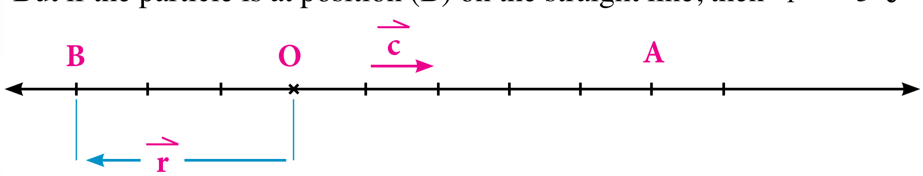
For example:

When the particle is at position (A) on the straight line, then $\vec{r} = 5\vec{c}$



where $\vec{c}$ is a unit vector in the direction $\overrightarrow{OA}$,

But if the particle is at position (B) on the straight line, then $\vec{r} = -3\vec{c}$



Note that the position of the particle is a vector quantity and it can be expressed as a function of time t.

i.e. $\vec{r}$ (t) and the magnitude of $\vec{r}$ in the international system is measured in metre.

Displacement

The displacement $\vec{s}$ of a particle is known as the change of its position.

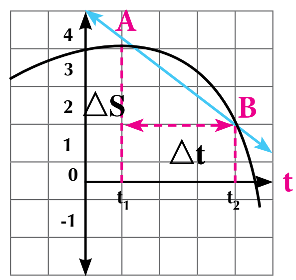
$\vec{s}(t) = \vec{r}(t) - \vec{r}(0)$

The distant is a scalar quantity (is determined) by a magnitude only), but displacement is a vector quantity (is determined by a magnitude and a direction).

The norm of displacement ⩽ the covered distant.

The symbols r, s are used to express the algebraic measures of r, s respectively

The average Velocity



If $\vec{s} = \Delta \vec{r}$ is the displacement of the particle during a period of time Δt, then the average velocity vector $\vec{V}_a = \dfrac{\Delta \vec{r}}{\Delta t}$

From the position - time graph, if $\Delta t = t_2 - t_1$, then the algebraic measure for the average velocity V_A equals the slope of the secant $\overrightarrow{AB}$ to the position - time curve

The instantaneous velocity

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t}$$

And so : $\int_{x_o}^{V} d\vec{r} = \int_{o}^{t} \vec{v}\, dt \qquad \vec{r} - \vec{r}_o = \int_{o}^{t} \vec{V}\, dt$

$\vec{s} = \int_{o}^{t} \vec{v}\, dt$

➢ The covered distant by the particle during the time interval $[t_1, t_2] = \int_{t_1}^{t_2} |v|\, dt$

I.E : The covered distant during the time interval $[t_1, t_2]$ equal the area of region included between time-axis and the velocity- time curve over this interval.

The average speed

during the time interval (t) equals the quotient of the covered distant by the interval tine"t"

$$\text{average speed} = \frac{\text{the covered distant}}{\text{interval time}}$$

Example

1. A stone is projected vertically upwards and its height x after t second from the projection is given by the relation $r = 49\,t - 4.9\,t^2$ where x is in meters.
   - **a** Find the maximum height that projected body can reach.
   - **b** Find the algebraic measure of the velocity vector when the stone is 78.4 meters high, then find its norm velocity.
   - **c** Graph both the position-time graph and the velocity-time graph and use them to analyze the motion.

Solution

In the coordinate system of the motion in a straight line, consider r measures the height (position) at the projection point.

v is positive in case of moving upwards.

$\because r(t) = 49t - 4.9t^2$ $\quad \because v(t) = \frac{dr}{dt}$ $\quad \therefore v(t) = 49 - 9.8t$

a The stone reaches the maximum height when V = 0

$\therefore 49 - 9.8t = 0$ $\quad \therefore t = 5$ sec

$\therefore$ the maximum height $r(5) = 49 \times 5 - 4.9 \times 5^2 = 122.5$ meters

b The stone is 78.4 meters high when r = 78.4

$\therefore 49t - 4.9t^2 = 78.4$ $\quad \therefore 4.9t^2 - 49t + 78.4 = 0$

By dividing both sides of the equation by 4.9 we find that: $t^2 - 10t + 16 = 0$

$\therefore (t - 2)(t - 8) = 0$ $\quad \therefore t = 2$ sec or $t = 8$ sec

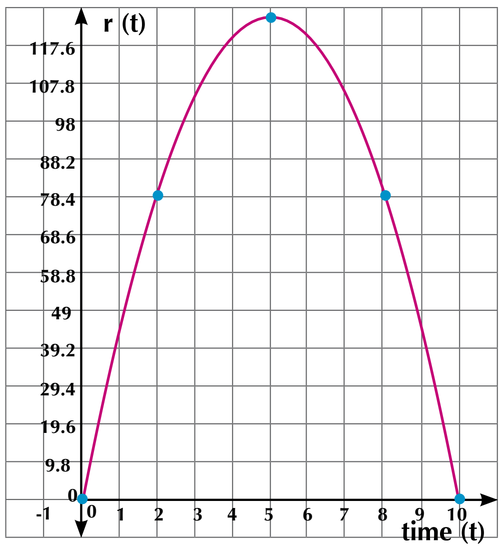
$\therefore v(2) = 49 - 9.8 \times 2 = 29.4$ meters / sec $\therefore v(8) = 49 - 9.8 \times 8 = -29.4$ meters / sec

i.e.: the stone is 78.4 meters once ascending after 2 sec and once descending after 8 sec

The algebraic measure of the velocity vector is either 29.4 or - 29.4

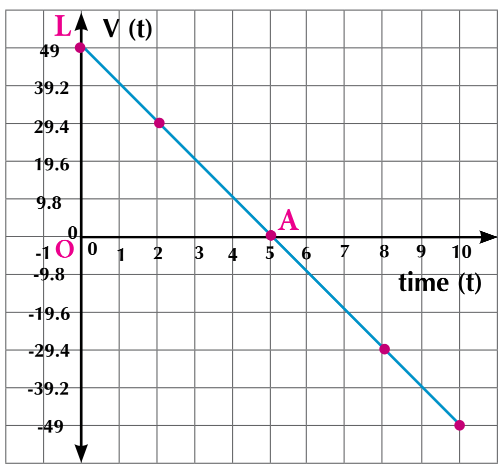
$\therefore$ the norm velocity of the stone in the two cases $= |\pm 29.4| = 29.4$ meters/sec

b From the position-time graph, we find that:



- The stone reaches the maximum height 122,5 meters when t = 5 sec **(the curve vertex point).**
- The stone gets back to the projection point once again when t = 10 sec **(point B (10 , 0)**
- The ascending stage took 5 seconds, and the descending stage took another 5 seconds.
- The stone was at 78,4 meters high when t = 2 sec, and t = 8 sec

From the velocity-time graph, we find that:



1- The starting(initial) velocity of the stone was 49 meters/sec and it gets to decrease during the time interval]0, 5[until the stone got instantaneously static when t = 5 when the stone reaches its maximum height, then its velocity increases in the opposite direction in the time interval]5, 10[until it turns back to the projection point when t = 10 sec at the same projection velocity 49 meters/sec.

2- The maximum height of the stone can be calculated by the velocity-time graph in two methods**:**

- The maximum height = area $\triangle$ OAL $= \frac{1}{2}$ OA $\times$ OL $= \frac{1}{2} \times 5 \times 49 = 122.5$ meters
- By calculating the integration. This method will be discussed in details later.
- The maximum height $= \int_0^5 v\, d\, t = \int_0^5 (49 - 9.8\, t)\, d\, t = 122.5$ meters

Critical thinking: How could you calculate from the velocity-time graph in **Example (1)** the distance traveled by the stone until the stone turns back to the throwing point and also its displacement during this time?

Try to solve

1 A particle moves in a straight line such that its position $\vec{r}$ at any time t is given by the relation $\vec{r}\,(t) = (t^2 - 4t + 3)\,\vec{c}$ where r is measured in meter, t in sec and $\vec{c}$ is the unit vector in the direction of the motion of the body.

- **a** Find the displacement of the particle during the first three seconds.
- **b** Find the average velocity vector of the particle when $t \in [0, 2]$
- **c** Find the velocity vector of the particle when $t = 4$
- **d** Through the velocity-time graph and the position -time graph, analyze the motion of the particle and show: when does the particle change the direction of its motion?

6- Acceleration

If $\triangle \vec{v}$ expresses the change of the velocity vector during time interval Δt then the average acceleration $\vec{a}_a$ is given by the relation

$$\vec{a}_a = \frac{\triangle \vec{v}}{\triangle t} \qquad \text{i.e} \quad \vec{a}_a = \frac{\vec{v}\,(t + \Delta t) - \vec{v}\,(t)}{\Delta t}$$

The instantaneous acceleration $\vec{a}$ **(at any time)** t is identified by

the relation $\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{v}\,(t + \Delta t) - \vec{v}\,(t)}{\Delta t}$

From the definition of the derivative, we can deduce that: $\vec{a} = \frac{d\,\vec{v}}{dt}$

i.e. the acceleration is the rate of change of the velocity vector with respect to the time (the slope of the tangent to the velocity-time graph)

$$\int_{V_o}^{V} d\,\vec{V} = \int_0^t \vec{a}\, d\, t \qquad \vec{V} - \vec{V}_o = \int_0^t \vec{a}\, d\, t$$

Acceleration motion and deceleration motion

the body moves in an acceleration motion if $\vec{v}$ and $\vec{a}$ have the same direction if $v\,a >$ zero

and moves retardation (deceleration) motion if $\vec{v}$ and $\vec{a}$ are in opposite directions if $v\,a <$ zero

Example

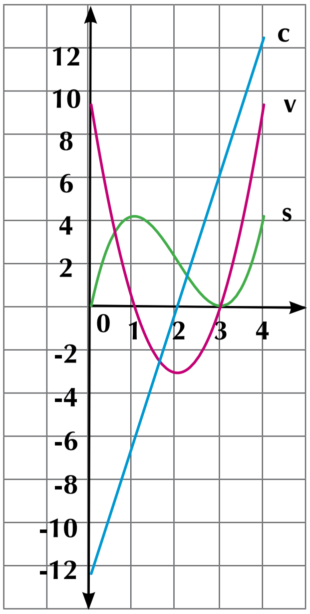
2) If the algebraic measure of the displacement of a particle moving in a straight line is given by the relation $s = t^3 - 6t^2 + 9t$ where s is measured in meter and t in second

a) Find the acceleration of the particle when the velocity vanishes.

b) Find the norm velocity of the particle when the acceleration vanishes.

c) Find the distance covered by the particle during the time interval from t = 0 to t = 2.

Solution



$\because s = t^3 - 6t^2 + 9t \qquad \therefore v = \frac{ds}{dt} = 3t^2 - 12t + 9$

$\therefore a = \frac{dv}{dt} = 6t - 12$

a) The velocity of the particle vanishes **when** $3t^2 - 12t + 9 = 0$

$\therefore t^2 - 4t + 3 = 0$

$(t - 1)(t - 3) = 0$ when $t = 1$, $t = 3$

a (1) $= 6(1) - 12 = -6$ m/sec^2

a (3) $= 6(3) - 12 = 6$ m/sec^2

b) The acceleration of the particle vanishes **when** $6t - 12 = 0$

$\therefore t = 2$

norm velocity $= |V(2)| = |3 \times 4 - 12 \times 2 + 9| = 3$ m/sec

c) From the study of the velocity-time graph of the motion of the particle or by studying the sign V (t), we find that the particle moves in the positive direction in the interval $0 \leqslant t < 1$ then it changes the direction of its motion to move in the opposite direction in the interval $1 < t < 3$.

$\therefore$ the distance traveled from $= {}_0\int^2 |v|\, dt = {}_0\int^2 |3t^2 - 12t + 9|\, dt = 6$ meters

Example

1) A particle starts to move in a straight line from the origin point with initial velocity of a magnitude 8m/sec and the acceleration of motion after t second is given by the relation (3 t -2), find each of the velocity of the particle and its displacement after 2 sec from the starting of motion.

Solution

$\because a = 3t - 2 \qquad \therefore v = \int (3t - 2)\, dt \qquad \therefore v = \frac{3}{2} t^2 - 2t + \text{sec}$

$\because V_\circ = 8$m/sec $\qquad \therefore v = 8$m/sec when $t = 0 \qquad \therefore c = 8$

$\therefore V = \frac{3}{2} v^2 - 2t + 8 \qquad \therefore v(2) = \frac{3}{2} \times 4 - 2 \times 2 + 8 = 10$ m/sec

$\because$ $V = \frac{3}{2} v^2 - 2t + 8$ $\quad\therefore$ $r = \int (\frac{3}{2} v^2 - 2t + 8)\, dt$

$\therefore$ $r = \frac{1}{2} v^3 - v^2 + 8v + c`$

$\because$ $r = 0$ when $t = 0$ $\quad\therefore$ $c` = 0$ $\quad\therefore$ $r = \frac{1}{2} v^3 - v^2 + 8t$

$s(2) = r(2) - r(0) = \frac{1}{2} (2)^3 - (2)^2 + 8(2) = 16$ meters

Another solution:

$\because$ $a = 3t - 2$ $\quad\therefore$ $\frac{dv}{dt} = 3t - 2$

$\therefore$ $\int_8^v dv = \int_0^t (3t - 2)\, dt$ $\quad\therefore$ $v - 8 = \frac{3}{2} t^2 - 2t$

$\therefore$ $v = \frac{3}{2} t^2 - 2t + 8$ $\quad\therefore$ $v(2) = \frac{3}{2} \times 4 - 2 \times 2 + 8 = 10$ m/sec

$\because$ $s = \int_{.}^{t} v\, dt$ $\quad\therefore$ $s(2) = \int_0^2 (\frac{3}{2} t^2 - 2t + 8)\, dt$

$\therefore$ $s(2) = [\frac{1}{2} t^3 - t^2 + 8t]_0^2 = \frac{1}{2} (2)^3 - (2)^2 + 8(2) = 16$ meters

Try to solve

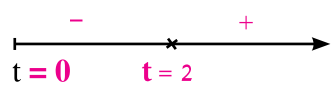
2) A particle moves in a straight line starting from rest and distant 8 meters from a constant point on the straight line. If $a = 6t - 4$ where a is measured in m/sec^2, find the relation between the velocity and the time and between the displacement and the time.

Example

2) A car starts moving from rest in a straight line from a constant point on the line and the algebraic measure of its velocity vector after time t is given by the relation $v = 3t^2 - 6t$ where v is measured in m/sec and t in second. Find each of the average velocity vector and the average speed during the time interval $0 \leqslant t \leqslant 3.5$

Solution

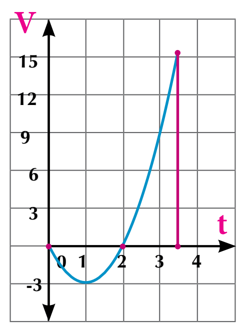
$\because$ $v = 3t^2 - 6t$ $\quad\therefore$ $v = 3t(t - 2)$



We find that the car changes the direction of its motion after 2 seconds and the process of investigating the sign of V (t) of the velocity-time graph shows that

$\therefore s = \int_0^{3.5} v\, dt = \int^{3.5} (3t^2 - 6t)\, dt$

$\therefore s = [t^3 - 3t^2]_0^{3.5} = (3.5)^3 - 3(3.5)^2 = \frac{49}{8}$



$\therefore$ the average velocity $\vec{V}_a = \frac{\frac{49}{8} \vec{c}}{3.5 - 0} = 1.75\, \vec{c}$

since $\vec{c}$ is a unit vector in the direction of motion and the algebraic measure of the average velocity vector is equal to 1.75m/sec

The distance traveled during the time interval $t \in [0, 3.5]$

$= |\int_0^2 v\, dt| + |\int_2^{3.5} v\, dt| = |[t^3 - 3t^2]_0^2| + |[t^3 - 3t^2]_2^{3.5}| = 4 + \frac{49}{8} + 4 = \frac{113}{8}$ meters

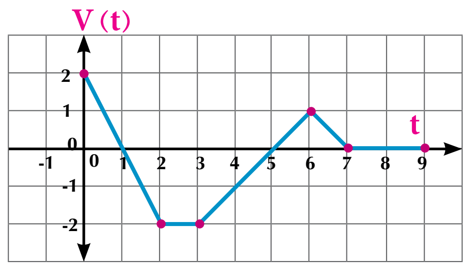
$\therefore$ The average speed $= \dfrac{\frac{113}{8}}{3.5 - 0} = \dfrac{113}{28} \simeq 4.04$ m/sec

Try to solve

3) A car starts moving from rest in a straight line from a constant point on this line and the algebraic measure of the velocity vector after time t is given by the relation $V = 4t - 3t^2$ where V is measured in m/sec and t in second. Find, through the time interval t, where $t \in [0, 4]$ each of the average speed and the average velocity. When does the velocity of the car reach the maximum value? Find the magnitude of the acceleration then.

Critical thinking:

The opposite figure shows the velocity V= v(t) of a particle moving in a straight line.



a) What time does the particle move forward? Backward?

b) What time is the acceleration of motion be positive? Negative? What time does the acceleration of motion vanish?

c) What time is the velocity of the particle be maximum?

d) What time does the particle stop (rest) for a period more than one second?

Deducing the acceleration when the velocity is a function of position:

If $V = f(r)$, $r = g(t)$

By using the chain rule, we can deduce that: $\dfrac{dv}{dt} = \dfrac{dv}{dr} \bullet \dfrac{dr}{dt}$

Use the definite integration with the proper limits of integration to find that:

$$\int_{v_\circ}^{v} v \, dv = \int_{r_\circ}^{r} a \, dx$$

$$\frac{1}{2}(v^2 - v_\circ^2) = \int_{r_\circ}^{r} a \, dx \qquad \textbf{(3 - 2)}$$

= The area under the acceleration-displacement graph

At the constancy of the acceleration a, then: $v^2 - v_\circ^2 = 2a \int_{r_\circ}^{r} dx$

$\therefore \; v^2 = v_\circ^2 + 2a(r - r_\circ)$ (notice that $r - r_\circ = s$) **(3 - 3)**

$\therefore \; v^2 = v_\circ^2 + 2as$

Example

3 A particle moves in a straight line such that the algebraic measure of its velocity vector $\vec{V}$ is given by the relation $V = \frac{1}{20}(400 - r^2)$ where x expresses the algebraic measure of the position $\vec{r}$. Find the algebraic measure of the acceleration of motion $\vec{a}$ when r = 15

Solution

$\because v = \frac{1}{20}(400 - r^2)$ $\therefore \frac{dv}{dr} = \frac{-1}{10} r$

$\because a = v \frac{dv}{dr}$ $a = \frac{-1}{200} \times (400 - r^2)$

when x = 15 unit length.

$\therefore a = \frac{-1}{200} \times 15 (400 - 225)$ $a = - \frac{105}{8}$ Acceleration unit

Try to solve

4 A particle moves in a straight line such that the relation between v and r is given in the form $v = \frac{5}{4 + r}$ where v is measured in m/sec, r is measured in meter. Find the acceleration of motion when r = 2 meter.

Example

4 A particle moves in a straight line such that the algebraic measure of its velocity vector $\vec{V}$ is given by the relation $V = \frac{1}{20}(400 - r^2)$ where x expresses the algebraic measure of the position $\vec{r}$. Find the algebraic measure of the acceleration of motion $\vec{a}$ when r = 15

Solution

$\because v = \frac{1}{20}(400 - r^2)$ $\therefore \frac{dv}{dr} = \frac{-1}{10} r$

$\because a = v \frac{dv}{dr}$ $a = \frac{-1}{200} \times (400 - r^2)$

when r = 15 unit length.

$\therefore a = \frac{-1}{200} \times 15 (400 - 225)$ $a = - \frac{105}{8}$ Acceleration unit

Try to solve

5 A particle moves in a straight line such that the relation between v and r is given in the form $v = \frac{5}{4 + r}$ where v is measured in m/sec, r is measured in meter. Find the acceleration of motion when r = 2 meter.

Example

5 A particle moves in a straight line such that it starts its motion from a constant point on the straight line and the algebraic measure of its acceleration a is given in terms of the algebraic measure of its position x by the relation $a = 2r + 5$, given that the initial velocity of the particle is 2m/sec. **Find:**

a v^2 in terms of r **b** the velocity of the particle when $r = 1$

c find r when $v = 4$ m/sec

Solution

a $\because a = 2r + 5$ $\because \int a\, dr = \int v\, dv$

$\therefore \int_0^r (2r+5)\, dr = \int_2^V v\, dv$ $\therefore [r^2 + 5r]_0^r = \frac{1}{2}[v^2]_2^v$

$\therefore r^2 + 5r = \frac{1}{2}(v^2 - 4)$ $\therefore v^2 = 2r^2 + 10r + 4$

b when $r = 1$, **we find that:**

$v^2 = 16$ $\therefore$ velocity $= |v| = 4$ m/sec

c when $V = 4$ m/sec, **we find that:**

$16 = 2r^2 + 10r + 4$ $\therefore r^2 + 5r - 6 = 0$

$(r + 6)(r - 1) = 0$ $r = -6$ meters or $r = 5$ meters

Try to solve

6 A car moves in a straight line with initial velocity 12 m/sec from a position distant 4 meters in the positive direction from a constant point on the straight line such that $a = r - 4$, find:

a v^2 in terms of r **b** the velocity of car when $a = 0$

Example

6 A particle moves in a straight line such that it starts its motion from a constant point on the straight line and the algebraic measure of its acceleration a is given in terms of the algebraic measure of its position r by the relation $a = 2r + 5$, given that the initial velocity of the particle is 2m/sec. **Find:**

a v^2 in terms of r **b** the velocity of the particle when $r = 1$

c find r when $v = 4$ m/sec

Solution

a $\because a = 2r + 5$ $\because \int a\, dr = \int v\, dv$

$\therefore \int_0^r (2r+5)\, dr = \int_2^V v\, dv$ $\therefore [r^2 + 5r]_0^r = \frac{1}{2}[v^2]_2^v$

$\therefore r^2 + 5r = \frac{1}{2}(v^2 - 4)$ $\therefore v^2 = 2r^2 + 10r + 4$

b when $r = 1$, **we find that:**

$v^2 = 16$ $\therefore$ velocity $= |v| = 4$ m/sec

c when $V = 4$ m/sec, **we find that:**

$16 = 2r^2 + 10r + 4$ $\therefore r^2 + 5r - 6 = 0$

$(r + 6)(r - 1) = 0$ $r = -6$ meters or $r = 5$ meters

Try to solve

7) A car moves in a straight line with initial velocity 12 m/sec from a position distant 4 meters in the positive direction from a constant point on the straight line such that $a = r - 4$, find:

a v^2 in terms of r **b** the velocity of car when $a = 0$

Example

7) A particle moves in a straight line with initial velocity of a magnitude 8m/sec from a constant point on the straight line such that $a = 40\, e^{-r}$, find:

a v^2 in terms of r **b** find r when $v = 10$ m/sec

c the maximum velocity of the particle.

Solution

a $\because a = 40\, e^{-r}$ $\because \int a\, dr = \int v\, dv$

$\therefore 40 \int_0^r e^{-r}\, dr = \int_8^V v\, dv$

$\therefore -40\, [e^{-r}]_0^r = \frac{1}{2}\, [v^2]_8^V$ $\therefore -80\, (e^{-r} - 1) = v^2 - 64$

$\therefore v^2 = 144 - 80\, e^{-r}$

b When $v = 10$ m/sec **we find that**: $80\, e^{-r} = 44$

$\therefore e^r = \frac{20}{11}$ $\therefore r = \ln \frac{20}{11}$ meters

c $\because v^2 = 144 - \frac{80}{e^x}$ $\because e^r > 0$ for all the values of r

$\therefore \frac{80}{e^r} \longrightarrow 0$ **when** $r \longrightarrow \infty$ $\therefore$ the maximum velocity $= 12$ m/sec

Try to solve

8) A particle moves in a straight line with initial velocity of a magnitude 2m/sec from a constant point on the straight line such that $a = e^r$. Find v^2 in terms of r, then find v when $r = 4$ meters and r when $V = 20$ m/sec.

Exercises 1 - 1

In all problems , let the particle move in a straight line, r, V and a be the algebraic measures for each of the position, velocity vector and acceleration respectively.

Choose the correct answer:

1. A particle moves in a straight line such that $V = 3e^{t+2}$ then its starting (initial) velocity is equal to ....

   a) 3 b) e
   c) $3e^2$ d) e^2

2. A particle moves in a straight line and the equation of its motion r = tan t, then the acceleration of motion a is equal to ....

   a) $\sec^2 t$ b) 2 sect
   c) 2v r d) vr

3. A particle moves in a straight line and the equation of its motion $r = 2 + \ln (t + 1)$ then ....

   a) Its velocity and the acceleration of motion always decrease.
   b) Its velocity and the acceleration of motion always increase.
   c) The velocity decreases and the acceleration of motion increases.
   d) The velocity increases and the acceleration of motion decreases.

4. If $v = 3t^2 - 2t$, r = 1 when t = 0 then:

   a) $r = 6t - 2$ b) $r = 3t^2 - 2t + 1$ c) $r = t^3 - t^2 + 1$ d) $r = t^2 - t - 1$

5. If $v = 1 + \sin t$, r = -2 when t = 0 then:

   a) $r = t + \cos t$ b) $r = t - \cos t$ c) $r = t - \cos t + 2$ d) $r = t - \cos t - 2$

6. If $v = 3t - 2$, then s in the interval [0 , 2] is

   a) 1 unit length b) 2 unit length c) 3 unit length d) 4 unit length

7. If $v = 3t^2 - 2t$, then the distance covered within the interval [0 , 2]

   a) $\frac{4}{27}$ unit length b) 4 unit length c) $\frac{112}{27}$ unit length d) $\frac{116}{27}$ unit length

8. If $v = t^3 - 3t^2 + 2t$, then the distance covered within the time interval [0 , 3]

   a) $\frac{1}{4}$ unit length b) $\frac{1}{2}$ unit length c) $\frac{9}{4}$ unit length d) $\frac{11}{4}$ unit length

9. If a = 3 , $v_0 = -1$, then s within the time interval [0 , 2].

   a) $\frac{1}{6}$ unit length b) 4 unit length c) $\frac{25}{6}$ unit length d) $\frac{13}{3}$ unit length

10 If $a = 3$, $V_{\circ} = -1$ then the distance covered within the time interval $[0 , 2]$.

a $\frac{1}{6}$ unit length b 4 unit length c $\frac{25}{6}$ unit length d $\frac{13}{3}$ unit length

11 Choose the appropriate chart in front of each sentence of the following sentences :

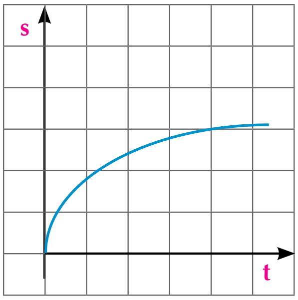


figure a

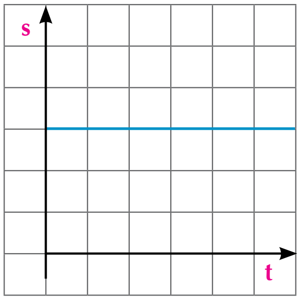


figure b

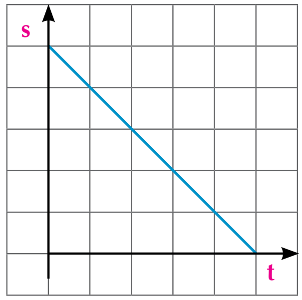


figure c

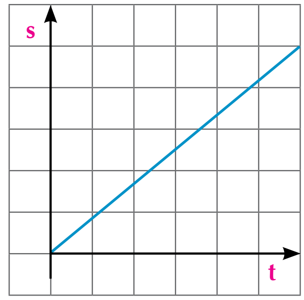


figure d

(1) the body stalled (2) Body back wards (3) The body is moving forward at a constant at speed (4) speed of the particle decreases

12 Choose the appropriate chart in front of each sentence of the following sentences:

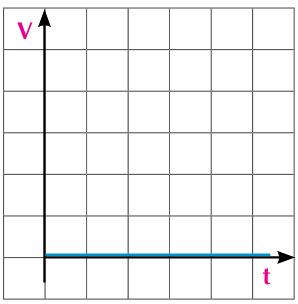
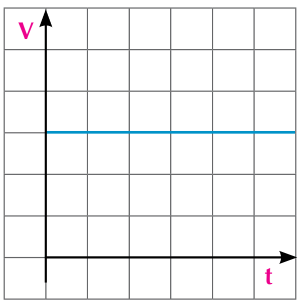


figure a



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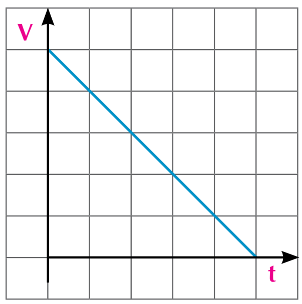


figure c

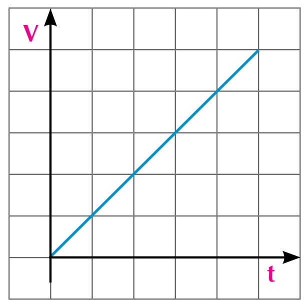


figure d

(1) The particle move declaration (2) The body is moving constant speed (3) the body stalled (4) the particle move at accelerated

13 In each of the graphs drawn **(position-time graph),** identify the sign of the algebraic measure of the velocity vector, then whether the particle moves in acceleration or deceleration.

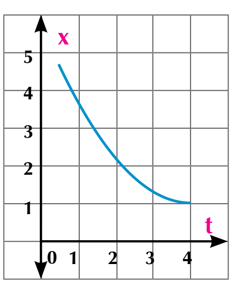


figure (1)

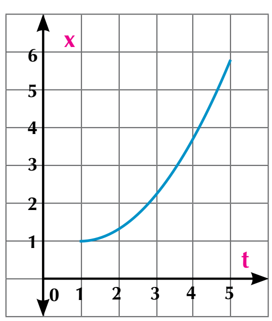


figure (2)

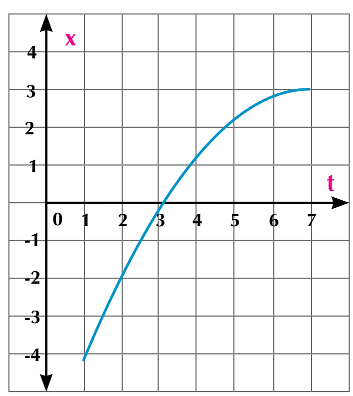


figure (3)

14 In each of the graphs drawn **(velocity-time graph),** identify the sign of the acceleration and tell whether the particle moves in acceleration or deceleration.

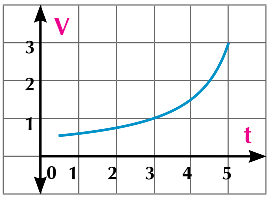


figure (1)

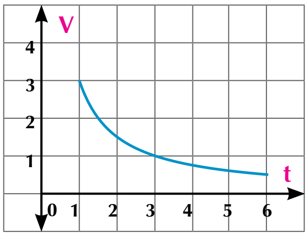


figure (2)

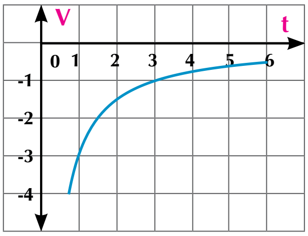


figure (3)

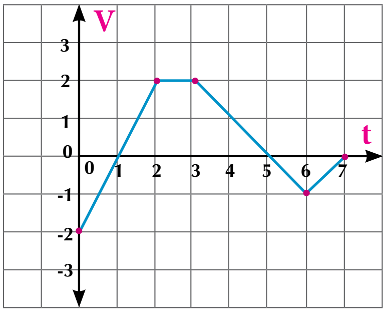
15 From the velocity-time graph in the opposite figure, the magnitude of displacement is equal to ....

a 3 unit length

b 5 unit length

c 7 unit length

d 8 unit length



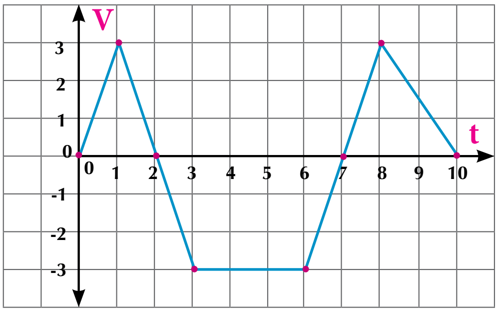
16 From the velocity-time graph in the opposite figure, then the distance traveled = ...

a 4.5 unit length

b 10.5 unit length

c 13.5 unit length

d 19.5 unit length



17 If V = 3r, find a in terms of r, then find a when r = 2

18 A particle moves in a straight line such that the algebraic measure of the velocity V is given in a relation with the algebraic measure of the position x in the form $V = r + \frac{1}{r}$. Find the acceleration of motion when r = 2 where r is measured in meter and, V is measured in m/sec.

19 A particle moves in a straight line such that the algebraic measure of its velocity is given in a relation with the algebraic measure of the position r in the form $V = \frac{1}{r^2}$. Find a in terms of r, then find a when $r = \frac{1}{2}$.

20 A particle moves in a straight line such that the algebraic measure of the velocity V is given in a relation with the algebraic measure of the position r in the form $V^2 = 16 - 9 \cos r$. Find the maximum velocity of the particle and the acceleration of motion then.

21 A particle is thrown vertically upwards with initial velocity of a magnitude 5.6 m/sec from a point 24.5 above the earth's surface. Find each of V and r in terms of t, then find the maximum height the particle can reach.

22 A particle moves in a straight line with initial velocity 2 m/sec from a constant point such that $a = 2t - 6$ where a is measured in m/sec^2. Find each of V and r in terms of t and r when $V = 18$ m/sec.

23 A particle moves in a straight line from a constant point starting at rest such that $a = 8 - 2t^2$ where a is measured in m/sec^2. Find the maximum velocity of the particle and the distance traveled maximum speed up.

24 A particle moves in a straight line from a constant point on the straight line starting at rest such that $a = \frac{3}{8} r^2$ where a is measured in m/sec^2 and r in meter. Find the velocity of the particle when $r = 2$ meters, then find its position when $V = 4$ m / sec.

25 A particle moves in a straight line with initial velocity 3 m/sec from a constant point such $a = 6r + 4$ where a is measured in m/ sec^2 and r in meter. Find v^2 in terms of r, then find the velocity of the particle when $r = 2$ and r when $v^2 = 87$.

Applicarions on Newton's Laws

Unit
2

Unit Introduction

Thanks to the discovery of the universal gravitation law by the British scientist, Isaac Newton (1642-1727), who is considered one of the icons of the scientific revolution in the field of modern mechanics, The German scientist, Johann Kepler (1571 - 1630), has stated some mathematical rules which control the motion of the planets around the sun. In addition to the work of the islamic Scientisits which had been translated during the previous centuries. The italian scientist, Galileo Galilit (1564 - 1642), had established the mechanics. He had conducted several experiments on the fallen or thrown bodies and the bodies moving horizontally, through his experiments, he had discovered a lot of the important properties of the motion of the bodies. Thanks to him it was discovered that the bodies moving on horizontal surfaces without resistance keep moving in a uniform velocity. It is thought that Galileo had discovered the first and second laws of Newton's laws of motion. Isaac Newton had collected his overall researches in a book named (Bernspaa) which means the mathematical principles of the natural philosophy. This book is considered one of the most important scientific books that appeared in the modern age. Newton had formulated his three laws. The Newton's law of universal gravitation had clarified the concept that the force can be occured under the action of a distance. Bodies attract each other even if they are not contacting. For example, the Earth attracts the bodies by a force called (the weight force). With respect to the mass, we notice that its static definition does not allow us to identify the mass of objects or bodies but to only compare the masses through the resistance of their weights. The mass can get a dynamic definition through studying the motion of bodies in this unit. You are going to learn the mass, momentum and Newton's laws of motion with application on these laws and you will also learn the motion on a rough or smooth plane and the motion of the simple pulleys.,

At the end of this unit and by doing all the activities involved, the student should be able to:

- Identify the relation between the force and acceleration:

  If the force F is a function of time t i.e. F = f(t) then:

  $F = m\frac{dV}{dt}$ i.e. $\int F\,dt = m\int dV$

  If the force F is a function of displacement S

  i.e. F = f(S) then:

  $F = mv\frac{dV}{dS}$ i.e. $\int F\,dS = m\int V\,dV$

- Apply Newton's laws of motion in daily life situations such as: a body is placed in a lift moving in s uniform acceleration (motion of the bodies connected by strings):
- Motion of the simple pulley.
- Motion of a system of two bodies connected by a string passing over a smooth pulley.
- Motion of a system of two bodies connected by a string. One moves on a smooth inclined plane and the other moves vertically.
- Motion of a system of two bodies connected by a string. One of them moves on a rough inclined plane and the other moves vertically.

Key Terms

- Momentum
- Equation of motion
- Weight
- Newtons third law
- Pressure
- Reaction
- Lift motion
- Spring scale
- Pressure scale
- Balance
- Pulley
- Impulse
- Impulsive force

Unit Lessons

(2 - 1): Motion of variable masses

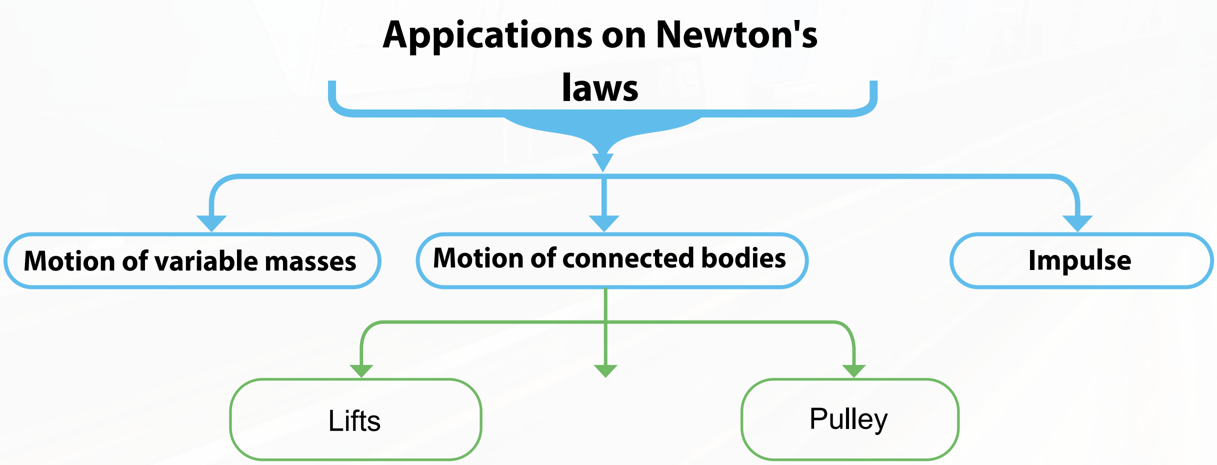
(2 - 2): Motion of connected bodies

(2 - 3): Impulse

Materials

Scientific calculator - Computer graphics

Unit Planing Guide



Unit Two

2 - 1

Applications on Newton's Laws and motion of variable Masses

You will learn

- Newton's second law
- Force units
- Weights and mass

Key terms

- Newton's second law
- Equqtion of motion
- Force
- Mass
- Weight

Materials

- Scientific calculator

Think and discuss

From Newton's first law, you know that the resultant of the forces acting on a uniformly moving body vanishes, but if the resultant of the forces acting on a body is not equal to zero, the body moves with an acceleration.

- Is there a relation between the magnitude of the resultant acting on a body and the magnitude of the acceleration of motion?
- Are you able to figure out such a relation?

Learn

1 - Newton's second law

The rate of change of momentum with respect to the time is proportional to the acting force and takes place in the direction in which the force is acting

But if the mass m of the body is variable, then the equation of motion is written in the form

$$\vec{F} = \frac{d}{dt}(m\,\vec{V})$$

$\therefore\ \Sigma F = \frac{d}{dt}(m\,V)$

where each of m and v are differentiable function in t

Equation of motion using differentiation

The equation of motion of a constant mass body is given in the form

$F = ma$

If $a = \frac{dV}{dt}$ **, then** $F = m\frac{dV}{dt}$

$\therefore \int_{t_1}^{t_2} F\,dt = m \int_{v_1}^{v_2} dV$

If $a = V\frac{dV}{dS}$ **, then** $F = mv\frac{dV}{dS}$

$\therefore \int_{s_1}^{s_2} F\,dS = m \int_{v_1}^{v_2} V\,dV$

Units of force and units of mass

As we deduce the equation of motion of a moving body, we choose certain units for each of the force, mass and acceleration until the constant of proportionality is equal to 1 (unity). The equation of motion takes the form m a = F. As a result, when we use the equation of motion, we use the absolute units of force such as Newton and dyne.

$m \times a = F$
$1kg \times 1\ m/sec^2 = 1$ Newton
$1gm \times 1cm/sec^2 = 1$ dyne

Remember

1 kg.wt = 9.8 Newton
1 gm.wt = 980 dyne

Weight and Mass

The weight of a body is the Earth's attraction force to the body. If we have a body of mass 1 kg, then its weight according to the equation of motion is equal to 1 kg.wt.

$\because\ m\,a = F \qquad \therefore\ 1 \times 9.8 = F \qquad F = 9.8$ Newton $= 1$ kg.wt

Example

1. A body of a unit mass under the action of three forces $\overrightarrow{F_1} = a\hat{i} + \hat{j}$, $\overrightarrow{F_2} = \hat{i} + b\hat{j} + 3\hat{k}$, $\overrightarrow{F_3} = \hat{i} + 2\hat{j} - e\hat{k}$, if the displacement vector $\overrightarrow{S}$ is given by the relation $\overrightarrow{S} = t\hat{i} + (\frac{1}{2}t^2 + t)\hat{j} + 5\hat{k}$ find the value for each of a, b and e

Solution

$\because\ \overrightarrow{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = (a + 2)\hat{i} + (b + 3)\hat{j} + (3 - e)\hat{k}$

$\because\ \overrightarrow{S} = t\hat{i} + (\frac{1}{2}t^2 + t)\hat{j} + 5\hat{k}$

$\overrightarrow{V} = \frac{dS}{dt} = \hat{i} + (t + 1)\hat{j}$

$\overrightarrow{a} = \frac{d\overrightarrow{V}}{dt} = \hat{j}$

$\because\ m\overrightarrow{a} = \overrightarrow{F} \qquad \therefore\ \hat{j} = (a + 2)\hat{i} + (b + 2)\hat{j} + (3 - e)\hat{k}$

$a + 2 = 0 \qquad ,b + 3 = 1 \qquad 3 - e = 0$

$a = -2 \qquad ,b = -2 \qquad e = 3$

Try to solve

1. A body of mass 3 kg moves under the action of three coplanar forces: $\overrightarrow{F_1} = a\hat{i} + \hat{j}$ $\quad \overrightarrow{F_2} = 2\hat{i} - \hat{j}$, $\overrightarrow{F_3} = 3\hat{i} + b\hat{j}$ where $\hat{i}$, $\hat{j}$ are two unit vectors perpendicular to each others. If the displacement vector is given as a function of time by the relation $\overrightarrow{S} = (t^2 + 1)\hat{i} + (2t^2 + 3)\hat{j}$, identify the value for each of a and b.

Example

(2) A body moves in a straight line under the action of three forces $\vec{F_1} = 4\hat{i} + 3\hat{k}$, $\vec{F_2} = -\hat{i} + 4\hat{j} - 15\hat{k}$ and $\vec{F_3}$ such that its displacement vector $\vec{S}$ is given as a function of time by the relation $\vec{S} = 2t\,\hat{i} - t\,\hat{j} + \hat{k}$ find the magnitude of $\vec{F_3}$.

Solution

$\because \vec{S} = 2t\,\hat{i} - t\,\hat{j} + \hat{k}$ $\quad\therefore \vec{V} = \frac{d\vec{S}}{dt} = 2\hat{i} - \hat{j}$, $\vec{a} = \frac{d\vec{V}}{dt} = \vec{0}$

$\therefore$ The body moves

$\therefore \vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0}$ $\quad\therefore \vec{F_3} = -\vec{F_1} - \vec{F_2}$

$\vec{F_3} = (-4, 0, -3) + (1, -4, 15)$ $\quad \vec{F_3} = -3\hat{i} - 4\hat{j} + 12\hat{k}$

$F_3 = \|\vec{F_3}\| = \sqrt{9 + 16 + 144} = 13$ force unit.

Try to solve

(2) A body moves with a uniform velocity under the action of a system of forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ where $\vec{F_1} = a\,\hat{i} - 5\hat{j} + 7\hat{k}$, $\vec{F_2} = -3\hat{i} + 4\hat{j}$, $\vec{F_3} = 2\hat{i} + 4\hat{j} + c\hat{k}$, find each of a , b and c.

Example

(3) A force $\vec{F}$ acts on a body of mass 1 kg at rest, moving in a straight line starting from the origin "o" on the straight line and F = 5r + 6 where r is the distance between the body and point "o" measured in meter and F in Newton.

Find:

First: the velocity of the body V when $r = 4$ meters

Second: the displacement of the body when $V = 9$ m/sec

Solution

$\because F = 5r + 6$

$\therefore ma = 5r + 6$

$\because a = v\frac{dv}{dr}$, $m = 1$ kg

Figure (1)

First:

$\therefore V\frac{dV}{dr} = 5r + 6$ $\quad\therefore \int_0^V V\,dV = \int_0^4 (5r + 6)\,dr$

$\therefore [\frac{1}{2}v^2]_0^v = [\frac{5}{2}r^2 + 6r]_0^4$ $\quad\therefore \frac{1}{2}v^2 = (\frac{5}{2} \times 16 + 6 \times 4) - 0$

$v^2 = 128$ $\quad\therefore V = \pm 8\sqrt{2}$ m/sec

Second:

$\because V\frac{dV}{dr} = 5r + 6$ $\therefore \int_0^9 V\,dv = \int_0^r (5r + 6)\,dr$

$\therefore [\frac{1}{2}v^2]_0^9 = [\frac{5}{2}r^2 + 6r]_0^r$ $\therefore \frac{81}{2} = \frac{5}{2}r^2 + 6r - 0$

$\therefore 5r^2 + 12r - 81 = 0$

$(5r + 27)(r - 3) = 0$ $\therefore r = 3 \quad , \quad r = -\frac{27}{5}$

Try to solve

3. A force F acts on a body of mass 3 kg, moving in a straight line with initial velocity of a magnitude 2m/sec. If $F = \frac{3}{2v + 1}$ where V is the velocity of the body after a time t, when is the velocity of the body 6 m/sec.

Example

4. A force acts upon a body of mass 250 gm moving in a straight line and starting from rest at the origin point "o" on the straight line where $\vec{F} = (5t - 2)\hat{i} + 4t\hat{j}$
If F is measured in Newton unit and t in second, find the velocity $\vec{V}$ and the displacement $\vec{S}$ in terms of t.

Solution

$\because \vec{F} = (5t - 2)\hat{i} + 4t\hat{j}$ $\because \vec{F} = m\vec{a}$ where $m = \frac{1}{4}$ kg

$\therefore \frac{1}{4}\vec{a} = (5t - 2)\hat{i} + 4t\hat{j}$ $\therefore \vec{a} = (20t - 8)\hat{i} + 16t\hat{j}$

$\because \vec{a} = \frac{d\vec{V}}{dt}$ $\therefore \frac{d\vec{V}}{dt} = (20t - 8)\hat{i} + 16t\hat{j}$

$\therefore \int_0^V d\vec{V} = \int_0^t [(20t - 8)\hat{i} + 16t\hat{j}]\,dt$

$\therefore \vec{V} = (10t^2 - 8t)\hat{i} + 8t^2\hat{j}$

$\because \vec{V} = \frac{d\vec{S}}{dt}$ $\therefore \frac{dS}{dt} = (10t^2 - 8t)\hat{i} + 8t^2\hat{j}$

$\therefore \int_0^S d\vec{S} = \int_0^t [(10t^2 - 8t)\hat{i} + 8t^2\hat{j}]\,dt$

$\therefore \vec{S} = (\frac{10}{3}t^3 - 4t^2)\hat{i} + \frac{8}{3}t^3\hat{j}$

Try to solve

4. A force $\vec{F}$ acts upon a body of mass $\frac{1}{2}$ kg at rest starting its motion from a fixed point "O" on the straight line and $\vec{F} = (4t - 1)\hat{i} + 4\hat{j}$ where t is the time measured in second an F in Newton. Find the velocity of the body when t = 2 second and the distance between the body and point "O".

Example

5 A body of a variable mass of $m = 2t + 1$ moves in a straight line and its displacement vector is given by the relation: $\vec{S} = (\frac{1}{2} t^2 + t)\hat{i}$ where $\hat{i}$ is the unit vector parallel to the straight line. Find the momentum of this body and deduce the magnitude of the force acting on it.

Solution

$m = 2t + 1$

The velocity vector $\vec{V} = \frac{d\vec{S}}{dt} = \frac{d}{dt}(\frac{1}{2} t^2 + t)\hat{i} = (t + 1)\hat{i}$

The momentum vector $\vec{H} = m\vec{V}$

$= (2t + 1)(t + 1)\hat{i} = (2t^2 + 3t + 1)\hat{i}$

From Newton's second law, we find that:

$$\vec{F} = \frac{d}{dt}(m\vec{V}) = \frac{d\vec{H}}{dt} = \frac{d}{dt}(2t^2 + 3t + 1)\hat{i} = (4t + 3)\hat{i}$$

i.e. the force acting upon the body is in the direction of vector $\hat{i}$ and its magnitude is equal to $(4t + 3)$

Try to solve

5 A metal ball of mass 100 gm moves with a uniform velocity 10 m/sec in a dusty medium such that the dust sticks to its surface at a constant rate equal to 0.6 gm per second. Find the mass of the ball and the force acting on it at any moment in dyne.

Example Using integration

6 A body moves in a straight line such that the acceleration of its motion a is given as a function of time t by the relation $a = 2t - 6$ where a is measured in m/sec^2, unit and time t in second. Calculate the momentum of the body in the time interval $3 \leqslant t \leqslant 5$ if the mass of the body is 8 kg.

Solution

$\because \Delta H = m \int_{t_1}^{t_2} a\,dt$

$\therefore \Delta H = 8 \int_3^5 (2t - 6)\,dt = 8\,[t^2 - 6t]_3^5$

$= 8\,[(25 - 30) - (9 - 18)] = 32 kg. m/sec$

$\therefore \Delta\vec{H} = 32\vec{C}$ where $\vec{C}$ is the unit vector in the direction of the motion of the body.

Try to solve

6 A car of mass 1.5 tons, moves in a straight line such that a (t) is given by the relation $a = 12t - t^2$ where a is measured in m/sec^2, unit and time t in sec, find:

a The change of momentum of the car during the first six seconds.

b The change of momentum of the car during the time interval [2,14]

Exercises 2-3

Choose the correct answer:

1. A body of mass unit moves under the action of the force $\vec{F} = 5\,\vec{C}$ if its velocity vector $\vec{V} = (a\,t^2 + b\,t)\,\vec{C}$, then a + b is:
   - a) 0
   - b) $\frac{5}{2}$
   - c) $\frac{7}{2}$
   - d) 5

2. If a body of mass m = (2t + 3) kg moves in a straight line and its displacement vector as a function of time is given by the relation $\vec{S} = (\frac{3}{2}t^2 + 2t)\vec{C}$, S is measured in meter and, t in second, then the magnitude of the force acting upon the body in Newton is:
   - a) 2 t + 3
   - b) 12 t + 3
   - c) 12 t + 13
   - d) 6t + 13

3. A body of unit mass moved in a straight line such that its acceleration is given by the relation a = 4t + 2, where a is measured in m/sec^2, t in second. Then the change in mementum during the interval time [2, 6] equals .... kg/sec :
   - a) 16
   - b) 32
   - c) 64
   - d) 84

4. A body of mass 3 kg moves under the action of 3 coplanar forces $\vec{F}_1 = 2\,\hat{i} - b\,\hat{j}$, $\vec{F}_2 = a\,\hat{i} + \hat{j}$, $\vec{F}_3 = 3\,\hat{i} + 2\,\hat{j}$ where $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors in the plane of the forces. If the displacement vector is given as a function of time by the relation $\vec{S} = (t^2 + 1)\,\hat{i} + (2t^2 + 3)\,\hat{j}$, identify the two constants a and b.

5. A body of mass m = (2 t + 5) kg and position vector $\vec{r} = (\frac{1}{2}t^2 + t - 5)\vec{C}$ where vector $\vec{C}$ has a constant direction,vector r is measured in meter and t is the time in second, find:

   First: the two vectors of velocity and acceleration of the body at any moment t.

   Second: the magnitude of the force acting upon the body when t = 10 seconds

6. A metal ball of mass 150 gm. moves with a uniform velocity 12 m/sec within a dusty medium such that the dust sticks to its surface at a constant rate of 0.5 gm per second. Find the mass of the ball and the force acting upon it at any moment t in dyne.

7. A metal ball of mass 10 gm moves in a straight line within a dusty medium such that the dust sticks to its surface at a rate of 1 gm per second. If the displacement of this ball at the end of a time interval is $\vec{S} = (t^2 + 3t)\,\hat{i}$ where $\hat{i}$ is a unit vector in the direction of its motion, find the force acting upon the ball at any moment t and calculate its magnitude when t = 3 seconds given that the magnitude of the displacement is measured in cm.

8. A body of variable mass moves in a straight line and its mass at any moment t is equal to m= (4t +1) gm and its displacement vector is given by the relation $\vec{S} = (t^2 + 2t)\,\hat{i}$ where $\hat{i}$ is a constant unit vector parallel to the straight line, t is the time in second and S is the distance in cm, find:

a the momentum vector of this body.

b the magnitude of the force acting upon the body when $t = 4$.

9 A force $F = 3t + 1$ acts upon a body at rest of mass, 4 kg starting its motion at the origin point "O" on the straight line.

a Find V when $t = 2$ seconds.

b Find S when $t = 2$ seconds , given that F is in Newton unit.

10 A body moves in a straight line with a uniform acceleration $a = - 3 m/sec^2$ and with initial velocity 5m/sec. If the mass of the body is 18 kg, find the magnitude of the change of the momentum in the following time intervals:

a [0 , 3] b [1, 2]

11 A body of mass 48 gm, moves in a straight line such that $a = (3t - 12) m/sec^2$. Calculate the change of momentum motion in the following time intervals:

a [1, 3] b [3, 5]

Motion of Connected Bodies
Lift motion - Simple Pulley

Unit Two

2 - 2

Cooperative work

Work with a classmate to bring a pressure scale and place it on the roof of a lift. Then stand on the scale while the lift is at rest and have your classmate record the scale readings as you stand on the pressure scale. Let the lift move upwards and have your classmate record any change occurring in the scale readings. After that, stop the lift and record the readings once more. Then let the lift descend and have your classmate record the readings of the scale when any change occurs in the readings. Repeat this experiment alternately with your classmate. Record the scale readings as you and your classmate stand on the scale in each phase of the scale; at rest, ascending and descending.


Figure (2)

What is your interpretation of the scale readings in each phase?

You will learn

- The pressure and reaction
- Lift motion
- Pulley
- Motion of a system of 2 bodies
- spended vertically by a string passes over a smooth pulley

Key terms

- Newtons third law
- Pressure
- Reaction
- Lift motion
- Spring scale
- Pressure scale
- Balance

Materials

- Scientific calculator
- Balance
- Spring scale
- Pressure scale

Learn

Newtons third law

To every action, there is a reaction equal in magnitude and opposite in direction.

Pressure and reaction

When we place a body of mass m on a rested horizontal plane, then the body acts on the plane with a pressure force equal, in this case, to the weight of the body and a force of reaction is generated for the plane acting upon the body completely equal to the pressure exerted by the body on the plane and the two forces are opposite in direction but equal in the magnitude completely. The pressure of the body on the plane changes as the plane moves up or down. The pressure in this case is known as the apparent weight.

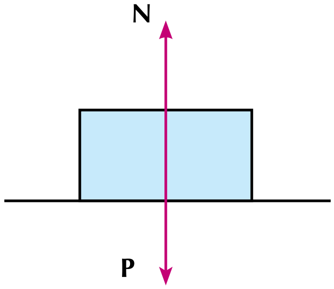


Figure (3)

Lift motion

The lift motion is considered the most well known application of the reaction when a person of mass m stands in a lift of mass m`, then there is a system of forces acting upon each of them.

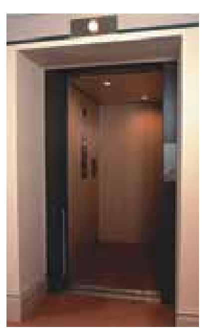


Figure (4)

The forces acting on the person inside the lift

There are two forces acting upon the person inside the lift:

1 - **The weight of the person** $= mg$ (**it acts vertically downwards whatever the direction of the lift is**)

2 - **The reaction of the lift on the person** $= R$

(**it acts vertically upwards whatever the direction of the lifts is**).

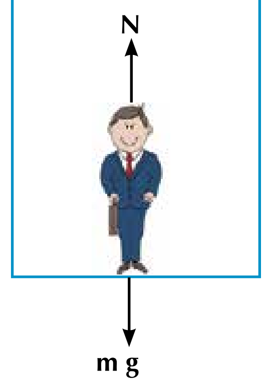


Figure (5)

The equation of a person's motion

When the lift is at rest or moves uniformly (constant velocity upwards or downwards) then $mg = N$

When the lift moves up with acceleration of a magnitude a, the equation of the person's motion is $ma = N - mg$

When the lift moves down with acceleration of a magnitude a, the equation of the person's motion is $ma = mg - N$

Critical thinking: What do you expect about the reaction of the lift on the person if the lift falls with an acceleration equal to the gravitational acceleration?

The forces acts on the lift only if the person is inside it (Figure 40)

There are three forces acting on the lift as a person is inside it:

1 - The weight of the lift only $= m` g$ (**it acts vertically downwards whatever the direction of the lift is**)

2 - The pressure of the person on the floor of the lift = **P**

(**it acts vertically downwards whatever the direction of the lift is**)

3 - The tension in the wire carrying the lift = **T**

(**it acts vertically upwards whatever the direction of the lift is**)

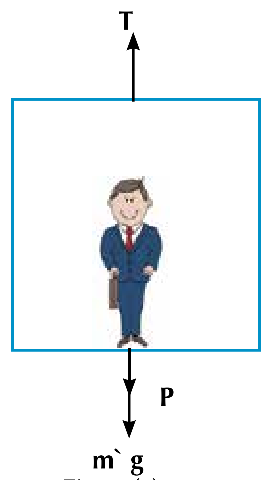


Figure (6)

The equation of the lift's motion

As moving up with an acceleration of a magnitude a, the equation of the lift's motion is $m` a = T - P - m` g$

As moving down with an acceleration of a magnitude a, the equation of the lift's motion is $m` a = m` g + P - T$

The forces acting on a system (the lift and the person together) (Figure 41)

There are two forces acting on both the lift and the person:

1 - The weight of the system (lift and person) $= (m + m`) g$

(**it acts vertically downwards whatever the direction of the lift is**)

2 - The tension in the wire carrying the lift = **T**

(**it acts vertically upwards whatever the direction of the lift is**)

Figure (7)

Note:

The pressure of the person on the floor of the lift is equal and opposite to the reaction of the lift on the person

The equation of the motion of the a system

As moving up with an acceleration of a magnitude a, the equation of the motion of the lift is

$$(m + m`)\, a = T - (m + m`)g$$

As moving down with an acceleration of a magnitude a, the equation of the motion of the lift is $(m + m`)\, a = (m + m`)\, g - T$

Spring Scale

When a body of mass m is suspended in a spring scale fixed in the ceiling of a lift, then the reading of the scale expresses the tension occurring in the spring.

Figure (8)

Pressure scale

When a body of mass m is placed on a pressure scale fixed in the floor of a lift, then the reading of the scale expresses the pressure of the body on the scale.

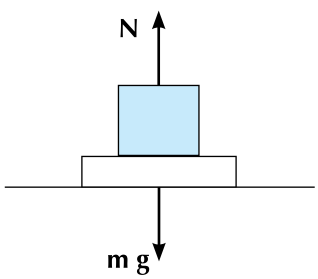


Figure (9)

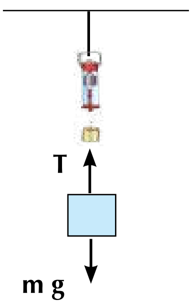


Figure (10)

1 - If the reading of the scale > the real weight $T > mg$, $N > mg$, then the lift is moving upwards with an positive acceleration or moving downwards with negative acceleration.

2 - If the reading of the scale < the real weight $T<mg$, $N < mg$, then the lift is moving downwards with an positive acceleration or moving upwards with a negative acceleration.

3 - If the reading of the scale = the real weight $T = mg$, $N = mg$, then the lift is either at rest or moving with a uniform velocity. The readings of the spring scale or pressure scale are called the apparent weight.

Notice that

If the lift moves upwards with a uniform acceleration and moves downwards with the same acceleration, then:

the reading of the scale when moving upwards + reading of the scale when moving down wards = twice the real weight.

The balance

The balance is the only machine that can measure the real weight in all conditions.

Figure (11)

Example

1. A man of mass 80 kg is inside a lift. Calculate in kg.wt the pressure of the man on the floor of the left in each of the following:

1 - Moving upwards with a uniform acceleration of magnitude 49 cm/ sec^2.

2 - Moving with a uniform velocity of magnitude 80 cm/ sec.

3 - Moving downwards with a uniform acceleration of magnitude 98 cm/ sec^2.

Solution

The pressure exerted by the man on the floor of the lift is equal in magnitude to the reaction of the lift on the man.

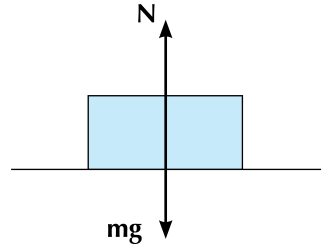
1 - The lift moves upwards with an acceleration of magnitude 0,49 m/ sec^2.

$\therefore ma = N - mg$

$80 \times 0.49 = N - 80 \times 9.8$

$\therefore N = 80 \times 0.49 + 80 \times 9.8$

$N = 823.2$ Newton $\qquad N = 84$ kg.wt



2 - The lift moves with a uniform velocity.

$\therefore a = 0$

$\therefore N = mg \qquad g = 80$ kg.wt

3 - The lift moves downwards with a uniform acceleration of magnitude 0.98 m/sec^2.

$m a = mg - N$

$80 \times 0.98 = 80 \times 9.8 - N$

$N = 80 \times 9.8 - 80 \times 0.98 \qquad N = 705.6$ Newton. $\qquad N = 72$ kg

Try to solve

1. A person of mass 60 kg is inside a lift. Calculate in kg.wt the pressure of the person on the floor of the left in each of the following cases:

1 - If the lift is at rest.

2 - The lift moves upwards with an uniform acceleration of magnitude 49 cm/sec^2.

3 - The lift moves downwards with an uniform acceleration of magnitude 49 cm/sec^2.

Example

(2) A body is suspended by a string in a spring scale fixed at the top of a lift moving upwards. If the magnitude of tension in the string during ascending with an uniform acceleration of magnitude 2,45 m/sec^2 is equal to 50 kg.wt, find the mass of the body. What is the magnitude of tension if the lift moves down with the same acceleration?

Solution

First: the lift moves upwards with an acceleration 2.45 m/sec^2

the equation of motion: $ma = T - mg$

$m \times 2.45 = 50 \times 9.8 - m \times 9.8$

$m(2.45 + 9.8) = 50 \times 9.8$ $\qquad m = 40$ kg

Second: the lift moves downwards with an acceleration 2,45 m/sec^2.

the equation of motion: $ma = mg - T$

$40 \times 2.45 = 40 \times 9.8 - T$

$T = 40(9.8 - 2.45)$ $\qquad T = 294$ Newton $\qquad T = 30$ kg.wt

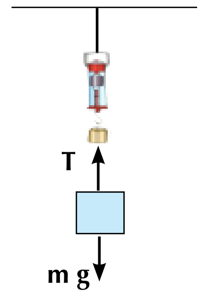


Figure (12)

Try to solve

(2) A body of real weight 240 kg.wt is suspended in a spring scale fixed at the top of a lift and its apparent weight is 276 kg.wt according to the reading of the spring scale. Show that the acceleration of motion of the lift has two values, then find them and determine the direction of the motion.

Example

(3) A lift moves vertically upwards with a uniform acceleration of a 140 cm/sec^2. and a person is inside the left. If the pressure of the person on the floor of the lift is equal to 72 kg.wt. **calcaulte the mass of the person, then find the magnitude of his pressure on the floor of the lift as he moves downwards with the same acceleration.**

Solution

First: The lift moves up with acceleration $a = 1.4$ m/sec^2.

the equation of motion: $m\,a = g - m\,g$

$m \times 1.4 = 72 \times 9.8 - m \times 9.8$

$\therefore m = 63$ kg

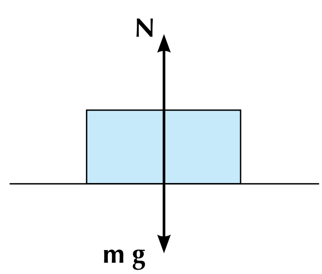
Second: the lift moves down with acceleration $a = 1.4$ m/sec^2.

the equation of motion: $m\,a = m\,g - N$

$63 \times 1.4 = 63 \times 9.8 - N$

$N = 63(9.8 - 1.4)$ $\qquad N = 529.2$ Newton

$N = 54$ kg.wt



Try to solve

(3) A man of mass 70 kg is inside an electrical lift of mass 420 kg. If the lift moves vertically upwards with an acceleration of uniform 70 cm/sec^2,
find in kg.wt the magnitude for each of the tension in the rope carrying the lift and the pressure of the man on the floor of the lift.

Example

(4) A body is suspended in a spring scale fixed at the top of a lift. It is noticed that when the lift moves up with acceleration a m/ sec^2, the reading of the scale is 8 kg.wt and when the lift moves down with acceleration 2 a m/sec^2 the reading of the scale is 5 kg.wt.
Calculate of a if the steel wire carrying the lift cannot stand tension more than 1.2 ton. wt. Find the maximum load that the lift can stand as it moves up with acceleration a given that the mass of the lift as it is empty is 600 kg.

Solution

First: The lift moves up with acceleration a

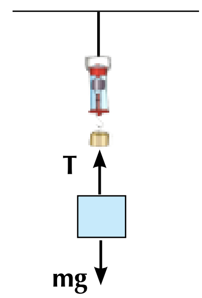


Figure (48)

the equation of motion: $m\,a = T - mg$

$m\,a = 8 \times 9{,}8 - m \times 9{,}8$

$m\,a = (8 - m) \times 9{,}8 \qquad (1)$

Second: The lift moves down with acceleration 2 a

the equation of motion $m\,a = mg - T$

$2ma = m \times 9{,}8 - 5 \times 9{,}8$

$2ma = (m - 5) \times 9{,}8 \qquad (2)$

From (1) and (2) we find that

$$\frac{2ma}{ma} = \frac{(m - 5) \times 9{,}8}{(8 - m) \times 9{,}8}$$

$$\frac{2}{1} = \frac{m - 5}{8 - K}$$

$m - 5 = 16 - 2m \qquad 3m = 21$

$\therefore m = 7$ kg

from (1) we find that

$7\,a = 9.8 \qquad a = 1.4\ m/sec^2.$

Third:

Suppose the maximum load that can be placed inside the lift of mass m kg

Hence, the tension in the wire carrying the lift is equal to 1200 kg.wt

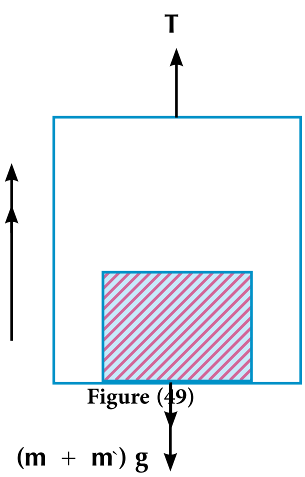


Figure (49)

Figure (13)

The equation of motion: $(m + m`)\,a = T - (m + m`)\,g$

$\therefore (m + 600) \times 1.4 = 1200 \times 9.8 - (m + 600) \times 9.8$

$\therefore (m + 600) \times 11.2 = 1200 \times 9.8$

$m + 600 = 1050 \qquad m = 450$ kg

Try to solve

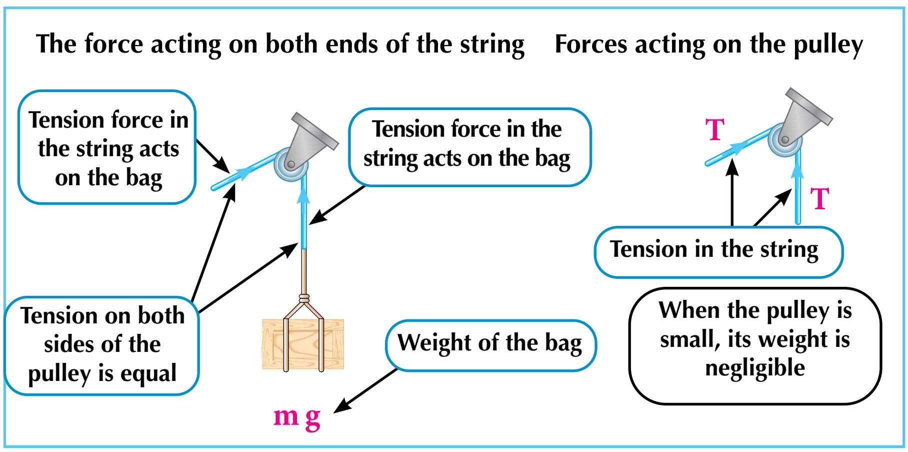
4. A body is suspended in a spring scale fixed at the top of a lift to record 17 kg.wt. When the lift moves up with a uniform acceleration 1.5 a m/ sec^2 and the scale records 16 kg.wt as the lift moves down with a the acceleration negative uniform of magnitude a m/sec^2. Find the mass of the body and the magnitude of a.

Second: Motion system of two bodies connected by a string passing over a smooth pulley

Introduction

Pulleys are used for a lot of purposes such as reducing the force needed to lift a body, ease the motion and to change the direction of a force. There are fixed pulleys and movable pulleys, in this unit. You are going to learn a system of pulleys of a constant pulley.

When the pulley is small and smooth, then the tension on both sides of the pulley is equal. The following Figure shows the forces acting as we lift a bag (body)



If two bodies of masses m_1, m_2 are connected by the two ends of an inelastic light string passing over a smooth small pulley, suspended vertically and $m_1 > m_2$, then the system starts to move from rest with a uniform acceleration of magnitude a.

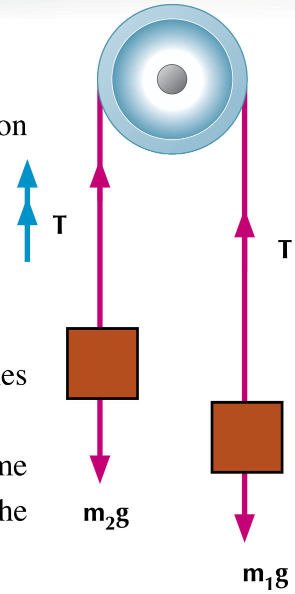
Equation of motion

$m_1 a = m_1 g - T$ $\qquad$ $m_2 a = T - m_2 g$

By adding the two equations by eliminating the tension, then acceleration of motion can be calculated:

$(m_1 + m_2) a = (m_1 - m_2) g$ $\qquad \therefore \quad a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

Thus, from any two equations, we find the tension T in the string



When the string is cut:

If the string connecting two bodies is cut after time t, then both bodies move in their same previous directions before the string is cut.

1) The mass m_1 moves downwards with initial velocity V (it is the same velocity at the moment the string is cut) and under the action of the gravitational acceleration.

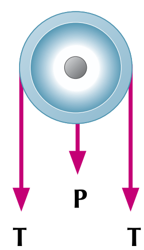
2) The mass m_2 moves upwards with initial velocity V(it is the same velocity at the moment the string is cut) until it reaches an instantaneous rest under the action of the gravitational acceleration, then it falls freely.

Notice that

If the system starts the motion and the two masses are in one horizontal plane and the distance covered after time of magnitude t is equal to S unit length, then the vertical distance between the two masses at the same time is equal to 2 S unit length

The pressure on the pulley

When the two masses are hanged up by the ends of the string passing over the pulley, the string gets tensioned due to the tension exerted on the string and a pressure force generated on the axis of the pulley is equal to the resultant of the two forces of the tension in the string. $\quad P = 2T$



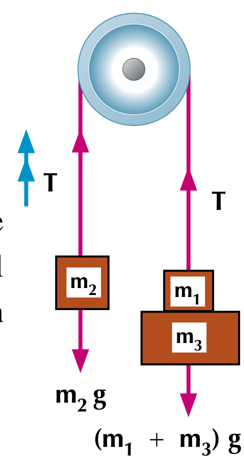
Similar cases (1)

In the drawn case:

$(m_1 + m_3) > m_2$, if $m_1 < m_2$ then the equation of motion

$(m_1 + m_3) a = (m_1 + m_3) g - T$

$m_2 a = T - m_2 g$



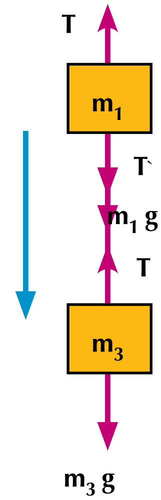
When the additional mass is separated

If the mass m_3 is separated after time t seconds, then the system moves in the same previous direction, but with a retarded acceleration (deceleration) until it instantaneously rests, then changes its direction to find the deceleration motion after the mass m_3 is separated, we find the equation of motion

$m_1 a` = m_1 g - T`$

$m_2 a` = T` - m_2 g$

After the mass m_3 gets separated, the system moves with initial velocity which is the velocity acquired at the moment of separation and reached the instantaneous rest, then changes the direction of its motion and rebounds to gets mass m_2 to be the leading.



The tension in the string between the two masses

In the previous Figure, if the two masses m_1 , m_3 are tied by another string then the tensions are as shown in the figure and the equations of motion are:

$m_1 a` = m_1 g + T` - T$

$m_3 a` = m_3 g - T`$

Similar cases (2)

If $m_1 = m_2 = m$

i.e. the two masses are equal in this case, the system does not move. But if a mass of magnitude m` is added to one mass of both, then the system moves in the direction of the two masses (m + m`) and the equations of motion

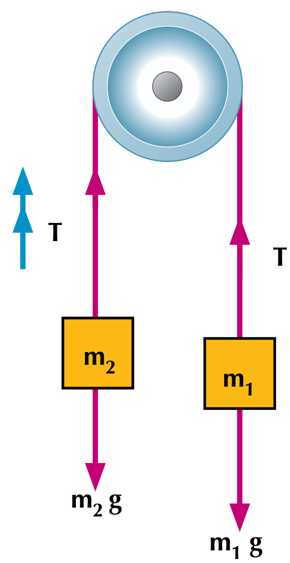
$(m + m`) a = (m + m`) g - T$

$m a = T - mg$

when the additional mass is separated:

If the additional mass m` is separated after time of magnitude t second, then the system moves in the same previous direction with a uniform velocity which is the velocity it acquired during t second t (the velocity at the moment of separating mass m`)

Similar cases (3)



If the two masses m_1 , m_2 are hanged up in the ends of a string and we do not know which of the two masses is heavier and we let the mass m_1 acquire a velocity of magnitude V downwards and the system moves, then we have three cases

1) If the system returns back to its original position after time t, then we deduce that $m_1 < m_2$ and the system moves with deceleration until it instantaneously rests, then changes the direction of its motion the acceleration of motion can be deduced from the given data where the initial velocity is the velocity which the mass m_1, acquired and the final velocity = 0, time = $\frac{t}{2}$

2) If the system moves uniformly with a constant velocity which is the velocity the mass m_1 acquired, we can deduce that the two masses are equal $m_1 = m_2$, and the motion belongs to Newton's first law.

3) If the system moves uniformly with an increasing acceleration, we deduce that $m_1 > m_2$, and the motion can be studied to find the equations of motion

$m_1 a = m_1 g - T$

$m_2 a = T - m_2 g$

Example

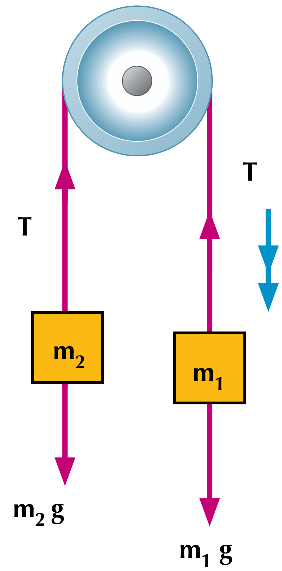
1 Two bodies of masses m_1, m_2, where $m_1 > m_2$ are connected by the ends of a string passing over a smooth pulley and the two bodies are at the same height from the ground at the beginning of motion, after one second, the vertical distance between them is 20 cm. Find $m_1 : m_2$

Solution

At the beginning of motion, the two bodies were in a one horizontal plane and after a second the vertical distance between them was 20 cm.

$\therefore S = \frac{20}{2} = 10$ cm $\qquad \because S = V_\circ t + \frac{1}{2} a t^2$

$10 = 0 + \frac{1}{2} \times a \times 1 \qquad a = 20$ cm/sec^2



equation of motion

$m_1 a = m_1 g - T \qquad m_2 a = T - m_2 g$

By adding, we find that

$(m_1 + m_2) a = (m_1 - m_2) g$

$20 (m_1 + m_2) = 980 (m_1 - m_2)$

$m_1 + m_2 = 49 (m_1 - m_2)$

$m_1 + m_2 = 49 m_1 - 49 m_2$

$50 m_2 = 48 m_1$

$\frac{m_1}{m_2} = \frac{25}{24} \qquad m_1 : m_2 = 25 : 24$

Try to solve

5 Two bodies of masses 21 gm and 28 gm are connected by the two ends of a string passing over a smooth small pulley. If the system moves from rest, find the acceleration of the set, the magnitude of the tension in the string, and the velocity of the system after two seconds from the beginning of motion.

Example

2 Two bodies of masses 105 gm and 70 gm are connected by the two ends of a light string of constant length passing over a smooth small pulley and suspended vertically. If the system starts to move from rest when the two masses are on one horizontal plane, find the magnitude of the acceleration of motion of the system. If the first body is impinged against the ground after it traveled 50 cm, find the total time taken by the second body from the beginning of motion until it instantaneously rests.

Solution

Equation of motion:

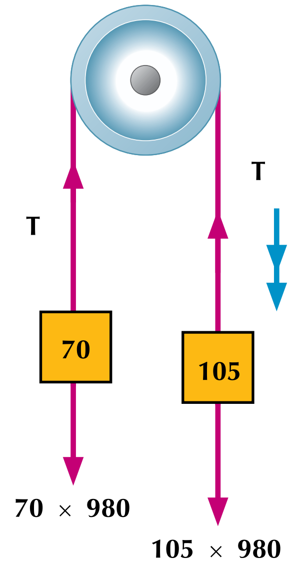
$105\,a = 105 \times 980 - T$

$70\,a = T - 70 \times 980$

By adding the two equations, we find that

$175\,a = 35 \times 980$

$a = 196 \text{ cm/sec}^2$



At the moment the body of mass 105 gm impings against the ground, it takes time t_1

$V2 = V_\circ^2 + 2\,a\,S$

$V2 = 0 + 2 \times 196 \times 50$

$V = 140$ cm/sec

$V = V_\circ + a\,t$

$140 = 0 + 196\,t$

$t = \frac{5}{7}$ seconds

When the body of mass 105 gm impings against the ground, the body of mass 70 gm, moves vertically upwards with a gravitational acceleration beginning with velocity $V_0 = 140$ cm/sec. to rest instantaneously after time t_2

$\because\ V = V_\circ + g\,t$

$\therefore\ 0 = 140 - 980\,t$

$t = \frac{1}{7}$ seconds

$\therefore$ The body of mass 70 gm takes time t to reach the instantaneous rest from the beginning of motion

where $t = t_1 + t_2 = \frac{5}{7} + \frac{1}{7} = \frac{6}{7}$ seconds

Try to solve

6. A light string passes over a fixed smooth pulley and a body of mass 90 gm is suspended in one end of the string while the other end is connected by a body of 70 gm. If the system starts moving from rest when the mass 90 gm is at a height of 245 cm from the ground, find:

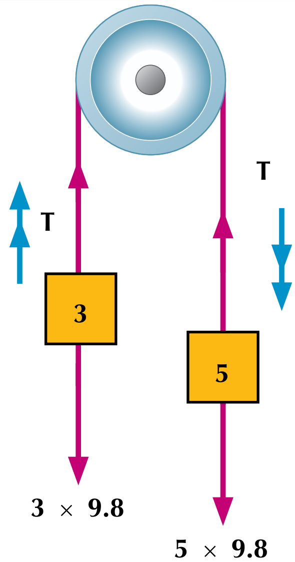
   a. The time taken until the mass 90 gm reaches the ground.

   b. The time taken until the string gets tensioned once more.

Example

3. Two bodies of mass 5 kg and 3 kg are connected by the two ends of a light string passing over a smooth pulley. The system starts to move from rest when the two bodies are in the same horizontal plane at a height of 245 cm on the ground and after one second from the beginning of motion, the string is cut. Find the acceleration of motion and the velocity of the two bodies as they reach the ground.

Solution



Equations of motion:

$5a = 5 \times 9{,}8 - T$ **(1)**

$3a = T - 3 \times 9{,}8$ **(2)**

By adding, we find that

$8a = 2 \times 9{,}8$

$\therefore a = 2{,}45 \text{ m/sec}^2$

at the moment when the string is cut

$V = V_\circ + at$

$= 0 + 2.45 \times 1 = 2.45$ m/sec

$S = V_\circ t + \frac{1}{2} a t^2$

$= 0 + \frac{1}{2} \times 2.45 \times 1 = 1.225$ meters

after the string is cut

the body of mass 5 kg moves vertically downwards

$V_\circ = 2.45$ m/sec , $g = 9.8 \text{ m/sec}^2$, $S = 2.45 - 1.225 = 1.225$ meters

$\because V^2 = V_\circ^2 + 2gS$

$\therefore V^2 = (2.45)^2 + 2 \times 9.8 \times 1.225$

$\therefore V = \frac{49\sqrt{5}}{20}$ m/sec

the body of mass 3 kg moves vertically upwards free from a point distant S from the ground surface to reach an instantaneous rest, then it turns back passing through the starting point of motion, then to the ground surface.

$V_\circ = 2{,}45$ m/sec , $g = -9{,}8 \text{ m/sec}^2$, $S = -(2{,}45 + 1{,}225) = -3{,}675$

$\because V^2 = v_\circ^2 + 2gS$

$= (2{,}45)^2 + 2 \times -9{,}8 \times -3{,}675$

$V = \frac{49\sqrt{13}}{20}$ m/sec

Try to solve

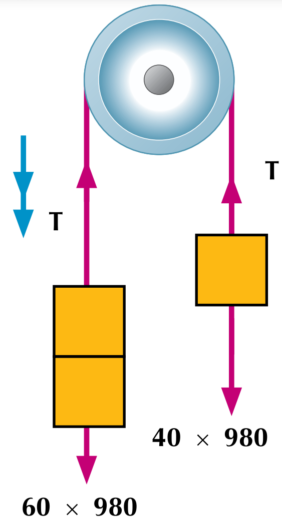
7) A light string of constant length passes over a fixed smooth small pulley and is connected to two masses 20, 12 gm suspended vertically. Find the acceleration of the motion of the system and the tension in the string if the system starts moving from rest and the string is cut after two minutes from the beginning of the motion. Determine the maximum height the mass 12 gm can reach from its original position at the beginning of motion.

Example

4) A light string passes over a smooth vertical pulley and a body of mass 40 gm is connected by one of its ends and two bodies the mass of each is 30 gm are connected by the other end of the string. The system is let to move from rest. After one second from the beginning of the motion, one of the two small masses is separated from the system. Find the distance traveled by the mass 49 gm from the beginning of the motion until it reaches an instantaneous rest.

Solution

Equations of motion:



$60\,a = 60 \times 980 - T$

$40\,a = T - 40 \times 980$

By adding the equations, we find that

$100\,a = 20 \times 980$

$a = 196 \text{ cm/sec}^2$

The moment of separating the small mass

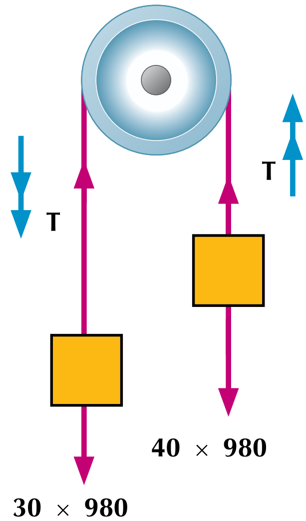
$V = V_\circ + a\,t$

$= 0 + 196 \times 1 = 196 \text{ cm/sec}$

$S_1 = V_0 t + \frac{1}{2} a\,t^2$

$= 0 + \frac{1}{2} \times 196 \times 1 = 98 \text{ cm}$

After the separation of the small mass - equations of motion



$40\,a` = T` - 40 \times 980$

$30\,a` = 30 \times 980 - T`$

By adding the equations, we find that

$70\,a = -10 \times 980$

$a = -140 \text{ cm/sec}^2$

i.e. the system moves in the same previous direction before the small mass is separated but with deceleration until it reaches the instantaneous rest after it covers a distance S_2, then changes its motion.

$\because V^2 = V_\circ^2 + 2\,a\,S$

$0 = (196)^2 - 2 \times 140\,S_2 \qquad \therefore S_2 = 137{,}2 \text{ cm}$

$\therefore$ the mass 40 gm moves upwards for a distance S before it rests instantaneously where S $= S_1 + S_2 = 235.2\text{cm}$

Try to solve

8) A light string passes over a smooth small pulley and carries two weights 235 gm and 20 gm connected together by a string in an end such that the weight 20 is placed below the weight 235 and in the other end of the string a weight of magnitude 235 gm is connected. Calculate the common acceleration if the system moves from rest. If the string carring the weight 20 gm is cut after the system covers a distance of 45 cm and the weight 235 descending at a distance of 90 cm above the ground then, calculate the time taken by this weight to reach the ground.

Exercises 2 - 2

Complete each of the following:

1. A body of mass 70 kg is placed on a pressure scale on the floor of a lift moving with a uniform acceleration 1.4 m/sec^2 downwards, then the reading of the scale is.................... kg.wt.

2. A body is suspended in the hook of a spring scale fixed at the top of a lift to record 390 kg.wt as the lift moves up:
   if the acceleration of motion is $-$ 70 cm/sec^2, then the mass of the body is.................... gm.
   If the mass of the body is 350 gm, then the acceleration of motion is.................... cm/sec^2.

3. A person stands on a pressure scale fixed at the floor of a lift the scale reads 75 kg.wt. As the lift moves up with acceleration a m/sec^2, and reads 69 kg.wt as the lift moves down with the same acceleration, then the real weight of the person is.................... kg.wt.

4. A child stands on a pressure scale inside a lift moving downwards with acceleration 1.4 m/sec^2.
   If the reading of the scale is 30 kg.wt, then the weight of the child is =....................kg.wt
   If the weight of the child is 49 kg.wt, then the reading of the scale is.................... kg.wt

Answer the following questions:

5. A person of mass 80 kg stands on a pressure scale fixed in the floor of a lift. Find the scale reading in each of the following cases:
   - a) The lift moves with a uniform velocity.
   - b) The lift moves upwards with negative acceleration of magnitude 44.1 cm/sec^2.
   - c) The lift moves downwards with an uniform acceleration of magnitude 29.4 cm/sec^2.

6. A body of mass m is suspended in a spring scale fixed at the top of a lift. Find m in each of the following cases:
   - a) The lift moves upwards with an uniform acceleration of magnitude 98 cm/sec^2 and the scale reading is 44 gm.wt.
   - b) The lift moves downwards with an uniform acceleration of magnitude 140 cm/sec^2 and the scale reading is 210 gm.wt.
   - c) The lift is at rest and the scale reading is 100 gm.wt.

7. An electric lift moves vertically upwards in a retarded motion with a uniform acceleration of magnitude a m/sec^2, and there is a spring scale fixed at its top carrying a body of weight 35 kg. If the apparent weight which the scale shows is of magnitude 30 kg.wt, find the value of a.

8. A body is placed on a pressure scale on the floor of a lift to record reading 14 kg.wt as the lift was at rest. Find in kg.wt the scale reading when it moves vertically upwards with a uniform acceleration of 70 cm/sec^2.

9. A body of mass 94.5 kg is placed in a box of mass 52.5 kg, then raised vertically upwards by a movable rope with an acceleration of magnitude 1.4 m/sec^2. Find the magnitude of the pressure of the body on the box base and the magnitude of the tension in the rope carrying the box. If the rope is cut, find the pressure of the body on the box base on that time.

10. An electrical lift of mass 350 kg.wt moves vertically downwards with a negative uniform acceleration of magnitude 49 cm/sec^2 and there is a person of weight 70 kg.wt inside it. Find the magnitude for each of the pressure of the person on the floor of the lift and the tension in the rope carrying the lift in kg.wt.

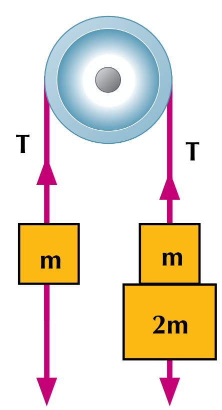
11. A body is suspended in a spring scale fixed at the top of a lift. The scale reading is 7 kg.wt when the lift is at rest. Then the scale reads 8 kg.wt when the lift moves vertically with a uniform acceleration. Find the magnitude and direction of the acceleration in which the lift moves.

12. A body is suspended in a spring scale fixed at the top of a lift if the scale reads 16 gm.wt, when the lift moves up with acceleration of uniform a cm/sec^2, and the reads 11 gm.wt when the lift moves downwards with acceleration of uniform 1.5 a cm /sec^2. Find the mass of the body and acceleration a, then calculate the scale reading when the lift moves downward with a acceleration uniform negative of magnitude $\frac{1}{2}$ a cm/sec^2.

Complete the following:

13. Two bodies the mass of each 3 kg, are connected by the ends of an inelastic light string passing over a smooth small pulley. If the system acquires a velocity of magnitude 2 m/sec, then:
   - **a** The acceleration a =..........
   - **b** The tension in the string T =.......... kg.wt
   - **c** The distance which one of the two masses travels within one second from the beginning of motion is.......... meters.

14. **In the opposite figure:** if the system moves from rest, then:
   - **a** The acceleration of the system =.......... m/sec^2
   - **b** The velocity of the system after 2 sec =.......... m/sec
   - **c** If the mass 2m is separated from the system after 2 seconds, then the systems moves after that with acceleration =..........
   - **d** The distance traveled by the mass m within 5 seconds from the beginning of motion =..........



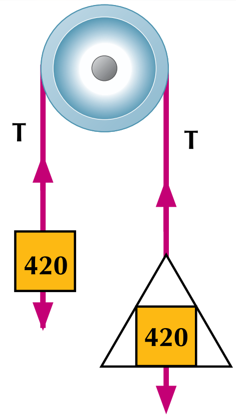
15 Two masses each of a magnitude 420 gm one of them is placed in a pan of mass 140 gm. and the system moves from rest, then:

a Acceleration =.................. cm/sec^2

b Tension in the string =.................. gm.wt

c Pressure on the axis of the pulley =.................. gm.wt

d Pressure on the pan =.................. gm.wt



16 In the opposite figure: two bodies of masses m and 2 m are connected by the ends of a string passing over a smooth small pulley and the system moves from rest when the two bodies are in a horizontal plane.

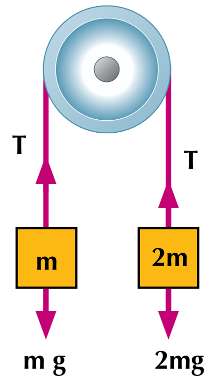
a Acceleration =.................. m/sec^2.

b Pressure on the pulley =.................. kg.wt.

c Velocity of the system after $\frac{3}{2}$ seconds from the beginning of the motion =.................. m/sec.

d Vertical distance between the two bodies after $\frac{3}{2}$ seconds from the beginning of motion =.................. meters.

e If the string is cut after $\frac{3}{2}$ seconds from the beginning of motion, then the mass m reaches the instantaneous rest after time of magnitude.................. seconds.



f If the distance between the two bodies after time t second as the string is cut becomes 12.25 meters, then t =.................. seconds.

If a smooth horizontal plane group moved from sleep

Answer the following questions:

17 In each of the following figures find:

a The acceleration.

b The tension in the string.

c The pressure on the pulley.

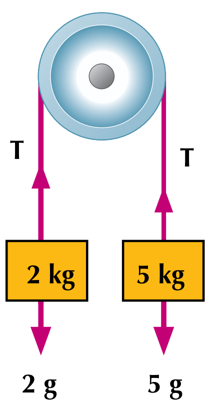


Figure (1)

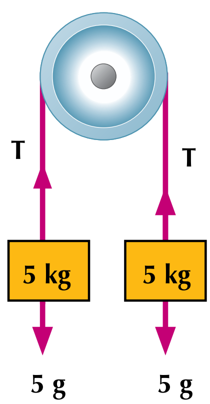


Figure (2)

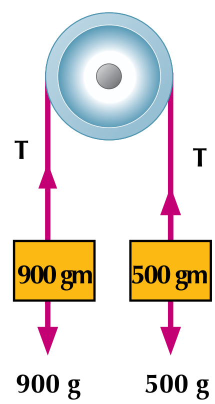


Figure (3)

18 Two bodies of masses 5 kg and 3 kg are connected by the ends of a string passing over a smooth small pulley and the system is kept in equilibrium and the two parts of the string are vertical. If the system is let to move, find the magnitude of its acceleration, and the pressure exerted on the pulley, then identify the velocity of the body whose mass is 5 kg when is lands 40 cm.

19 Two bodies of masses m_1, m_2 where $m_1 > m_2$ are connected by the ends of a string passing over a smooth pulley. If the system moves with acceleration 196 cm/sec^2, find $m_1 : m_2$.

20 Two masses 3m, m are connected by the ends of a light string passing over a smooth pulley and the system is kept in an equilibrium state and the two parts of the string are vertical. If the system is let to move from rest when the vertical distance between the two masses is 160 cm and the mass m is below the mass 3 m, find the time at which the two masses are in one horizontal plane.

21 Two pans each of mass 210 gm are connected by the ends of a light string passing on a smooth small pulley and suspended vertically. A body of mass 700 gm is placed in a pan and a body of mass 840 gm is placed in the other pan. Find the acceleration of the system and the pressure on both pans.

22 Two masses 5m, 2m are connected by the ends of a string passing over a smooth small pulley and the system is kept in an equilibrium state and the two parts of the string are vertical. If the system is let to move from rest, find the acceleration of the system. If the pressure on the axis of the pulley is equal to 112 Newtons, find the value of m.

23 Two bodies of masses 420 gm and 560 gm are connected by the ends of a light string passing over a smooth small pulley. The system starts to move from rest when the two bodies are in the same horizontal plane. After passing one second, the string connecting them is cut. Calculate the distance between the two masses after passing another second from the moment of cutting the string.

Unit Two

2 - 3

Impulse

You will learn

- The concept of impulse.
- Deduce the relation between the impulse and the change of momentum.

Key terms

- Impulse
- Momentum
- Impulsive forces

Materials

- Scientific calculator

Preface:

- Throwing a ball in the direction of a vertical wall.
- Colliding of cars on the highways.
- Colliding the wheel of planes like landing in airports.

In such cases, the study of the motion of the body is an extremely difficult process due to the overlapping of the acting factors on them and the shortness of the infinitesimal time intervals. In this unit, you are going to learn some information specially related to this topic to relate the position of a body before and after the occurrence of change of its velocity vector through this activity.

Activity

Materials: a wooden ruler of length more than 1 meter and a group of different balls such as a golf ball, tennis ball, billiard ball, clay ball,...

Construction: Let these balls fall one after another from a constant height. Let it be 2 meters on the roof of marble or ceramics, then record the height which each ball rebounds back.

Type of ball	Height	Maximum rebound
Glass ball	2 m	....
Billiard ball	2 m	....
Tennis ball	2 m	....
Wood ball	2 m	....
Golf ball	2 m	....
Bowling ball	2 m	....
Tennis ball	2 m	....
Play ball	2 m	....

Observation and deduction: Have you noticed differences in the heights in which the different balls rebounded? Can you order the balls according to the rebound for each in a descending order?

The difference of the distance of rebound is related to several factors such as the change of momentum due to the impact (collision) of the ball with the ground.

Search in the internet for impulse and momentum.

First: impulse

If force $\vec{F}$ of a constant magnitude acts on a body during a time interval t , then the impulse of this force

It is denoted by the symbol t - is known as the product of force vector by the time of its action i.e

$$\vec{I} = \vec{F}\ t$$

According to such a definition, it is clear that the impulse $\vec{I}$ is a vector in the same direction as the force vector $\vec{F}$:

$$I = F t$$

The relation between the algebraic measure of impulse $\vec{I}$ and the algebraic measure of the force $\vec{F}$ can be written as follows:

The measuring units of the magnitude of impulse:

From the definition of the impulse, we find that:

The measuring unit of magnitude of impulse = measuring unit of magnitude of force × time measuring unit

In the international system of units, the magnitude of the impulse is measured in Newton. sec (N. sec) It can also be measured by the product of any force by any time unit.

Furthermore, the measuring units of the magnitude of impulse can be expressed in a different method with regarding that:

kg.wt.sec , gm.wt.sec , ...

So, we find that: if the mass is in kg and the velocity is in m /sec, then the impulse magnitude unit is kg.m/sec and it is the same N. sec unit.

When the mass is in gm and the velocity is in cm/sec then the impulse magnitude unit is gm.cm/sec and it is the same dyne.sec unit

Example Definition of the impulse

1 A force of magnitude 25 kg.wt acts on an object for a time interval of magnitude $\frac{1}{10}$ of second. Find the impulsive force on the body in N.sec unit.

Solution

Impulse $= F \cdot t = 25 \times 9{,}8 \times \frac{1}{10} = 24.5$ N.sec

Try to solve

1 A force of magnitude 10^{12} dyne acts on a body for a time interval 10^{-5} seconds, find the force impulse on the body in N.sec

Example Finding the magnitude of impulse

2 The forces $\vec{F_1} = 4\hat{i} - 3\hat{j} + \hat{k}$, $\vec{F_2} = \hat{i} + 2\hat{k}$, $\vec{F_3} = 4\hat{j} - \hat{k}$ act on a body for a time interval of magnitude 5 seconds. Find the magnitude of the impulsive force on the body if the magnitude of the forces is measured in Newton unit.

Solution

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$= (4\hat{i} - 3\hat{j} + \hat{k}) + (\hat{i} + 2\hat{k}) + (4\hat{j} - \hat{k}) = 5\hat{i} + \hat{j} + 2\hat{k},$$

$$\because \vec{I} = \vec{F} \times t = 5(5\hat{i} + \hat{j} + 2\hat{k})$$

Magnitude of impulse $= \sqrt{(25)^2 + (5)^2 + (10)^2} = 5\sqrt{30}$ N.sec

Try to solve

(2) The force $\overrightarrow{F_1} = 2\hat{i} + 3\hat{j}$, $\overrightarrow{F_2} = \hat{j} - 5\hat{i}$ act on a body for one second. Find the force impulse on the body if the force magnitude is measured in Newton unit.

Second: impulse and momentum

Remember

$\because$ $v = v_o + a t$

$\therefore$ $v - v_o = a t$

Since the impulse of a force F of a constant magnitude on a body for a time interval t is equal to F t and from Newton`s second law, we find that:

impulse = m a • t

$\therefore$ impulse = m (v - v_o)

Where v_o and v are the two algebraic measures to the two vectors of the initial velocity and velocity after time t respectively.

i.e the impulse is equal to the change of momentum. But if the force is variable, then the impulse is given by the following integration :

$$\text{impulse} = \int_0^t F\,dt$$

$$\therefore \int_0^t F\,dt = \int_0^t m a \,dt$$

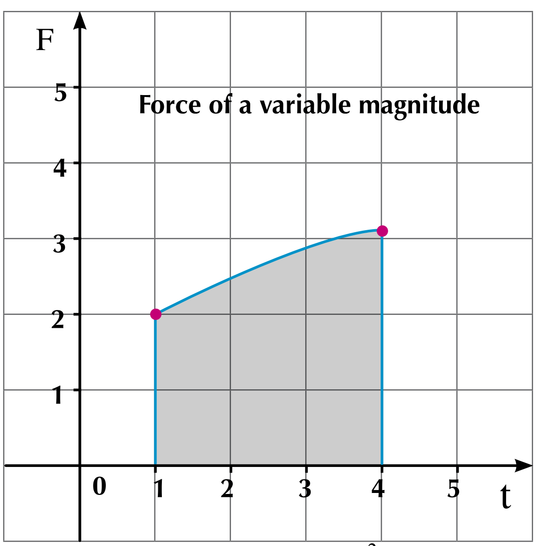
$$\int_0^t F\,dt = m \int_0^t \left(\frac{dv}{dt}\right)dt$$

$$= m \int_{v_o}^{v} dv$$

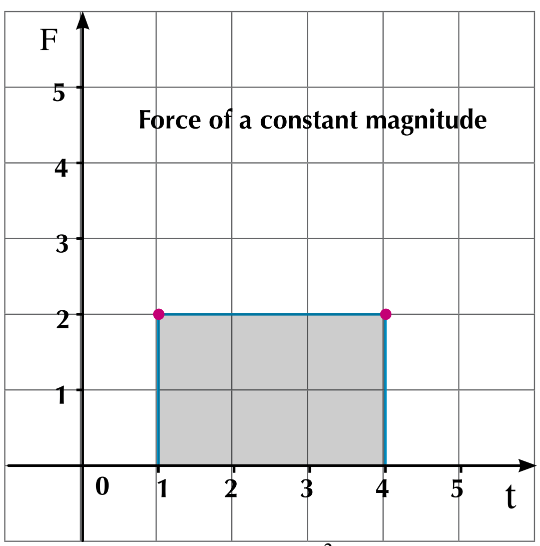
$$\int_0^t F\,dt = m\,[\,v\,]_{v_o}^{v}$$

$$\int_0^t F\,dt = m\,(v - v_o)$$

In general, impulse is equal to the change of momentum



Impulse = $\int_{t_1}^{t_2} F\,dt$



Impulse = $\int_{t_1}^{t_2} F\,dt$

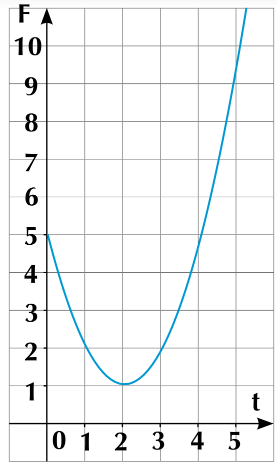
Example

3 The opposite figure represents the force-time graph where $F = 1 + (t - 2)^2$. Find:

a The impulse of the force F within the first three second.

b The impulse of the force F in the fifth second.

Where the magnitude of the force F is in Newton and the time t in second



Solution

$F = 1 + (t - 2)^2$ $\qquad F = t^2 - 4t + 5$

a Impulse within the first three seconds

$$= \int_0^3 F\,dt = \int_0^3 (t^2 - 4t + 5)\,dt = \left[\frac{1}{3}t^3 - 2t^2 + 5t\right]_0^3 = 6 \text{ N.sec}$$

b Impulse within the first fifth seconds

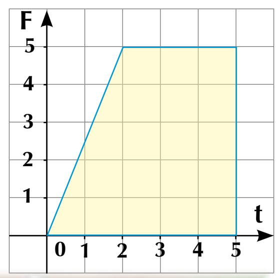
$$= \int_4^5 F\,dt = \int_4^5 (t^2 - 4t + 5)\,dt = \left[\frac{1}{3}t^3 - 2t^2 + 5t\right]_4^5 = \frac{22}{3} \text{ N.sec}$$

Try to solve

3 The opposite figure represents the force-time graph. Find using the integration:

a The impulse of the force F within the first second.

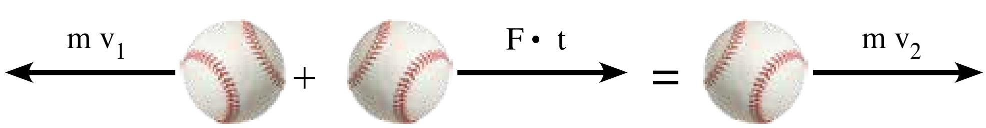
b The impulse of the force F within the first five seconds where the force F is in Newton and the time t is in second.



The impulsive forces

The impulse forces are extremely tremendous forces acting for infinitesimal time interval and causing an extremely tremendous change in the momentum of the body without any acting on its position. The resulted motion at the action of these forces is called an impulsive motion. For example, the baseball when hit by a bat, the contact time between the bat and the ball is extremely infinitesimal although the average of the force acting on the ball is extremely tremendous. The impulse is tremendous enough to change the momentum of the ball with out any change in the position of the ball.

When an impulsive force acts on a body then $m\,v_1 + F \cdot t = m\,v_2$ where t is an extremely infinitesimal time interval.



Example — Impulse and momentum

4 A body of mass 4 kg placed at rest on a smooth horizontal plane and is acted upon by a horizontal force of magnitude 5 Newtons for 8 seconds. Find the impulse magnitude on the body and the velocity magnitude of the body after 8 seconds.

Solution

$\because$ impulse $= F \cdot t$

$\therefore$ impulse $= 5 \times 8 = 40$ N.s

$\because$ impulse = The change of momentum

$40 = m(v - v_\circ)$

$40 = 4(v - 0)$

$v = 10$ m/sec

Try to solve

4 A constant force of magnitude F acts on a body of mass m for $\frac{1}{49}$ of a second to change its velocity from 3 m/sec to 54 km/h in the force direction and the force impulse is equal to 4.8 N.sec. Find the mass of the body and the force magnitude in kg.wt.

Example — Expressing the impulse and momentum using vectors

5 The force $\vec{F} = 2\hat{i} + 7\hat{j}$ acts on a body of mass 5 kg for 10 seconds when its velocity vector $\vec{v} = \hat{i} - 2\hat{j}$. Find its velocity after the action of the force if the force magnitude is in Newton unit and the velocity is in m/sec unit.

Solution

$\because$ impulse = change of momentum

$\therefore \vec{F} \cdot t = m(\vec{v} - \vec{v}_\circ)$

$\therefore 10(2\hat{i} + 7\hat{j}) = 5(\vec{v} - \vec{v}_\circ)$ $\qquad \therefore \vec{v} - \vec{v}_\circ = 2(2\hat{i} + 7\hat{j})$

$\therefore \vec{v} = 2(2\hat{i} + 7\hat{j}) + (\hat{i} - 2\hat{j})$

$\therefore \vec{v} = 4\hat{i} + 14\hat{j} + \hat{i} - 2\hat{j}$ $\qquad \therefore \vec{v} = 5\hat{i} + 12\hat{j}$

$\therefore \|\vec{v}\| = \sqrt{25 + 144} = 13$ m/sec

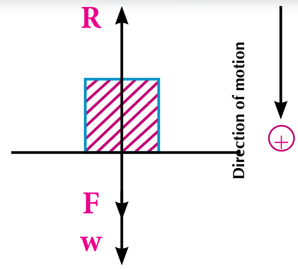
Try to solve

5 A body of mass 3 kg moving with velocity $\vec{v} = 5\hat{i} - 2\hat{j}$, is acted upon a constant force for a time interval t and the force impulse on the body is equal to $6\hat{i} + 9\hat{j}$. Find the velocity of the body after the action of the force if the velocity is in m/sec, unit and the impulse magnitude is in N.sec unit

Notice that

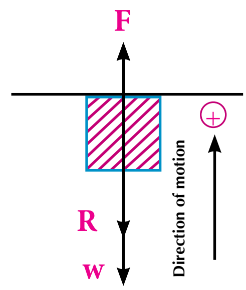
- When a body of weight «w» falls vertically on the ground, **then:** at the moment of contact

  The pressure of the body on the ground = reaction of the ground on the = F + w



- When a body of weight «w» is projected vertically and collides with the ceiling of a room, **then:**

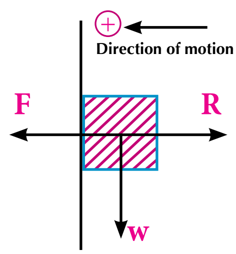
  The pressure of the body on the ceiling = reaction of the ceiling on the body = F - w



- When a body of weight « w » is projected horizontally and collides with a vertical wall, **then:**

  The pressure of the body on the wall = reaction of the wall on the body = F

  Where F is the magnitude of the impulsive force in each of the cases above.



Example **The vertical motion**

6 A rubber ball of mass $\frac{1}{4}$ kg fell down from a height of 10 meters above the ground and rebounded after collided with the ground for a height of 2.5 meters. Find the impulse resulted from the collision (impact) of the ball with the ground and identify the reaction of the ground on the ball if the contact time of the ball with the ground is $\frac{1}{10}$ of second.

Solution

Studying the phase of falling down

$\because v^2 = v_{.}^{2} + 2g\,s$

$\therefore v_1^2 = 0 + 2 \times 9.8 \times 10$

$\therefore v_1 = 14$ m/sec

It is the velocity of the ball be for it contacts directly with the ground.

Studying the phase of rebound back

$\because v^2 = v2. + 2g\,s$

$\therefore 0 = v_2^2 - 2 \times 9{,}8 \times 2{,}5$

$\therefore v_2 = 7$m/sec

$\therefore$ The velocity of rebound back = 7m/sec vertically up wards

impulse = Change of momentum = $m(v_2 - v_1)$

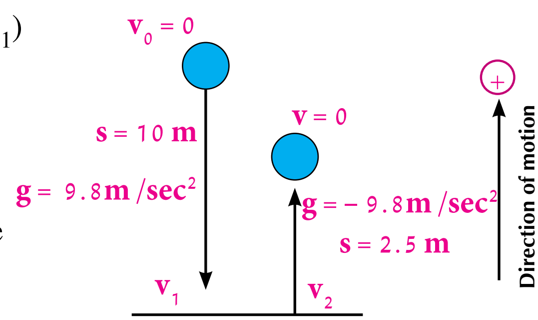
$= \frac{1}{4}[7 - (-14)] = 5.25$ kg . m/sec

$\because$ impulse = F . t $\quad \therefore 5.25 = F \times \frac{1}{10}$

$\therefore F = 52.5$ Newton

The reaction of the ground on the ball = impulsive force + weight of the ball

$= 52.5 + \frac{1}{4} \times 9.8 = 54.95$ Newton



Try to solve

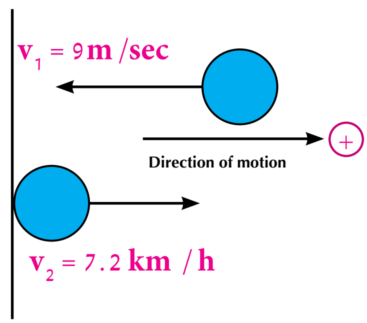
6 A body of mass 300 gm is projected vertically upwards with velocity 840 cm/sec from a point placed under the ceiling of a room of magnitude 110 cm to collide (impact) with the ceiling and bounds down to the roof of the room after $\frac{1}{2}$ of second of the rebound. Find the impulse of the ceiling to the body given that the ceiling height is 272.5 cm. If the contact time is $\frac{1}{10}$, find the impulsive force.

Critical thinking: A clay ball of mass 1kg fell down from a height of 40cm on a pressure scale and the collision (impact) time is $\frac{1}{7}$ of seconds, find the scale reading given that the ball did not rebound after the impact.

Example Horizontal motion

7 A ball of mass 100 gm moves horizontally with velocity 9 m/sec to collide with a vertical wall and rebound back with velocity of magnitude 7.2 km/ h If the contact time of the ball with the wall is $\frac{1}{10}$ of second, find the impulse of the wall to the ball and then find the pressure of the ball on the wall.

Solution



Let the rebound direction be the positive direction of motion

$\therefore\ v_1 = -9\text{m/sec}\ ,\ v_2 = 7.2 \times \frac{5}{18} = 2\text{m/sec}$

$\because\ I = m\,(v_2 - v_1)$

$\therefore\ I = \frac{100}{1000}\,[\,2 - (-9)] = 0{,}11\ \text{gm. m/sec}$

$\because\ I = F \times t$

$\therefore 0.11 = F \times \frac{1}{10}$

$\therefore\ F = 1{,}1$ Newton = the pressure of the ball on the wall

Try to solve

7 A tennis ball of mass 40 gm moves horizontally with velocity of 50 cm/sec to collide with the bat and rebounds in the opposite direction with velocity of 110 cm/sec . Find the impulse magnitude of the bat on the ball. What is the magnitude of the impulsive force of the bat on the ball. If the contact time of the ball with the bat is $\frac{1}{49}$ of a second?

Exercises 2 - 3

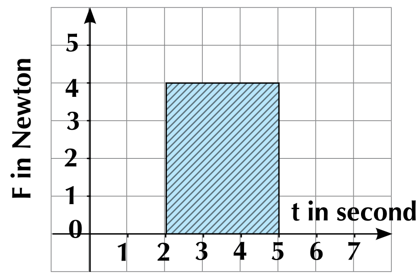
First : Choose the correct answer:

1. If a force of magnitude 16 kg.wt acts on a body for four seconds, then the magnitude of the force impulse in N.sec unit is equal to.
   - a) 4
   - b) 39.2
   - c) 49
   - d) 64

2. If the magnitude of the impulse of the force F on a body for 10^{-4} seconds is equal to 10 N.sec, then the magnitude of F is equal to:
   - a) 10^3 dynes
   - b) 10^5 dynes
   - c) 10^3 Newton
   - d) 10^5 Newton

3. If the two forces $\overrightarrow{F_1} = \hat{i} + 5\hat{j} + 7\hat{k}$ and $\overrightarrow{F_2} = 2\hat{i} - \hat{j} - 2\hat{k}$ which F_1, F_2 of measured Newton unit act on a body for a time interval of magnitude 2 seconds, then the magnitude of the impulse of the force in N.sec unit is equal to:
   - a) $5\sqrt{2}$
   - b) $10\sqrt{2}$
   - c) $50\sqrt{2}$
   - d) $100\sqrt{2}$

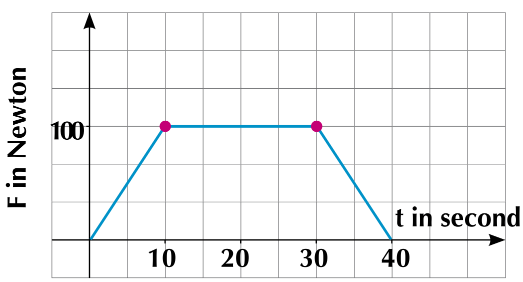
4. If a force of constant magnitude acts up on a body for a time interval as given in the figure, then the impulse magnitude in N.sec unit is equal to:



   - a) 8
   - b) 12
   - c) 20
   - d) 50

5. If a force of magnitude 90 Newton acts upon a body of mass 10 kg for five seconds, then the magnitude of the change of the velocity of the body in the same direction of the force is equal to:
   - a) 45m/sec
   - b) 50m/sec
   - c) 90m/sec
   - d) 120 m/sec

6. A body of mass 20 kg is placed on a smooth horizontal plane. If it moves under the action of a force in a constant direction and its magnitude changes over time as shown in the figure, then the impulse magnitude of this force after 40 seconds in N.sec unit is equal to:



   - a) 1000
   - b) 2000
   - c) 3000
   - d) 4000

Second : Answer the following questions :

7 A bullet of mass 20 gm is shot horizontally from a gun. If its path inside the gun continues for 0.5 of a second and the magnitude of the impulsive force of the gun on the bullet is 20 Newton, find the velocity of the bullet`s exit from the barrel of the gun.

8 A fast shooting mortar shoots bullets each of mass 500 gm vertically upwards. If the average of the impulsive force of the gas in the mortar`s cylinder on the ballet is 250 Newton and acts on the bullet for 0.2 of a second until the moment the bullet exits from the barrel of the mortar, calculate the speed of the bullets of the barrel of a gun.

9 A rubber ball of mass 20 gm fell down from a height of 6.4 meters above the ground to rebound vertically upwards. If the average force which the ground exerts (does) on the ball is 182×10^4 dynes and the contact time of the ball with the ground is 0.02 of seconds, find:

a The magnitude of the impulse of the ground to the ball

b The maximum height the ball reaches after it rebounds

10 A smooth ball of mass 200 gm moves in a straight line on a smooth horizontal ground with velocity 10 m/sec. If the ball collided with a smooth vertical wall and rebounded with velocity 4m/sec, Find:

a The magnitude of impulse of the wall on the ball.

b The magnitude of the impulsive force of the wall if the contact time of the ball on the wall is 0.05 of a second.

11 A train car of mass 10 tons moves with velocity 18 km/h to collide (impact) with a barrier and rebounds with velocity 9 km/ h. Find the magnitude of the impulse of the barrier on the train car.

12 A car at rest, of mass 1 ton is pushed in the direction of its motion with a force of 200 kg.wt for 5 seconds, then it is released freely to become at rest again after 15 seconds. Find the magnitude of the resistance supposing it is constant in the two cases. Then find the maximum velocity that the car reached using the relation between the impulse and momentum.

13 A ball of mass 1 gm is projected vertically upwards and in the direction of a ceiling of height 360 cm from the projection point with velocity of magnitude 14 m/sec. If the ball collides with the ceiling and rebounds back with velocity 10 m/sec, find the impulsive force of the ceiling on the ball if the contact time of the ball with the ceiling is 0.02 of a second.

14 A fast shooting mortar shoots horizontal 600 bullets each of mass 39.2 gm a minute with velocity 1260 km/h. Calculate the force of reaction acting on the mortar in kg.wt.

15 A ball of mass 1500 gm falls down from a height of 2.5 meters on a viscous liquid surface to embed in it with a uniform velocity and travels for a distance of 70 cm in 0.2 of a second. Calculate the magnitude of the impulse of the liquid on the ball.

16 The forces $\overrightarrow{F_1} = a\,\hat{i} - \hat{j}$, $\overrightarrow{F_2} = 3\,\hat{i} + b\,\hat{j}$, $\overrightarrow{F_3} = a\,\hat{i} + 2\,\hat{j}$ act on a body for $\frac{1}{2}$ of a second and the impulse of this force on the body is given by the relation $\overrightarrow{I} = 2\,\hat{i} + 4\,\hat{j}$, Find the value of a , b.

17 A body of mass 20 gm falls down from a height of 40 cm above a pond surface to embed in water for a distance 210 cm within one second with acceleration 2.1m/sec^2. Find the impulse of water on the body. As a result of colliding surface water.

Work, Energy & Power

Unit 3

Introduction

In our study to the previous units, you found that when the resultant of a system of forces acts on a body, it moves in different forms. If you ask about the benefits of moving a body, the answer would be in two parts; first, humans are naturely curious and always eager to interpret the natural phenomena, their reasons and results.

Second, the humans need to benefit from the blessing which God gifts the people. People for example need cars to travel from a place to another, they need the light bulbs to lighten cities and villages and so on. Of course, these matter can only be achieved when people know how to control the objects and benefit from their motion whether these objects are electric or electronic devices, means of transportation or even celestial bodies cause the Earth`s revolution and rotation and the succession of day and night. In this unit, you are going to learn the motion of the bodies and identify the work to know how to get the best benefit of moving the bodies. You also identify the kinetic energy and potential energy to relate these scalar quantities (non - vectored) of different measuring units and the relation among them. Finally you identify the force which conserve the energy and the forces which do not conserve it to explore the principle of work and energy, then you identify the simple machines which humans had used and compare them by calculating the power resulted in each one and their different benefits in your daily life.

Unit objectives

At the end of the unit and carrying out the involved activities, the student should be able to:

- Identify the work done by a force and the work measuring units.
- Identify the concept of the power and its measuring units.
- Identify the kinetic energy of a particle and its measuring units.
- Identify the principle of work and energy.
- Identify the potential energy، its measuring unit، and applications.

Key terms

- Work
- Constant force
- Scalar quantity
- Displacement vector
- Position vector
- Joule
- Variable force
- Erg
- Kinetic energy
- Potential energy
- Change in potential energy
- The work – energy principle
- Power
- Horse power
- Conservation of energy

Lessons of the unit

(3 -1): Work

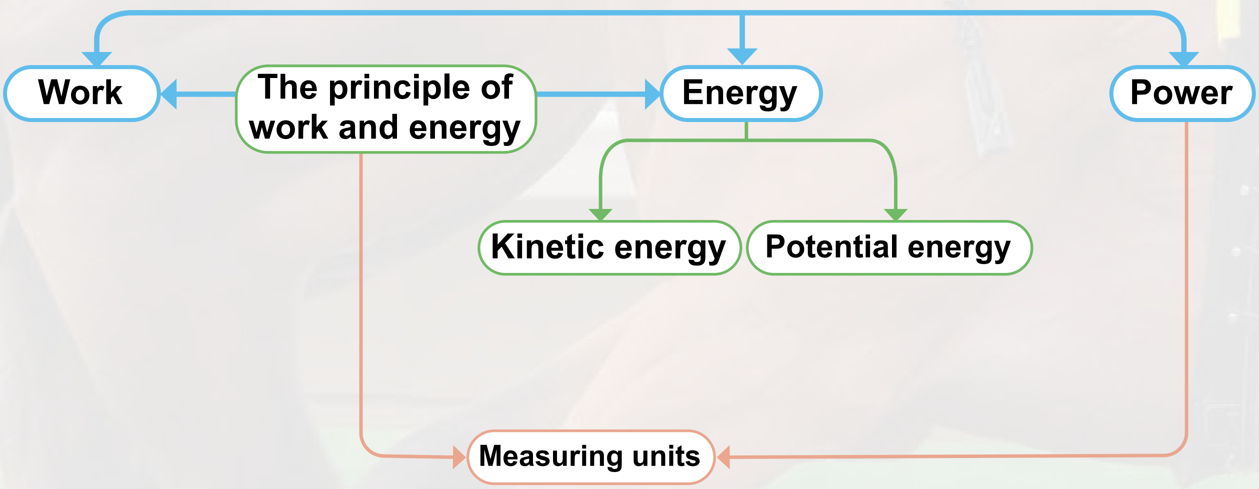
(3 -2): Energy

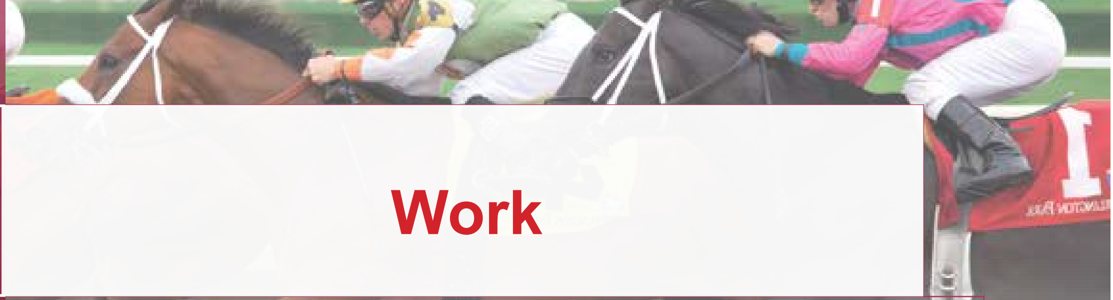
(3 -3): Power

Materials

Scientific Calculator .

Unit planning guide





Unit Three

3 - 1

Work

You will learn

- The work done by a constant force.
- Some different cases of the two vectors of force and displacement.
- Work measuring units.
- Work done by a variable force.

Key terms

- Work
- Constant force
- Scalar quantity
- Displacement vector
- Position vector
- Joule
- Erg

Materials

- Scientific calculator

Interoduction:

The concept of work is one of the important topics in Kinetic since it relies on the concept of the force stated by Newton in his three laws. It is worth to mention that the work and energy are scalar quantities and then it is easier to deal with them than using Newton`s laws of motion, especially when the force vector is variable and in turn, the acceleration vector is also variable. In this lesson, we are going to clarify the concept of work which is considered the link between the force and energy. work may be resulted from a constant force or a variable force. in this lesson, you are going to learn both types.

First: The work done by a constant force

Let a body move in a straight line under the action of a constant force $\overrightarrow{F}$ and it moves from position A to position B and its displacement vector is $\overrightarrow{AB} = \overrightarrow{S}$ as shown in figure (1)

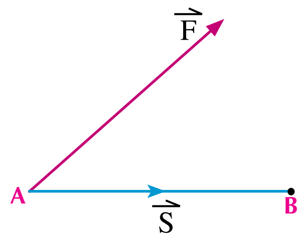


Figure (1)

Definition

The work done by the constant force $\overrightarrow{F}$ to move a particle from an initial position to a final position and denoted by the symbol (W) is known as the scalar product of the force vector by the displacement vector between the two position

$$W = \overrightarrow{F} \cdot \overrightarrow{S}$$

It is clear that the work is a scalar quantity that may be positive, negative or zero according to the magnitude and direction of both vectors $\overrightarrow{F}$, $\overrightarrow{S}$

Example

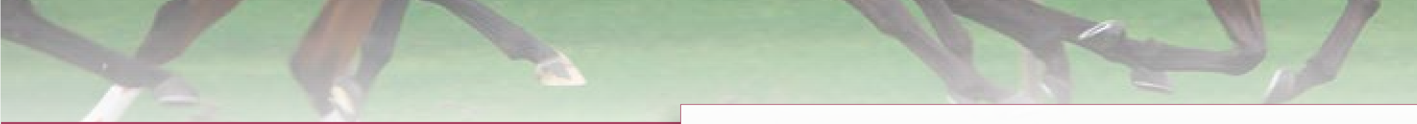
(1) A particle moves in a straight line under the action of the force $\overrightarrow{F} = 6\hat{i} + 8\hat{j}$ from point A (3 , -4) to point B (7 , 2), calculate the work done by this force.

Solution

The displacement vector $\overrightarrow{S} = \overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$

$= (7\hat{i} + 2\hat{j}) - (3\hat{i} - 4)$

$$\overrightarrow{S} = 4\hat{i} + 6\hat{j}$$



Apply the definition of work and notice that the force given is constant

$$W = \overrightarrow{F} \cdot \overrightarrow{S}$$

$$W = (6\,\hat{i} + 8\,\hat{j}) \cdot (4\,\hat{i} + 6\,\hat{j}) = 6 \times 4 + 8 \times 6 = 72 \text{ work measuring unit}$$

Try to solve

1. A body moves in a straight line under the action of the force $\overrightarrow{F} = 5\,\hat{i} + 2\,\hat{j}$ from the point A (5 , 2) to the point B (3 , 1), calculate the work done by this force

Some different cases of the vectors of the force and displacement

Since the equation of defining the work $W = \overrightarrow{F} \cdot \overrightarrow{S}$ can be rewritten in another form which is $W = \| \overrightarrow{F} \| \, \| \overrightarrow{S} \| \cos \theta$ where θ is the measure of the angle between the force vector $\overrightarrow{F}$ and the displacement vector $\overrightarrow{S}$ by considering them drawn from the same point.

a If the force is constant and its direction is parallel to the direction of displacement i.e θ = zero, on that moment, the work $W = \| \overrightarrow{F} \| \, \| \overrightarrow{S} \| \cos \text{zero} = \| \overrightarrow{F} \| \, \| \overrightarrow{S} \|$

and written: $W = F \times S$

Figure (2) illustrates that

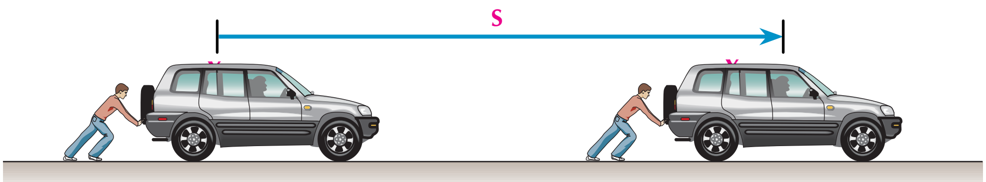


Figure (2)

b If the force is constant and its direction inclines on the direction of the displacement with an angle of measure less than 90°, then the work

$$W = \| \overrightarrow{F} \| \, \| \overrightarrow{S} \| \cos \theta$$

The work in this case is positive and equal to the horizontal component of the force F multiplied by the distance S.

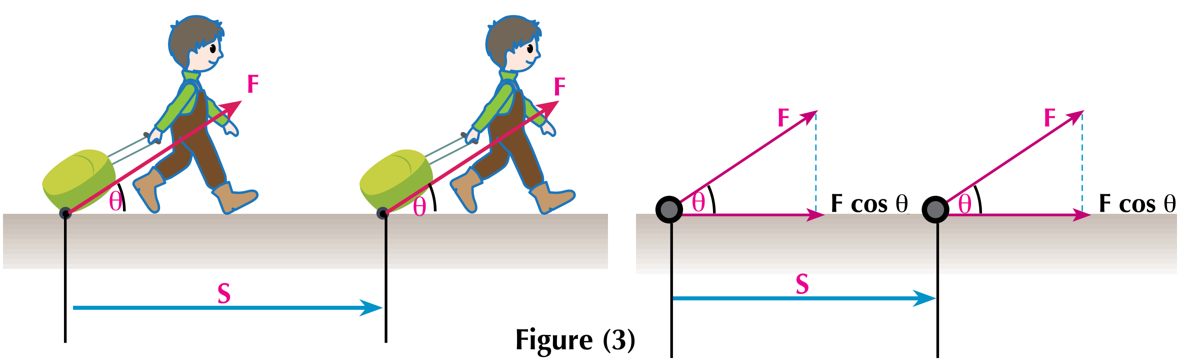


Figure (3)

a If the force is constant and its direction is perpendicular to the direction of displacement i.e $m(\angle\theta) = 90^\circ$, then the work $W = \|\overrightarrow{F}\| \, \|\overrightarrow{S}\| \cos 90 = \text{zero}$

Figure (4) illustrates that.

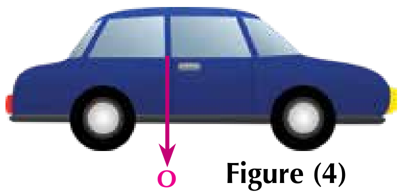


Figure (4)

The car moves horizontally, its weight does not do any work in the motion pathway

d If the force is constant and its direction inclines at 90° to the direction of displacement, then the work $W = \|\overrightarrow{F}\| \, \|\overrightarrow{S}\| \cos\theta$

and the work is negative and called work resistant. For example, the work done by the resistance force and friction force.

Example

(2) A particle moves in a straight line under the action of the force $\overrightarrow{F} = 5\hat{i} - 3\hat{j}$ from point A (1 , 0) to point B (3 , 3) where coordinates are referred to a system of rectangular Cartesian coordinates $\overrightarrow{OX}$, $\overrightarrow{OY}$. identify the work

Solution

Figure (5) illustrates the position of the two point A , B referred to the axes.

To calculate the displacement vector $\overrightarrow{S}$:

$\overrightarrow{S} = \overrightarrow{OB} - \overrightarrow{OA}$ **(Rule of subtracting vectors)**

$\therefore \overrightarrow{S} = (3 - 1)\hat{i} + (3 - 0)\hat{j}$

$= 2\hat{i} + 3\hat{j}$

$\because W = \overrightarrow{F} \cdot \overrightarrow{S}$ **(Work definition)**

$= (5\hat{i} - 3\hat{j}) \cdot (2\hat{i} + 3\hat{j})$

$= 5 \times 2 + (-3) \times (3) = 1$ Work unit.

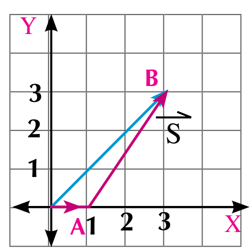


Figure (5)

Try to solve

(2) A particle moves under the action of two forces $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = 5\hat{i} + \hat{j}$ from point A (2 , 1) to point B (3 , 0) where $\hat{i}$, $\hat{j}$ are the main two unit vectors. Calculate the work done.

Critical thinking:

Prove that if two subsequent displacements occur to a body under the action of a force, then the work done within the resultant displacement is equal to the sum of the two works done within the two displacements.

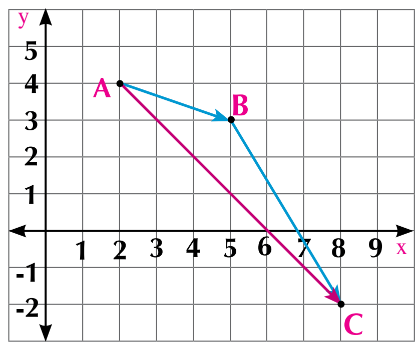
Example

(3) The force $\overrightarrow{F} = 3\hat{i} + 5\hat{j}$ acts on a body and moves from point A (2, 4) on a straight line to point B (5 , 3), then to point C (8, -2). Calculate the work done by this force during each of the two displacements, then check if the sum of the two works is equal to the work done during the resultant displacement.

Solution

First: The first displacement vector $\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = (5, 3) - (2, 4) = (3, -1)$

The work done during the first displacement



$W_1 = \overrightarrow{F} \cdot \overrightarrow{S_1} = (3\hat{i} + 5\hat{j}) \cdot (3\hat{i} - \hat{j})$

$W_1 = 9 - 5 = 4$ a work unit

The second displacement vector $\overrightarrow{BC} = \overrightarrow{C} - \overrightarrow{B}$

$= (8, -2) - (5, 3) = (3, -5)$

The work done during the second displacement

$W_2 = \overrightarrow{F} \cdot \overrightarrow{S_2} = (3\hat{i} + 5\hat{j}) \cdot (3\hat{i} - 5\hat{j})$

$W_2 = 9 - 25 = -16$ a work unit

The resultant work = sum of two works

$W = W_1 + W_2 = 4 - 16 = -12$ a work unit

Second: The resultant displacement $\overrightarrow{AC} = \overrightarrow{C} - \overrightarrow{A} = (8 , -2) - (2 , 4) = (6 , -6)$

$\therefore$ **The work during the resultant displacement**

$W = \overrightarrow{F} \cdot \overrightarrow{S} = (3\hat{i} + 5\hat{j}) \cdot (6\hat{i} - 6\hat{j})$

$W = 18 - 30 = -12$ work unit

Try to solve

(3) The force $\overrightarrow{F} = 5\hat{i} - 7\hat{j}$ acts on a body and moves it from point A (5, -1) on a straight line to point B (-1, 3), then to point C (4, 6). Calculate the work done by this force during each of the two displacement, then check if the sum of the two works is equal to the work done during the resultant displacement.

Oral expression: What is the magnitude of the work done in the pathway if a particle moves on a straight line from a position, then it turns back to this position under the action of the same force?

Example

(4) The force $\overrightarrow{F} = 2\hat{i} + 3\hat{j}$ acts on a particle and the position vector at a moment t is identified by the relation: $\overrightarrow{r}(t) = (t + 5)\hat{i} + (t^2 + 4)\hat{j}$ where $\hat{i}$, $\hat{j}$ are the main unit vectors, Calculate the work done by this force from t = 1 to t = 5

Solution

The displacement occurring from t = 1 to t = 5 is

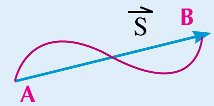
$\overrightarrow{S} = \overrightarrow{r_5} - \overrightarrow{r_1}$

$\therefore \overrightarrow{S} = (10\hat{i} + 29\hat{j}) - (6\hat{i} + 5\hat{j}) = 4\hat{i} + 24\hat{j}$

$\because W = \overrightarrow{F} \cdot \overrightarrow{S}$ **(From work definition)**

$\therefore W = (2 , 3) \cdot (4 , 24) = 8 + 72 = 80$ work unit.

Note that



The work is not based on the pathway which the body take from position A to position B but it is based on the displacement $\overrightarrow{AB}$

Try to solve

(4) If the position vector of a particle is given as a function of time by the relation: $\overrightarrow{r}(t) = (t + 4)\hat{i} + (t^2 + 3)\hat{j}$ where $\hat{i}$, $\hat{j}$ are the basic unit vectors. A force $\overrightarrow{F} = 3\hat{i} + 2\hat{j}$ acts on the particle, calculate the work done by the force $\overrightarrow{F}$ from t = 1 to t = 3

Work units:

From the work definition, we deduce that:

The work unit = the magnitude of the force measuring unit × displacement unit

From the work units are:

- **Joule:** It is known as the magnitude of work done by a force of magnitude 1 Newton to move a body a distance one meter .

  If we take $\|\overrightarrow{F}\| = 1$ Newton , $\|\overrightarrow{S}\| = 1$ mater then:

  Joule = 1 Newton × 1 mater i.e Joule = Newton . mater (N.m)

  The Joule is the international unit of the work

- **Erg:** It is known as the magnitude of the work done by a force of magnitude one dyne to move a body a distance one centimeter.

  If we take $\|\overrightarrow{F}\| = 1$ dyne ,$\|\overrightarrow{S}\| = 1$ cm then,

  Erg = 1 dyne × 1cm i.e Erg = dyne . cm

Kg.wt.m: it is the magnitude of the work done by a force of a magnitude 1 kg.wt to move a body a distance one meter.

If we take $\|\overrightarrow{F}\| = 1$ kg.wt , $\|\overrightarrow{S}\| = 1$ m, then kg.wt.m = 1 kg.wt × 1 m

You can convert the work units as follows:

1 kg.wt. m = 1 kg.wt × 1 m = 9.8 N . m **kg.wt.m = 9.8 Joule**	1 Joule = 1 Newton × 1 m = 10^5 dyne × 100 cm = 10^7 dyne × cm **Joule = 10^7 Erg**

Example

5 A body moves on a straight line. A resistance force of magnitude 100 Newton acts on it. Calculate the work done by this force during a displacement of magnitude 300 m.

Solution

Since the force is a force of resistance, then it acts in an opposite direction to the displacement vector, and if $\hat{c}$ is a unit vector in the direction of the displacement, we can express each of the displacement and force vectors in terms of their algebraic measures.

$\overrightarrow{S} = S\hat{c}$, $\overrightarrow{F} = -F\hat{c}$

In our case:

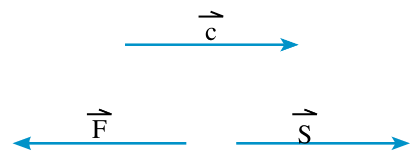
S = + 300 m , F = 100 Newton

From figure (6) W = - F S

= (-100) × (300)

= -3×10^4 N . m

= -3×10^4 Joule



(figure 6)

Example Work of weight , perpendicular reaction and friction

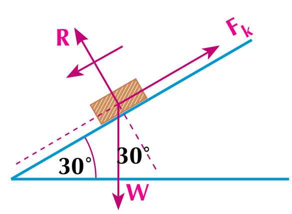
6 A body of mass 10kg slides a distance 6 m on a rough plane and the coefficient of the kinetic friction between them is 0.2 and inclined at 30° to the horizontal, Find in kg.wt.m unit the work done by:

First: The force of the weight of the body

Second: The normal reaction of the plane

Third: Friction force

Solution



First: the work done by the force of weight

Weight of the body (W) = 10 kg.wt

∵ The angle included between $\overrightarrow{W}$, $\overrightarrow{S}$ is equal to 60°

From the work definition:

$W = W \times S \cos 60^\circ$

$\therefore \quad W = 10 \times 6 \times \frac{1}{2} = 30$ kg.wt. m

Another Solution:

The weight component acting in the displacement direction, so the work $W = W \sin \theta \times S$

$\therefore \quad W = 10 \times \frac{1}{2} \times 6 = 30$ kg.wt. m

Second:

$\because$ The normal reaction force of the plane (R) is always perpendicular to the plane on which the body moves, then the angle between R , S is equal to 90°.

$\therefore$ The worked done from R = zero.

Third: The work done by the friction force:

We know that the force of the kinetic friction $\mu_k R$ (where μ_k is the coefficient of the kinetic friction)

$\therefore \quad F_K = 0.2 \times 10 \times 9.8 \times \cos 30^\circ = 49\sqrt{3}$ Newton

$\because$ The work done by friction force $W = -\mu_k R \times S$

$\therefore \quad W = -49\sqrt{3} \times 6 = -294\sqrt{3}$ Joule $= -30\sqrt{3}$ kg.wt.m

Try to solve

5) A car of mass 6 tons ascends a slope making an angle of sine $\frac{1}{98}$ to the horizontal against resistances equal to 10 kg.wt per each ton of the mass of the car. If the car acquires velocity 54 km/ h within 30 seconds and starts to move from rest, calculate the magnitude of the work done in juole by:

First: The engine force

Second: Resistance force

Third: Weight of car

Work done by a variable force

You have previously studied the concept of work as you deal with the motion when the force is uniform. This can be clear through the following example:

Explanatory Example:

Let a constant force of magnitude 10 Newtons act on a body to move from A to B as illustrated in figure (8)

Then, the displacement from A to B = 20 m. To represent that graphically, draw the force axis and displacement axis as illustrated in the figure, then the force represented on a horizontal straight line parallel to the displacement axis S.

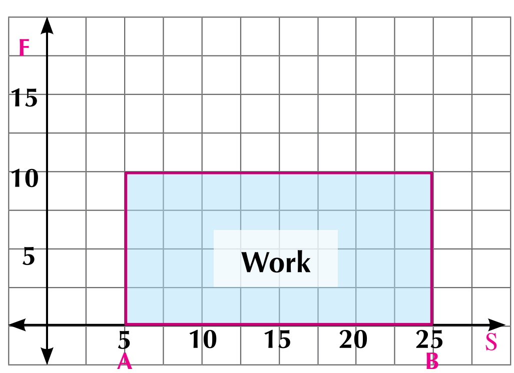


Figure (8)

$$Work = F\,S = 10\,(25-5) = 200 \text{ Joules}$$

It is the area under the curve and represented by the area of the rectangle whose width is 10 Newton and length is 20 meters.

In figure (9), then the area under the curve is determined by the relation:

$$W = \int_A^B F\,d\,S$$

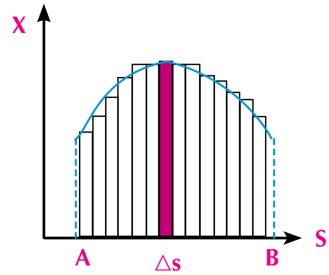


Figure (9)

In this case, we take a small displacement of magnitude △ s until the force acting on this displacement is uniform and the work done is given by the relation:

$$\triangle W = F_S \triangle s$$

If we divide the force curve into small parts and calculate the work done during each part, and find their sum, it can be expressed by the relation:

$$W = \sum_A^B F_s \triangle s$$

When the displacement △x is as smallest as possible (it reduces to zero) to get accurate values in the previous equation, then the previous equation is converted into:

$$W = \int_A^B F_S\,d\,s$$

This is the general form of work (notice that: $F_S = F\cos\theta$ (represents the component of the force in the direction of the displacement)

$$W = \int_A^B F_S\,d\,s$$

Example

7 Figure (10) illustrates the action of a variable force on a body, calculate the work done in Erg by this force in the following cases: where the force magnitude with dayn, S by cm

First: When the body moves from

S = 0 to S = 8

Second: When the body moves from

S = 8 to S = 12

Third: When the body moves from

S = 0 to S = 12

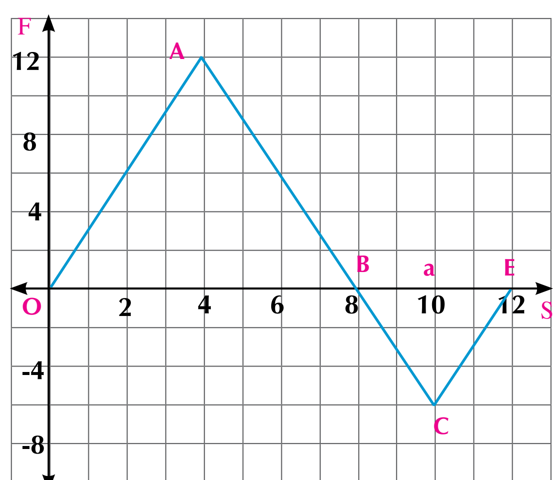


Figure (10)

Solution

First : $W_1 = \int_0^8 F\,ds$ = the area under the curve from S = 0 to S = 8

$= \text{area of } \triangle OAB = \frac{1}{2} \times 8 \times 12 = 48 \text{ Erg}$

Second: $W_2 = \int_8^{12} F\,ds$ = -the area under the curve from S = 8 to S = 12

$= -\text{area of } \triangle BCD = \frac{1}{2} \times 4 \times 6 = -12 \text{ Erg}$

Third: $W_3 = \int_0^{12} F\,ds$ = the area under the curve from $= \int_0^{8} F\,ds + \int_8^{12} F\,ds$

$= \text{area of } \triangle OAB - \text{area of } \triangle BCD$

$= \frac{1}{2} \times 8 \times 12 - \frac{1}{2} \times 4 \times 6 = 36 \text{ Erg}$

Try to solve

6) The opposite figure illustrates the action of a variable force on a body, calculate the total work done by this force in the following cases:

First: from S = 0 to S = 10

Second: from S = 8 to S = 14

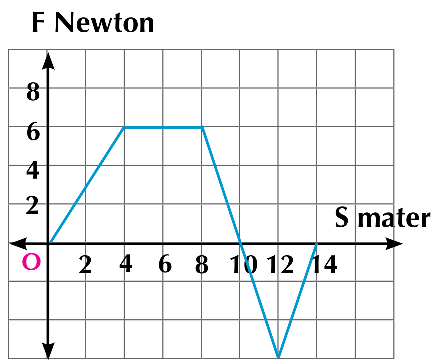


Figure (11)

Example

8) A variable force F (measured in Newton) acts up on a body where $F = 3s^2 - 4$, S measured in meter S Algebraic measured of displacement find the work done by this force in the interval from S = 2 m to S = 5 m.

Solution

$\because S = 3s^2 - 4 \quad , \quad W = \int_A^B F\,ds$

$\therefore W = \int_2^5 (3s^2 - 4)\,ds = [s^3 - 4s]_2^5$

$\therefore W = [(125 - 20) - (8 - 8)] = 105 \text{ Joules}$

Try to solve

7) A variable force F (measured in dyne) acts upon a body where F is given by the relation: $F = 4S^3 - 2S + 1$, S The algebraic measured of displacement at measured in with cm find the work done by this force in the interval from S = 0 to S = 4

Exercises 3 - 1

First: Choose the correct answer:

1. If a body moves in a straight line from the origin point to the point A (3 ,2) under the action of the force $\overrightarrow{F} = 3\hat{i} - 5\hat{j}$, then the work done by this force = ... work unit .

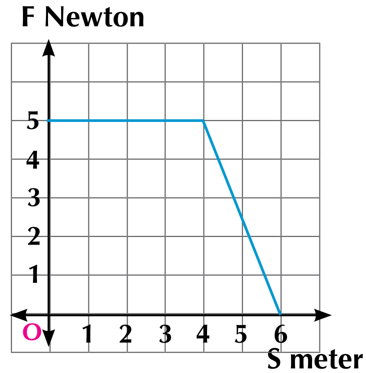
   a -4 b -1 c zero d 1

2. If a body moves in a straight line from point A (-3, 2) to point B (5, -3) under the action of the force $\overrightarrow{F} = 5\hat{i} + 8\hat{j}$,then the work done by this force = ... work unit

   a zero b -40 c 40 d 80

3. The opposite figure illustrates the action of the force (F) on a body moving a distance (S), then the work done by this force to make the body move from S = 0 to S = 6 m is equal to .... Joule

   a Zero b 40
   c 80 d 25



4. The work done to lift a mass of magnitude 200 gm placed on the ground surface a distance of 10 meters above the ground is equal to .... Joules.

   a zero b 9.8 c 19.6 d 29.4

5. If a body moves in a straight line and a resistance force of magnitude 400 Newton acts on it, then the work done by this force during the displacement $\overrightarrow{S}$ where $\|\overrightarrow{S}\| = 350$ m is equal to ...... Joule.

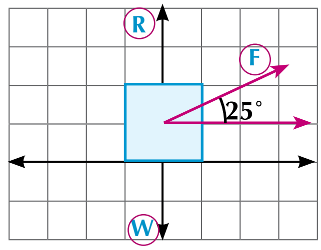
   a -14×10^4 b -7×10^4 c 7×10^4 d 14×10^4

Second: Complete:

6. A person goes shopping and pushes a shopping cart with a force of magnitude 35 Newtons inclined at 25° to the horizontal to move the cart a distance 50 meters, then the work done by the person = ... Erg

7. The work done to move a mass of magnitude 600 gm a distance 4m with acceleration of magnitude 20 cm / w^2 is equal to .... Joule

8 The opposite figure illustrates a force of magnitude 16 Newtons inclined at 25° to the horizontal acts on a body of magnitude 2.5 kg to move on a smooth horizontal table a distance 220 cm then:



- a The work done by the force = ... Joule
- b The work done by the reaction of the table = ...
- c The work done by the weight of the body = ...
- d The total work done by the forces acting on the body = ...Joule

Third: Answer the following questions:

9 A particle moves in a straight line under the action of the force $\overrightarrow{F} = 6\hat{i} - 3\hat{j}$ from point A (-1 , 2) to point B (3 , 4) where $\hat{i}$, $\hat{j}$ are the main unit vectors, calculate the work done by this force.

10 The forces $\overrightarrow{F_1} = 4\hat{i} + 3\hat{j}$, $\overrightarrow{F_2} = 2\hat{i} - 4\hat{j}$, $\overrightarrow{F_3} = 3\hat{i} - \hat{j}$ act up on a body to move from point A (2 , 3) to point B (4 , 4), calculate the work done by the resultant of these forces during the displacement $\overrightarrow{AB}$

11 A body of mass 1kg and its displacement vector $\overrightarrow{S} = (3t^2)\hat{i} + (3t^2 + t)\hat{j}$ moves. what is the moving force? Calculate the work done by the moving force during five second from the beginning of motion given that S is measured in meter , F in Newton and t in second.

12 The position vector of a particle of mass 3 kg is given as a function of time by the relation $\overrightarrow{r} = (3t^2 + 2)\hat{i} + (4t^2 + 3)\hat{j}$ where $\hat{i}$, $\hat{j}$ are two unit vectors perpendicular to the plane. Prove that the particle moves under the action of a constant force, then calculate the work done by this force from t = 1 to t = 5

13 A tram car is pulled by a rope inclined at 60° to the railroad. If the tension force is 500 kg.wt and the car moved with acceleration 5 cm/sec^2 for 30 seconds, calculate the work done by the tension force.

14 A construction worker of mass 70 kg carries an amount of bricks and ascends a ladder whose top is 12 meters high from the ground. If work done with magnitude 11760 Joules until the worker reaches the ladder top, find the mass of bricks.

15 A force acts on a body at rest of mass 50 kg to a acquire it a uniform acceleration 0.7 m/sec^2 to move distance S in direction. If the work done by this force is equal to 350 kg.wt. m, find the distance which the body moves.

16 A stone of mass 4 kg is projected vertically upwards from the ground surface. If the work done to reach the maximum height is 1176 Joule , find the maximum height the stone reached.

17 Calculate in Joule the magnitude of the work needed to be done to lift 5 cubic meters of water to a height of 10 meters.

18 A woman pushes a baby stroller with a baby from rest on a horizontal road with a force of magnitude 2 kg.wt inclined at 60° to the horizontal downwards against resistances of magnitude 0.95 kg.wt. If the mass of the stroller and the baby is 18 kg, find in kg.wt.m the magnitude of the work done during one minute from :

a The weight of the stroller and the baby

b The force of the woman

c The resistance of the road.

19 A train of mass 200 tons ascends a an inclined plane with an angle whoses sine $\frac{1}{100}$ to the horizontal with a uniform velocity. If the work done by the train engine is equal to 15×10^5 kg.wt.m unit the train reaches the top of the plane and the work done against resistances is 5×10^5kg.wt.m find:

First: The plane length

Second: The resistance per each ton of the train's mass

20 A car of mass 4 tons ascends an inclined plane at an angle of sine $\frac{1}{100}$ to the horizontal against a resistance of magnitude 5 kg.wt per each ton of the car's mass to acquire velocity 54km/h during $\frac{1}{2}$ minute. If the car starts moving from rest, calculate in Joule the work done by:

First: the force of the car's engine

Second: the resistance force

Third: the car's weight

Fourth: against the car's weigh

21 A particle moves in a straight line under the action of the force F(Newton) where F = 0.4 S and S is measured in meter, calculate the work done by the force when the particle moves from :

a $S = 0$ to $S = 10$

b $S = 1$ to $S = 5$

22 A particle moves in a straight line under the action of the force F(Newton) where F = sin 2 S where S is measured in meter, Calculate the work done by the force F when the particle moves from:

a $S = 0$ to $S = \frac{\pi}{2}$

b $S = \frac{-\pi}{4}$ to $S = \frac{\pi}{4}$

c $S = \frac{\pi}{4}$ to $S = \frac{3\pi}{4}$

Unit Three

3 - 2

Energy

You will learn

- Kinetic energy
- Measuring units of kinetic energy
- Energy-work principle

Key terms

- Kinetic energy
- Potential energy
- Energy-work pricipale

Materials

- Scientific calculator

First: Kinetic energy

The kinetic energy of the body is the energy which the body acquires due to its velocity and it is estimated at a moment as the product of half the mass of the particle times the square of the magnitude of its velocity and it is denoted by the symbol T.

If m is the mass of a particle, $\overrightarrow{V}$ is its velocity vector and V is the algebraic measure of this vector, then:

$$T = \frac{1}{2} m \| \overrightarrow{v} \|^2 = \frac{1}{2} m v^2 \qquad (1)$$

Since $\| \overrightarrow{v} \|^2 = \overrightarrow{v} \cdot \overrightarrow{v}$, then the kinetic energy can be expressed a s fellows:

$$T = \frac{1}{2} m (\overrightarrow{v} \cdot \overrightarrow{v}) \qquad (2)$$

From the definition above, it is quite clear that the kinetic energy of a particle is a non-negative scalar quantity and vanishes when the velocity vanishes. According to the definition, the kinetic energy of a particle may change from an instant to another during the particle`s motion in regard to the magnitude of its velocity.

Measuring units of kinetic energy:

Since the work is a form of the energy forms:

Unit of kinetic energy = Measuring unit of work

For example. if the mass is measured in kg and velocity in meter/ sec. then:

The unit of kinetic energy = kg $\times \frac{\text{mater}}{\text{sec}} \times \frac{\text{mater}}{\text{sec}}$

= kg $\frac{\text{mater}}{\text{sec}^2} \times$ mater = Newton. mater = Joule

If the mass is measured in gm and velocity in cm/sec then:

The measuring unit of kinetic energy = gm $\times \frac{\text{cm}}{\text{sec}} \times \frac{\text{cm}}{\text{sec}}$ = gm $\times \frac{\text{cm}}{\text{sec}^2}$ $\times$ cm = deyan $\times$ cm = Erg

Example

1. A body of mass 100 gm moves with velocity $\overrightarrow{v} = 5\hat{i} + 12\hat{j}$ where $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors and the magnitude of the velocity is measured in cm/sec, unit, calculate the kinetic energy of this body. **First:** in Erg **Second:** Joule

Solution

Find the $\vec{v} = 5\hat{i} + 12\hat{j}$

$\|\vec{v}\| = \sqrt{5^2 + 12^2} = 13$cm/sec $\qquad \therefore \|\vec{v}\|^2 = 169$

First: The kinetic energy of the body $= \frac{1}{2} m \|\vec{v}\|^2 = \frac{1}{2} \times 100 \times 169 = 8450$ erg

Second: The kinetic energy $= \frac{8450}{10^7} = 8{,}45 \times 10^{-4}$ Joule

Try to solve

1. A body of mass 200 gm moves with velocity $\vec{v} = 60\hat{i} - 80\hat{j}$ where $\hat{i}, \hat{j}$ are two perpendicular unit vectors and the magnitude of the velocity is measured in cm/sec unit, calculate the kinetic energy of this body.
   First: In Erg **Second:** In Joule.

Example

2. A body of mass 1kg is projected vertically upwards with velocity 49m/sec, find
   a. The kinetic energy of the body after 6 seconds of throwing
   b. The kinetic energy of the body when it is at a height of 102.9 meters from the point of projection

Solution

a. $\because v = v_\circ + g t$ $\qquad \therefore v = 49 - 9.8 \times 6 = -9.8$m/sec

$\therefore$ The body discends with velocity of magnitude 9.8 m/sec^2 ,$T = \frac{1}{2} m V^2$

$T = \frac{1}{2} \times 1 \times (9.8)^2 = 48.02$ Joule

b. $\because v^2 = v_\circ^2 + 2 g s$ $\qquad \therefore v^2 = (49)^2 - 2 \times 9.8 \times 102.9$

$\therefore v^2 = 384.16$ $\qquad T = \frac{1}{2} m v^2 = \frac{1}{2} \times 1 \times 384.16 = 192.08$ Joule

Try to solve

2. A body of mass 500 gm fell vertically downwards from a height of 78.4 meters above the ground surface, find:
   a. The kinetic energy of the body after 2 seconds from the instant of fell.
   b. The kinetic energy of the body at the instant it contacts the ground surface.

The Principle of work and energy

If F is a constant :

Let a body of mass (m) move a distance (S) under the action of the resultant of forces (F) such that its velocity changes from (v_1) to (v_2) then the work done by the resultant of these force:

$W = F \times S$

$\because v^2 = v_\circ^2 + 2 a s$ and let v_1 , v_2 be the initial and final velocities respectively.

$\therefore v_2^2 - v_1^2 = 2 a s$ by multiplying both sides of the relation by $\frac{1}{2}$ m

$\frac{1}{2} m (v_2^2 - v_1^2) = m a s$

$\therefore \quad \frac{1}{2} m (v_2^2 - v_1^2) = F \cdot S$ where F is a constant force

$\therefore$ The change of the kinetic energy is equal to the work done

If F is a variable force, then:

$\because \quad T = \frac{1}{2} m v^2$

$\therefore \quad \frac{d}{dt}(T) = m v \frac{dv}{dt} \qquad \frac{d}{dt}(T) = m a v \qquad \frac{d}{dt}(T) = F \frac{dS}{dt}$

$\therefore \quad \int_{T_\circ}^{T} d(T) = \int_{S_\circ}^{S} F\, ds \qquad$ i.e $T - T_\circ = W$

$\therefore$ The change of the kinetic energy = work done

The last relation expresses the principle of work and energy which states the following:

«The change of the kinetic energy of a particle as it transfers from an initial position to a final position is equal to the work done by the force acting on it during the displacement between these two positions».

It is noticed that when the previous relations are used, the measuring units of kinetic energy are to be the same measuring units of work.

Critical thinking:

Prove that If a particle starts its motion from a certain position, then it turns back to the same position, then its final kinetic energy is equal to its initial kinetic energy. Deduce that in the motion of the vertical projectile under the action of the gravitational force, the velocity of the projectile during the ascending phase at a point is equal to its velocity during its descending phase at the same point.

Example

3 A bullet of mass 200 gm is fired with velocity 400 m/sec on a thick barrier to embed in depth 20 cm. Find the magnitude of the resistance force of the barriers material to the motion of the bullet supposing this force is constant.

Solution

Let A be the position at which the bullet entered the barrier, B the position at which the bullet rested in the barrier and R is the resistance force measured in dyne unit.

we obtain AB = 20 cm, since the resistance force works in the opposite direction of the displacement,

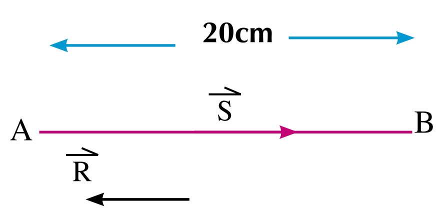


Figure (6)

Then the work done by this force is negative and be calculated as follows:

$W = -AB \times R = -20\, R$

The kinetic energy as it enters the barrier :

$T_A = \frac{1}{2} \times 200 \times (400 \times 100)^2 = 1.6 \times 10^{11}$ Erg

(**Notice the velocity is converted into cm/sec**).

The kinetic energy of the bullet at position B: T_B = Zero since the bullet is at rest in this position.

The change in the kinetic energy of the bullet : $T_B - T_A = -1.6 \times 10^{11}$ Erg

$\because\ T_B - T_A = W$

$\therefore\ -1.6 \times 10^{11} = -20\ R$

$\therefore\ R = \dfrac{-1.6 \times 10^{11}}{-20} = 8 \times 10^{9}$ Dyne

Try to solve

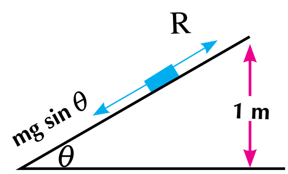
3. A bullet is fired on a target of thickness 9 cm and exits from the other side with half velocity before it enters the target. What is the least thickness needed for a target of the same material in order the bullet not to exit from if it is fired with its previous velocity?

Example

4. A body of mass 300 gm is placed at the top of an inclined plane whose height is 1m. Find the velocity which the body reaches the bottom of the plane if the work effort against by the resistance force of the plane to the motion is equal to 1.59 Joule.

Solution

Let S be the length of the plane measured in meters, θ the inclination of the plane to the horizontal. Two forces acting on the body in a direction parallel to the direction of motion, the component of weight acting in a line of greatest slope downwards and of magnitude mg sin θ and the resistance force, let its magnitude be R and acting along a line of greatest slope upwards.



Fiugre (8)

The work done during the motion of the body from the top of the plane to its base:

$W = (m\ g \sin\theta - R) \times S = (0.3 \times 9.8 \times \dfrac{1}{S} - R) \times S = 0.3 \times 9.8 - R\ S$

But RS = 1.59 Joule is work effort against the resistance.

$\therefore\ W = 0.3 \times 9.8 - 1.59 = 1.35$ Joule

$\therefore\ T - T_{\cdot} = W$ $\qquad\qquad$ $\therefore\ \dfrac{1}{2} \times 0.3\ v^2 - \text{zero} = 1.35$

$\therefore\ v^2 = 9$ $\qquad\qquad$ $\therefore\ v = 3$ m /sec

Try to solve

4. A body of mass 200 gm is placed at a top of an inclined plane of height 3 meters. Calculate the velocity by which the body reaches the bottom of the plane given that the work effort against the resistance force of the plane to the motion is 4.48 Joule.

Example

5 A body of mass 1 kg moves with a uniform velocity of magnitude 12 m/sec. A resistance force of magnitude $6x^2$ (Newton) where x is the distance which the body travels under the action of the resistance (meter) acts on it.

a Find the work done by the resistance when x = 4

b Find the velocity of the body and its kinetic energy x = 2

Solution

a
$$W = \int_0^4 F\,dx$$
$$= \int_0^4 -6x^2\,dx = [-2x^3]_0^4$$
$$= -128 \text{ Joul}$$

b $\because$ Change in kinetic energy = Work done
$$\frac{1}{2} m (v^2 - v_\circ^2) = \int_0^2 F\,dx$$
$$\frac{1}{2} \times 1 (v^2 - 144) = \int_0^2 -6x^2\,dx$$
$$\frac{1}{2} (v^2 - 144) = [-2x^3]_0^2$$
$$\frac{1}{2} (v^2 - 144) = -16$$
$$v^2 = 112$$
$$v = 4\sqrt{7} \text{ m/sec}$$
$$T = \frac{1}{2} m v^2 = \frac{1}{2} \times 1 \times 112 = 56 \text{ Joul}$$

Second: Potential energy

When a particle moves in a straight line under the action of a constant force $\overrightarrow{F}$ parallel to this line, then the potential energy of the particle P at an instant is the work done by the force acting on this particle if it's motion from its position to another constant position on the straight line $\overleftrightarrow{AB}$ as shown in the opposite figure. If the force $\overrightarrow{F}$ is parallel to $\overleftrightarrow{AB}$, (O) is the constant position , A, B are two different positions of the body on this line, then:

the potential energy at A is $P_A = \overrightarrow{F} \cdot \overrightarrow{AO}$, and the potential energy at B is $P_B = \overrightarrow{F} \cdot \overrightarrow{BO}$, by using the symbol P to express the potential energy, We find that:

- Potential energy at (O) = 0 since potential energy at O = $\overrightarrow{F} \cdot \overrightarrow{0} = 0$
- Let A , B be the initial and final positions of the moving body. P_A, P_B , are the two potential energies at A , B respectively, then:

$$P_B - P_A = (\overrightarrow{F} \cdot \overrightarrow{BO}) - (\overrightarrow{F} \cdot \overrightarrow{AO})$$
$$= \overrightarrow{F} \cdot (\overrightarrow{BO} - \overrightarrow{AO}) = (\overrightarrow{F} \cdot \overrightarrow{BA})$$
$$= -\overrightarrow{F} \cdot \overrightarrow{AB} \quad (1)$$

But: $\overrightarrow{F} \cdot \overrightarrow{AB} = W$ (2) from (1) (2)

$$P_B - P_A = -W$$

i.e.: The change in the potential energy of a body when it moves from an initial position to a final position is equal to negative the work done by the force during the motion.

Conservation of energy

If a body moves from position A to another position B without encountering any resistance, then the sum of the kinetic and potential energies at A is equal to the sum of the kinetic and potential energies at B.

From the principle of work and energy, we find that:

$$T_B - T_A = W$$

From the previous relation relating the work to the potential energy, we find that:

$$P_B - P_A = -W$$

$$\therefore\ T_B - T_A = -[P_B - P_A]$$

$$\therefore\ T_B + P_B = T_A + P_A$$

The sum of the kinetic and potential energies remains constant during the motion.

Units of measuring the potential energy: in regard to the definition of the potential energy, we find that its unit is the same measuring units of work and kinetic energy.

Example

1 The force $\vec{F} = 6\hat{i} + 2\hat{j}$ acts on a body to move it from position A to position B in two seconds and the position vector of the body is given by the relation: $\vec{r} = (3t^2 + 2)\hat{i} + (2t^2 + 1)\hat{j}$. Calculate the change in the potential energy of the body where the magnitude of F is measured in Newton and r in m and t in second.

Solution

$\because\ \vec{S} = \vec{r} - \vec{r_\circ}$ $= (3t^2 + 2)\hat{i} + (2t^2 + 1)\hat{j} - (2\hat{i} + \hat{j})$

$= 3t^2\hat{i} + 2t^2\hat{j} = \overrightarrow{AB}$

$\therefore$ The change in the potential energy $= \vec{F} \cdot \overrightarrow{BA} = -(\vec{F} \cdot \overrightarrow{AB})$

$= -(6, 2) \cdot (3t^2, 2t^2)$

$= -(18t^2 + 4t^2) = -22t^2$

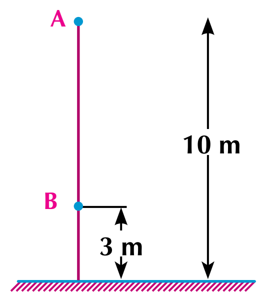
$= -22 \times 4 = -88$ Joule

Try to solve

5) The force $\vec{F} = 4\hat{i} + 5\hat{j}$ acts on a particle to move it from position A to position B within two seconds and the position vector of the body is given as a function of time by the relation $= (2t^2 + 3)\hat{i} + (4t + 1)\hat{j}$. Calculate the change in the potential energy of the particle where the magnitude of F is measured in Newton and r in m and t in second.

Example

2) A body of mass 300 gm is placed at a height of 10 m above the ground surface. Find the potential energy of the body. If it fell vertically downwards, find the sum of the kinetic and potential energies of the body at any instant during its fall, then find the energy of its motion when it is 3 m above the ground surface.



Solution

The potential energy of the body at A:

The potential energy of the body at A $= mg \times h$

$= 0.3 \times 9.8 \times 10 = 29.4$ Joule

$\because$ The body rests at A $\therefore$ its kinetic energy = zero

$\therefore$ $T_A + P_A = 29.4$ Joule

$\because$ The sum of kinetic and potential energies remains constant during motion

$\therefore$ The sum of kinetic and potential energies of the body at any instant during its falling = 29.4 Joule

Kinetic energy and potential energy at B:

$\therefore$ The potential energy of the body $= mg \times h$

$= 0.3 \times 9.8 \times 3 = 8.82$ Joule

$\because$ $T_B + P_B = T_A + P_A$

$\therefore$ $T_B + 8.82 = 29.4$ $\therefore$ $T_B = 29.4 - 8.82 = 20.58$ Joule

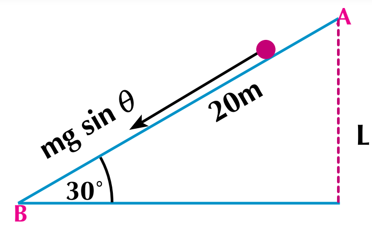
Try to solve

6) A particle of mass 100 gm is let to fall down from a height of 4 m above the ground surface. Find the sum of kinetic and potential energies of the body at any instant of its falling, then find the energy of its motion when it is 1 m above the ground surface.

Example

3) A body of mass 3 kg is placed at A which is the highest point of a smooth inclined plane of length 20 m and inclined at 30° to the horizontal. Calculate the potential energy of the body and if it descends in the direction of the line of the greatest slope of the plane. Calculate the velocity of the body at the instant it reaches the lowest point of the plane.

Solution



The potential energy of the body at A:

$P_A = mg \times h$

$= 3 \times 9.8\ (20 \sin 30^\circ)$

$= 294$ Joule

$T_A + P_A = 0 + 294 = 294$ Joule (since the body rests at A)

The kinetic and potential energies at B:

$T_B + P_B = 294$ Joule

$\frac{1}{2} mv^2 + 0 = 294$ $\qquad \therefore \frac{1}{2} \times 3 \times v^2 = 2994$

$\therefore V^2 = \frac{294 \times 2}{3} = 196$ $\qquad \therefore v = 14$ m/sec

Try to solve

7. A, B are two points on the line of the greatest slope in a rough inclined plane such that B is down A. A body of mass 500 gm starts its motion from rest from point A. If the vertical distance is equal to one meter and the velocity of the body as it reaches B is equal to 4 m/sec, find in Joule:

First: the lost potential energy

Second: the work done by the resistances

Example

4. A simple pendulum is made up of a light rod of length 80 cm carrying a body of mass 4 kg suspended vertically and is oscillatory an angle of measure 120°. Find:

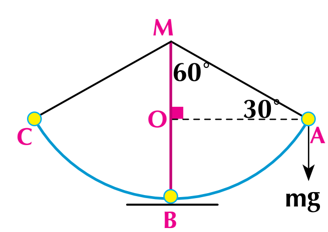
First: the increase of the potential energy at the end of the pathway more than at the middle of the pathway.

Second: the velocity of the body at the middle of the pathway.

Solution

From the geometry of the figure:

The mass moves in a circular arc whose center is point M and its radius length = 80 cm.



$\because m(\angle AMC) = 120^\circ$

$\therefore m(\angle AMO) = 60^\circ$

$\because$ Triangle AOM is thirty -sixty triangle

$\therefore MO = 40$ cm , $BO = 40$ cm

The increase of the potential energy at A is more than at B:

$P_A - P_B = mg\,h_1 - mg\,h_2 = mg\,(h_1 - h_2) = mg \times BO$

$= 4 \times 980 \times 40 = 156800$ Erg

To find the velocity of the body at the middle of the passway:

from the principle of the conservation of energy $T_B + P_B = T_A + P_A$

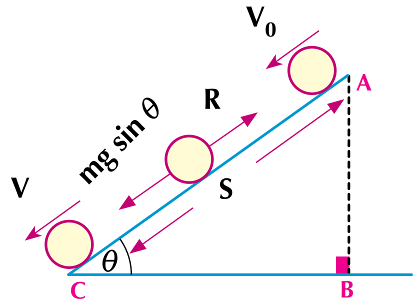
$\therefore \quad \frac{1}{2} \times 4 \times V^2 + 0 = 0 + 156800$

$\therefore \quad V^2 = 78400$

$\therefore \quad V = 280$ cm/sec

Motion on a rough inclined plane

If a body descends on a rough inclined plane under the action of its weight only from position A to position C, then the change in the potential energy = the change in kinetic energy + the work done against resistances.



Proof:

Let the distance traveled by the body on the plane be (S), then the vertical distance AB which the body descends AB = S sin B

The change in the kinetic energy from A to B =

the work done by (mg sin θ – R)

$\frac{1}{2}$ m $(V^2 - V_\circ^2)$ = (mg sin θ - R) $\times$ S $\qquad \frac{1}{2}$m $(V^2 - V_\circ^2)$ = mg sin $\theta \times$ S - R $\times$ S

$\frac{1}{2}$m $(V^2 - V_\circ^2)$ = mg $\times$ AB - R $\times$ S $\qquad$ mg $\times$ AB = $\frac{1}{2}$ m $(V^2 - V_\circ^2)$ + R $\times$ S

The change in the potential energy = the change in the kinetic energy + the work done against the resistances.

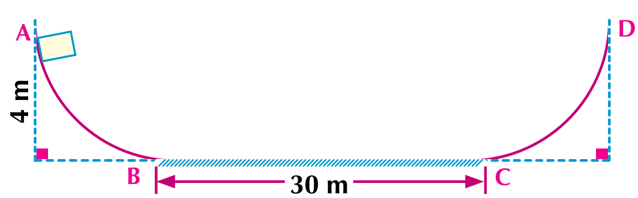
Note: the previous rule can be generalized whether the motion is vertical or on an inclined plane as follows:

If a body is fallen or thrown vertically in a medium containing a resistance or descended on a rough inclined plane, then:

The change in potential energy = change in kinetic energy + the work against resistance

Example **Motion on a rough plane**

5 In the opposite figure , a cube of wood of mass 2 kg at A, slides on a surface (as illustrated in the figure) where $\overset{\frown}{AB}$, $\overset{\frown}{CD}$ are two smooth surfaces. The horizontal plane BC is rough, its length is 30 m, and its coefficient of the kinetic friction between the cube and the horizontal surface



$\frac{1}{5}$. If the cube starts motion from rest and it is 4 m high, at which distance does the cube rest on $\overline{BC}$.

Solution

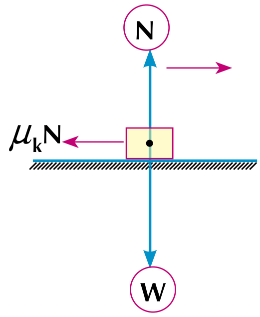
The cube slides on the arc $\overset{\frown}{AB}$

According to the principle of conservation of energy $T_A + P_A = T_B + P_B$

zero + $2 \times 9.8 \times 4 = T_B$ + zero

$\therefore T_B = 78.4$ Joule.

Since the cube moves on the rough plane $\overline{BC}$.



The change in potential energy = change in kinetic energy + the work against resistances

zero $= (0 - 78.4) + \mu_K N \times S$

$\frac{1}{5} \times 9.8 \times 2 \times S = 78.4$ $\therefore$ S = 20 m

Try to solve

8. A car descends a slope from rest. The length of 180 m, it is height of 10 m. If it is given that $\frac{3}{4}$ of the potential energy is lost due to overcoming the resistances against motion and these resistances remain constant during the motion of the car, find the velocity of the car after it traveled the previous distance of 180 m.

Exercises 3 - 2

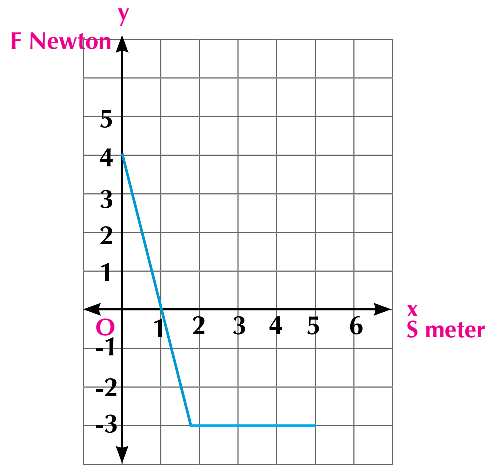
First: Complete

1. The kinetic energy of a projectice of mass $\frac{1}{3}$ kg and moving with velocity 300 m/sec is equal to .... Joule.
2. The kinetic energy of a body of mass 40 gm and moving with velocity 20 m/sec is equal to .... Joule
3. A car of mass 1.5 tons and its kinetic energy is 168750 Joule , then the velocity of the car is .... m/sec
4. A body of mass 200 gm is moving with velocity $\vec{v} = 30\hat{i} + 40\hat{j}$ where $\hat{i}, \hat{j}$ are the two perpendicular unit vectors and the magnitude of the velocity is measured in cm/sec unit, then the kinetic energy of this body = ... Erg
5. A body moves with velocity $\vec{v} = 50\hat{i} + 100\hat{j}$ where $\vec{v}$ is measured in cm/sec unit, $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors in the directions of $\overrightarrow{OX}$, $\overrightarrow{OY}$ and the kinetic energy of this body is equal to 3.9 Joules then the mass of the body = ... gm.
6. If a body of mass 30 gm is let to fall from a height of 10 meters above the ground surface, then the kinetic energy of this body = ... Joules when it is about to collide with the ground.

Second:

7. A force of magnitude 12 Newtons with constant direction does work on a body moved. If its displacement is given by the relation $\overrightarrow{S} = 3\hat{i} - 4\hat{j}$ where S is in meter, calculate the measure of the angle between $\overrightarrow{F}$ and $\overrightarrow{S}$ if the change in the kinetic energy of the body is equal to:
   First: 30 Joules **Second:** -30 Joules

8. The opposite figure illustrate the effect of the component of a force in the positive direction of x-axis upon a body body of mass 2 kg. If the speed of the body at S = 0 equals 4 m/sec.



First: find the change in kinetic energy from x = 0 to S = 5 m.

Second: calculate the magnitude energy of the body at S = 3 m.

Third: at what value of S is the magnitude of kinetic energy equals 8 Joule?

9. A body of mass 200 gm is let to move from rest from the top of a smooth plane of length 25 m and inclined at angle whose sine is $\frac{1}{10}$ to the horizontal. Find the kinetic energy of this body when it reaches the bottom of the plane.

10. A particle of mass 5 kg is thrown on the line of the greatest slope inclined to the horizontal at an angle whose sine is $\frac{1}{10}$, upwards with velocity 4 m/sec. Calculate the change in the kinetic energy of the particle after passing one second from throwing instant, then when it returns to the throwing position.

11. A rough inclined plane of length 20 m and height 5 m. Find the minimum velocity by which a body is thrown from the lowest point of the inclined plane and in the direction of the line of the greatest slope to the plane in order to hardly reach the highest point in the plane given that the body encounters resistances equal to $\frac{1}{4}$ of its weight.

12. A cannon shell is fired with velocity $\overrightarrow{v} = 105\hat{i} + 360\hat{j}$ where $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors and the magnitude of the velocity is measured in m/sec unit. If the kinetic energy of the shell is equal to 1.125×10^6 Joules, Find the mass of the shell in kg.

13. A body of mass 2 kg moves under the action of the forces $\overrightarrow{F_1} = \hat{i} + 2\hat{j}$, $\overrightarrow{F_2} = 2\hat{i} + \hat{j}$, $\overrightarrow{F_3} = 3\hat{i} + 5\hat{j}$ each is measured in Newton where $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors. If the displacement vector as a function of time is given by the relation $\overrightarrow{S} = At^2\hat{i} - B(t^2 - t)\hat{j}$ and the magnitude of the displacement is in meter. Find:

First: The value of the two constants A , B

Second: The work done by such resultant force after 2 seconds from the beginning of motion.

Third: The kinetic energy at the end of time of magnitude 2 seconds

14. A ballet is horizontally fired with velocity 540 km/h on a piece of wood to embed inside it at depth 20 cm. If the same bullet is fired with the same velocity on a constant target of the same kind of wood of thickness 15 cm. What is the velocity by which the bullet exits from the target supposing that the resistance is constant.

15. A ball of mass 100 gm is let to fall down of a height 3.6 m on horizontal ground to collide with it and rebounds vertically upwards. If the loss in the kinetic energy of the ball due to the collision with the ground is 1.96 Joules, calculate the distance by which the ball rebounded back after it collided with the ground.

16. A rubber body is let to fall from rest from a top of a tower. If its momentum reaches 1092 gm. m/sec and kinetic energy reaches 1014 gm.wt.m directly before collision, calculate the mass of this body and the height of the tower. If the body rebounds back as it collides with the ground a distance of 4.9 m, find the impulse magnitude of the ground on the body.

Complete:

17. A body of mass 0.2 kg is let to fall from a height 5 m above the ground .
   - a) The potential energy of the body at the instant it fell = ... Joule
   - b) The kinetic energy of the body at the instant it fell = ... Joule
   - c) The sum of the kinetic and potential energies at the instant the body reached the ground = ... Joule

18. A body of mass 350 kg and height 20 m above the ground, then its potential energy = ... Joule.

19. A helicopter of weight 3500 kg.wt descends vertically downwards from height 250 m to height 150 m above the ground, then the magnitude of loss in its potential energy = ... Joule.

20. A body of weight 2 kg.wt ascends a distance of 200 cm on the line of the greatest slope to a smooth plane inclined at 30° to the horizontal, then the increase of its potential energy = ... Joule

21. A body is placed on the top of a smooth inclined plane of height 90 cm , then its velocity as it reaches the bottom of the plane =.......................... m/sec

22. A body moves from position A(2, 3) to position B (7, 6) under the action of the force $\overrightarrow{F} = 3\hat{i} + 4\hat{j}$, then the change in the potential energy of the body = ... Erg; where S is measured in cm and $\overrightarrow{F}$ in dyne.

23. The force $\overrightarrow{F} = 4\hat{i} + 5\hat{j}$ acts on a body to move it from position A to position B in two seconds and the position vector of the body is given as a function of time by the relation $\overrightarrow{r} = (2t^2 + 3)\hat{i} + (4t + 1)\hat{j}$, then the change in the potential energy of the body = ...Joule; where F in Newton and $\|\overrightarrow{r}\|$ in m and t in seconds.

Answer the following questions:

24. A body of mass 300 gm is placed on a height of 10 m above the ground. Find the potential energy of the body. If the body falls vertically, find its kinetic energy when it is at a height of 3 m above the ground.

25. A body of mass 140 gm is projected vertically upwards from the top of a tower whose height is 25 m above the ground. Calculate the change in the kinetic energy of the body from the instant it is projected until it reaches the ground in Joule.

26. A body of mass 2 kg from the Earth's surface is projected vertically upwards with velocity 70 m/sec . Find the sum of the kinetic and potential energies after 5 seconds. If the kinetic energy of the body after time is 125.44 Joule. Find this time and find its potential energy.

27. A body of mass 100 gm is let to fall from a height of 5 m above mire ground and embeds in it 20 cm. Find:

First: the magnitude of what is lost of the potential energy in Joule before the instant the body collides with the ground directly.

Second: the average resistance of the ground in kg.wt.

28. A person of mass 72 kg ascends a road inclined at an angle of sine $\frac{1}{6}$ to the horizontal. he covers a distances of 120 m. Calculate the change in the potential energy of the person.

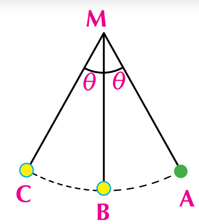
29. Calculate the velocity of a body of mass 300 gm placed at the top of an inclined plane of height 2 m reaches the bottom of the plane if the magnitude of work done against the resistance is equal to 2.13 Joule.

30. A, B are two points on the line of the greatest slope to a rough inclined plane such that B is under A. A body of mass 500 gm starts moving from rest from point A. If the vertical distance is equal to 1 m and the velocity of the body when it reaches B is equal to 4 m/sec Find in Joule:

First: the lost potential energy.

Second: the work done by the resistances.

31 **In the figure opposite:** A simple pendulum of a string of string length 130 cm, If the pendulum starts its motion from rest from point A to move freely to oscillate at angle of measure 2θ where $\tan\theta = \frac{5}{12}$. Find the velocity of the sphere at the midpoint of the pathway.



32 A ring of mass $\frac{1}{2}$ kg slides on rough vertical cylindric pool. If its velocity is 6.3 m/sec after it traveled 4.8 m from the beginning of its motion, use the work-energy principle to calculate the work done by the resistance during the motion.

Unit Three

3 - 3

Power

You will learn

- Power

Key terms

- Power
- Horse power

Materials

- Scientific calculator

Think and Discuss

A machine does work of magnitude 200 kg.wt. m in 4 minutes and another machine does another work of magnitude 100 kg.wt.m in a minute.

Which machine is more efficient?

You may think that the first machine is more efficient than the second machine since it did more work.

But what the first machine did in a minute = $\frac{200}{4}$ = 50 kg.wt.m and what the second machine did in a minute = 100 kg.wt.m . Thus, we can deduce that when we measure the power of a machine, it is necessary to know the work this machine does in a time unit.

Definition Power : the time rate to do work

This definition is formulated as follows:

«The Power is the work done in a time unit»

$$\text{Power} = \frac{d}{dt}(W)$$

$$\because \quad W = \int \vec{F} \cdot d\vec{S}$$

$$\therefore \quad \frac{d}{dt}(W) = \frac{d}{dt}\int \vec{F} \cdot d\vec{S}$$

$$= \frac{d}{dt}\int (F \cdot \frac{d\vec{S}}{dt})\, dt$$

$$\frac{d}{dt}(W) = \frac{d}{dt}\int (\vec{F} \cdot \vec{V})\, dt$$

$$\frac{d}{dt}(W) = \vec{F} \cdot \vec{V} =$$

If $\vec{V}$ has the same direction of $\vec{F}$ then Power = F V

From this relation; we can find that the power is a scalar quantity identified at any instant in terms of F, V and its value is determined by the time rate to do work at this instant.

Notice that: the power is identified instantaneously (at a certain instant) otherwise the work which is always calculated between two instants.

The average power:

If the force does work of magnitude w during a time interval $\Delta t = t_2 - t_1$ then:

The average power $= \frac{w}{\Delta t} = \frac{w}{t_2 - t_1}$

Using the integration to find the work

$\because$ Power $= \frac{d}{dt}(w)$, $\therefore w = \int_{t_1}^{t_2} (\text{power})\, dt$

The variable power and maximum power

When the magnitude of the force F is constant , then the magnitude of power changes directly with the magnitude of the velocity V of the body and F is the arbitrary constant where

power = F V power $\propto$ V when F is constant

As long as the magnitude of the velocity changes, the magnitude of the power changes and we obtain the maximum power when the velocity is as maximum as possible . In this case, the power is called the power of machine (in general)

Measuring units of power:

Since the power is equal to the time rate to do work.

$\therefore$ The measuring unit of power $= \frac{\text{measuring unit of work}}{\text{measuring unit of time}}$ = measuring unit of force $\times$ measuring unit of velocity

The measuring units of power are : watt (Newton. m/s), kg.wt .m/sec - erg /sec, horse

- **Newton - meter/second :** is known as the power of a force doing work at a constant time rate measured in Newton one meter per second. The Newton-meter /second (Joule/second) are called "Watt".

- **(Kg.wt .meter /sec) :** is known as the power of a force doing work at a constant time rate of magnitude kilogram-one meter per second.

- **Erg/second :** is known as the power of a force doing work at a constant time rate of magnitude one erg per second.

- **Horse:** It is known as the power of the machine doing work of magnitude 75 kg.wt.m per second.

Here are the rules to convert the measuring units of power.

- 1 kg.wt. meter/sec = 9.8 Newtons. meter/sec
- 1 Newton. meter/sec = 1 watt = 10^7 Erg/sec

There are other measuring units for the power such as kilowatt and horse.

- 1 kilowatt = 1000 watt = 1000 Newton. meter/sec = 10^{10} erg/sec
- 1 horse = 75 kg.wt. meter/sec
  = 75×9.8 Newton. meter/sec
  = 735 Newton. meter/sec (watt)
  = 0.735 kilowatt

Example

1. A person of mass 50 kg ascends the stairs of a tower of height 441 meters in time of magnitude 15 minutes. Calculate the average power in watt unit.

Solution

Force (F) = mg = 50×9.8 = 490 Newton

The person's average velocity = $\frac{\text{distance}}{\text{time}} = \frac{441}{15 \times 60}$ = 0.49 m/sec

The average power = force $\times$ velocity = F $\times$ V = 490×0.49 = 240.1 watt

Try to solve

1. A plane engine produces a force of magnitude 32.2×10^4 Newtons when the plan's velocity is 900 km/h. Calculate the power of the engine in horse.

Example

2. A car of mass 2 tons moves on a horizontal road with a uniform velocity of magnitude 108 km/h against resistances equivalent to 150 kg.wt per each ton of the mass. Calculate the power of its engine in horse.

Solution

The body moves with a uniform velocity (according to Newton's first law) then F = R = 150×2 = 300 kg.wt

Car's velocity = $108 \times \frac{5}{18}$ = 30 m/sec

$\therefore$ Power = F $\times$ V = 300×30 = 9000 kg.wt. m/sec

$\therefore$ Power = $\frac{900}{75}$ = 12 horse

Try to solve

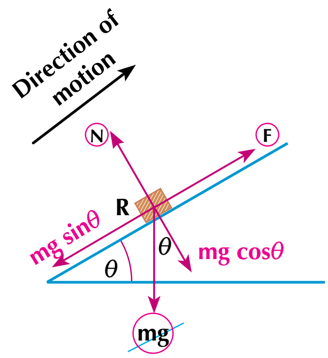
2. A truck of mass 6 tons moves on a horizontal road with a uniform velocity of magnitude 54 km/h when the power of its engine is 300 horses. Calculate the resistance of the road in kg.wt per ton of mass.

Example

(3) A car of mass 9 tons ascends a slope inclined at an angle of sine $\frac{1}{125}$ to the horizontal with maximum velocity of magnitude 45 km/h against resistances equivalent to 200 kg.wt per each ton of the mass. Calculate the power of its engine in horse.

Solution

The motion is upwards the plane



Equation of motion $F = R + mg\sin\theta$

$F = 200 \times 9 \times 9.8 + 9 \times 10^3 \times 9.8 \times \frac{1}{125}$ Newtons

$F = 2520$ kg.wt

The maximum velocity the car can ascend the slope

$V = 45 \times \frac{5}{18} = \frac{25}{2}$ m/sec

$\therefore$ The maximum power of the car $= F \times V = \dfrac{2520 \times \frac{25}{2}}{75} = 420$ horse

Try to solve

(3) In the previous example, if the car descends on the same plane after loading it with goods of mass 3 tons, calculate the maximum velocity to descend in km/h given that the resistance per ton of mass does not change.

Note: If the rate of doing work is uniform (constant), then:

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}}$$

Example

(4) A worker whose job is to load boxes each of mass 30 kg on a truck. If the height of the truck is 0.9 meter, calculate the number of boxes which the worker can load in time of magnitude 1 minute if his average power is equal to 0.6 horse.

Solution

$$\text{Power} = \frac{\text{Total work}}{\text{Time}} = \frac{\text{Number of boxes} \times \text{Work needed to load a box}}{\text{Time}}$$

$\therefore$ Number of boxes needed to be loaded in a minute $= \dfrac{\text{Power} \times \text{Time}}{\text{The work to one box}}$

Number of boxes $= \dfrac{0.6 \times 735}{30 \times 9.8 \times 0.9} = \dfrac{5}{3}$ boxes per second

Number of boxes $= \dfrac{5}{3} \times 60 = 100$ boxes per minute

Try to solve

4) In the previous example, calculate the number of boxes if the power of the man is 352.8 watt

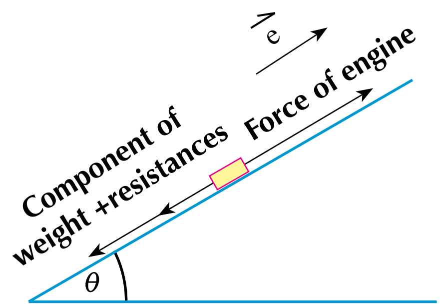
Example

5) A train of mass 200 tons ascends a slope whose inclination to the horizontal is an angle of sine $\frac{1}{200}$ with a uniform velocity of magnitude 27 km/h against resistances to the motion parallel to the direction of the line of the greatest slope of the plane at a rate of 18 kg.wt per each ton of the mass. What is the power of the engine in horse? If the train descends on the same slope with the same velocity; what is the power of the engine in this case supposing the resistances are constant in both cases?

Solution

First: when the train ascends the slope:

we take the unit vector $\vec{e}$ in the direction of motion.
i.e. upwards the plane



$\therefore$ Motion resistances = 200 × 18 = 3600 kg.wt

component of the train weight in the direction of the plane $= 200 \times 1000 \times \frac{1}{200} = 1000$ kg.wt

$\because$ The train ascends with a uniform velocity

$\therefore$ Force of the engine = resistances + component of weight = 3600 + 1000 = 4600 kg.wt

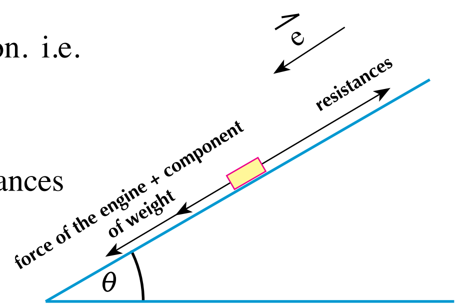
$\because$ Power $= F_1 v$ where F_1 force of the engine , v velocity

$\therefore$ Power $= 4600 \times 27 \times \frac{5}{18}$ kg.wt . meter/sec

$= 4600 \times 27 \times \frac{5}{18} \times \frac{1}{75} = 460$ horses

Second: when the train descends the slope:

We take the unit vector $\vec{e}$ in the direction of motion. i.e. downwards the plane



$\because$ The train descends with a uniform velocity

$\therefore$ Force of the engine + component of weight = resistances

$\therefore$ Force of the engine + 1000 = 3600

$\therefore$ Force of the engine = 2600 kg.wt

$\because$ Power $= F_2 v$ where F_2 is the force of the engine, v is the velocity (since it doesn't change)

$\therefore$ Power $= 2600 \times 27 \times \frac{5}{18} \times \frac{1}{75} = 260$ horses

Try to solve

5) A locomotive of mass 28 tons pulls a train car of mass 56 tons with a uniform acceleration downwards a plane inclined at sin $\frac{1}{100}$ to the horizontal. When the power of the engine reaches 84 horses, its velocity becomes 21m/sec. Calculate the acceleration if given that the resistance is 10 kg.wt per each ton of mass.

Example

6 A particle of mass 1 kg moves under the action of a force $\vec{F} = 3\hat{i} + 4\hat{j}$ such that its displacement $\vec{S}$ is given as a function of time by the relation $\vec{S} = 3t^2\hat{i} + 6t\hat{j}$ where $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors. Find the work done by the force then find the power when t = 2 seconds if F is measured in Newton, S in m and t in second.

Solution

$\because$ $w = \vec{F} \cdot \vec{S}$

$\therefore$ $w = (3, 4) \cdot (3t^2, 6t) = 9t^2 + 24t$

$\because$ power $= \frac{d}{dt}(W)$ $\therefore$ power = 18 t + 24

when t = 2 second power = 60 watt

Try to solve

6 A constant force $\vec{F}$ acts on a particle such that its displacement vector is given as a function of time t by the relation $\vec{S} = (3t^2 + t)\hat{i} - 4t\hat{j}$ where $\hat{i}$, $\hat{j}$ are two perpendicular unit vectors. Find $\vec{F}$ if the power of the force $\vec{F}$ is equal to 75 erg/sec when t = 4 seconds and the power of force $\vec{F}$ is equal to 165 erg/sec when t = 9 seconds given that S is measured in cm and F in Dyne.

Example

7 If the power of an engine at any time measured in seconds is equal to $(9t^2 + 4t)$, find the work done by the engine during the first three minutes, then find the work done during the fourth second.

Solution

$\because$ Power $= \frac{dw}{dt}$

$\therefore$ $w = \int_{t_1}^{t_2} (\text{power})\, dt$

The work done during the first three seconds $= \int_0^3 (9t^2 + 4t)\, dt$

$= [3t^3 + 2t^2]_0^3$

$= 99$ work unit

The work done during the fourth second $= \int_3^4 (9t^2 + 4t)\, dt$

$= [3t^3 + 2t^2]_3^4$

$= 125$ work unit

Example

8) Find the time taken by a car of mass 1200 kg to reach the velocity 126 km/h from rest. If the power of the engine is constant and equal to 125 horses.

Solution

$\because \quad w = \int_0^t (\text{power})\, dt \qquad \therefore \quad w = \int_0^t (125 \times 735)\, dt$

$w = 125 \times 735\, t$

$\because$ work = change in kinetic energy $\qquad \therefore \quad \frac{1}{2} m (v^2 - v_\circ^2) = 125 \times 735\, t$

$\therefore \quad \frac{1}{2} \times 1200 \left((126 \times \frac{5}{18})^2 - 0 \right) = 125 \times 735\, t$

$\therefore \quad 735 \times 1000 = 125 \times 735\, t \qquad \therefore \quad t = 8$ sec

Try to solve

7) The force of a car's engine does work at a rate given during the time interval $t \in [0, 5]$ by the relation $144\, t - 26\, t^2$. If the mass of the car is 980 kg and its velocity by the end of the third second is 90 km/h, find its velocity by the end of the fourth second.

Exercises 3 - 3

Complete

1. A particle moves under the action of force $\overrightarrow{F} = 3\hat{i} + 4\hat{j}$ where its displacement $\overrightarrow{S} = t\,\hat{i} + (t^2 + t)\,\hat{j}$ where F in dyne, and S in cm then the power of the force $\overrightarrow{F}$ at instant t = 3 seconds is equal to .... dyne. cm/sec.

2. A train of mass 375 tons and the power of its engine is 625 horses moves on horizontal ground with maximum velocity of magnitude 90 km/h, then the resistance which it encounters per ton of the train's mass = ... kg.wt.

3. A car of mass 4 tons and the power of its engine is 10 horses moves in a straight line on horizontal ground. If the maximum velocity of the car is 75 km/h, then the resistance magnitude of the road to the car's motion = ...kg.wt.

4. A train of mass 108 tons moves with a uniform velocity on a horizontal railroad with velocity 30 km/h. If the resistances are equivalent to 10.5 kg.wt per each ton of mass, find the power of the engine in horse.

5. The power of a train's engine is 504 horses and its mass is 216 tons moves on a horizontal railway with its maximum velocity against resistances equivalent to 5 kg.wt per each ton of its mass, find its maximum velocity in km /h.

6. A horizontally train moves under the action of a resistance proportional to the square of its velocity. If the resistance is equivalent to 800 kg.wt when its velocity is 20 km/h and the power of the train is 200 horses when it moves with a maximum velocity, find this velocity in km/h.

7. A car of mass 1500 kg and the power of its engine is 120 horses moves on a horizontal straight road with maximum velocity of magnitude 72 km/h. What is the maximum velocity this car can ascend a straight road inclined to the horizontal with an angle of sine $\frac{1}{10}$ given that the resistance is the same on the two roads?

8. A car of mass 3 tons moves on a horizontal road with a uniform velocity of magnitude 37.5 km/h to reach the top of a slope inclined at an angle of sine 0.03 to the horizontal . The driver stops the engine and the car moved down the slope with its previous velocity. If the slope resistance is $\frac{2}{3}$ the horizontal road resistance, find:

   first: the slope resistance in kg.wt.

   second: the engine power on the horizontal road.

9. A car of mass 6 tons moves with maximum velocity of magnitude 27 km/h ascending a slope inclined at an angle of sine $\frac{1}{10}$, to the horizontal , then the car starts to descend the same road with maximum velocity of magnitude 135 km/h. Identify the magnitude of the resistance force of the road to the motion supposing it did not change along the time, then find the power of the car's engine.

10 The engine power of a plane is 1350 horses when it moves horizontally with a uniform velocity of magnitude 270 km/h. Find the air resistance to the plane's motion. If the air resistance is proportional to square of its velocity. Find the power of its engine when it fly horizontally with a uniform velocity of magnitude 180 km/h.

11 A train engine of power 400 horses pulls a train with a maximum velocity of magnitude 72 km/h on horizontal ground. Calculate the resistance to the train's motion. If the mass of the engine and the train was 200 tons. Find the maximum velocity by which the train ascends a slope inclined at an angle of sine $\frac{1}{200}$ to the horizontal, supposing the road resistance to the motion didn't change.

12 The mass of a cyclist and the bike together is 80 kg, and the greatest power to the cyclist is $\frac{4}{5}$ of horse. If the maximum velocity for this cyclist in a horizontal road is 18 km/h, calculate the road resistance in kg.wt. If given that the cyclist ascends a slope inclined at an angle of sine $\frac{3}{40}$ to the horizontal with maximum velocity, calculate this velocity in km/h.

13 A truck of mass 5 tons moves on a horizontal road with a uniform velocity of magnitude 144 km/h when the power of its engine is 120 horses. Find the road resistance per ton of the truck's mass in kg.wt. If the resistance is proportional to the velocity, find the power of the engine in horse when the truck ascends a slope inclined at an angle of sine $\frac{3}{200}$ to the horizontal with a uniform velocity of magnitude 96 km/h.

14 A truck of 2 tons descends a road inclined at an angle of sine $\frac{1}{100}$ to the horizontal from site (A) to site (B) with maximum velocity of magnitude 90 km/h. Calculate the power of the truck's engine if given that the road resistance to its motion is estimated with ratio 13 % of the truck's weight. If the car is loaded with a mass of $\frac{1}{2}$ ton as it reaches site (B), then it ascends the road to site (A) with maximum velocity, find this velocity if the resistance remains with its same ratio of the weight.

15 A train of mass (m) ton moves on a horizontal road with the maximum velocity of magnitude 60 km/h. The last car of mass 15 tons is separated from the train and the maximum velocity of the train increases at a magnitude of 7.5 km/h. Find the power of the engine in horse and the mass of the train given that the resistance is equal to 9 kg.wt per each ton of mass.

16 A particle moves under the action of the force $\vec{F} = 3\hat{i} + 4\hat{j}$ and its displacement vector $\vec{S}$ is given as a function of time by the relation $\vec{S} = t\hat{i} + (\frac{1}{2}t^2 + t)\hat{j}$, if F is measured in Newton,s in meter and t in second, find:

a The work done during the first three seconds

b The average power during the first three seconds

c The power of the force $\vec{F}$ when t = 3 sec

17 A particle of mass the unity moves under the action of the force

$\vec{F} = (2t - 1)\hat{i} + (5t + 2)\hat{j}$ where its displacement vector is given as a function of time by the relation $\vec{S} = (3t^2 + t)\hat{i} + 4t\hat{j}$, if F is measured in Newton, S in meter and t in seconds, find:

a The work done during the third, fourth and fifth seconds

b The average power during the third, fourth and fifth seconds.

c The force power when t = 5 sec

18. A body of 3 kg moves under the action of the force $\vec{F}$ and the position vector of the body at any instant t is given by the relation $\vec{r}(t) = t^3 \hat{i} + t^2 \hat{j}$. If F is measured in Newton, r in meter and t in seconds, find:

- a) The acting force $\vec{F}$ in terms of t.
- b) The power of force $\vec{F}$ in terms of t.
- c) The work done by force $\vec{F}$ during the time interval $0 \leqslant t \leqslant 2$

19. If the power of an engine (in horse) is equal to $(6t - \frac{1}{20} t^2)$ where t is time in second, $t \in [0, 120]$ find:

- a) Power of engine when t = 90 sec.
- b) The work done during the time interval [0, 30].
- c) The maximum power of the engine.

20. A body of mass 5 kg moves under the action of force $\vec{F}$ such that its position vector at the time t is given buy the relation $\vec{r}(t) = t\hat{i} + t^2\hat{j}$. If F is measured in Newton, r in meter and t in seconds, find :

- using the integration the work done by force F in the time interval [0, 2] .

21. A body of mass 3 kg moves under the action of force $\vec{F}$ such that its velocity vector $\vec{V}$ is given by the relation $\hat{v} = (1 - \sin 2t)\hat{i} + (-1 + \cos 2t)\hat{j}$. If F is measured in Newton, V in meter /second, find:

- a) Force $\vec{F}$ in terms of t.
- b) The kinetic energy at time t.
- c) Prove that the rate of change of T is equal to the power resulted form force $\vec{F}$.