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Introduction

بسم الله الرحمن الرحيم

Today's world live an age of continuous scientific progress. Tomorrow's generation needs to be well prepared with the materials of the future in order to be able to match with the massive progress of different science. According to this principle, the Ministry Of Education does its best to develop the curricula via placing the learners in the position of being explorer to the scientific truth besides, training the students on the scientific researches as a way of thinking to make the minds the real materials to the scientific thinking and not to be stores for the scientific facts.

We introduce this book " Mathematics Applications" for second secondary grade to be assisting tool to lighten the scientific thoughts of our students and motivate them to search and explore.

In light of what was previously mentioned, the following details have been considered:

★ This book contains three domains: mechanics, geometry and measurements and probability. The book has been divided into related and integrated units. Each unit has an introduction illustrating the learning outcomes, the unit planning guide, and the related key terms. In addition, the unit is divided into lessons where each lesson shows the objective of learning it through the title You will learn. Each lesson starts with the main idea of the lesson content. It is taken into consideration to introduce the content gradually from easy to hard. The lesson includes some activities, which relate Math to other school subjects and the practical life. These activities suit the students' different abilities, consider the individual differences throughout Discover the error to correct some common mistakes of the students, confirm the principle of working together and integrate with the topic. Furthermore, this book contains some issues related to the surrounding environment and how to deal with.

★ Each lesson contains examples starting gradually from easy to hard and containing various levels of thoughts accompanied with some exercises titled Try to solve. Each lesson ends in Exercises that contain various problems related to the concepts and skills that the students learned through the lesson.

Last but not least. We wish we had done our best to accomplish this work for the benefits of our dear youngsters and our dearest Egypt.

Contents

First: Mechanics

Contents

Second: Geometry and Measurement

Geometry and Measurement

Unit Two

Mechanics

Introduction to the development of the science of mechanics

Mechanics, as a general concept, is the science that studies the motion or the balance of bodies through using its own laws; for example there are laws which are applied to the Earth's rotation around the sun and the firing of rockets, a cannon ball or otherwise. It is intended to the change that happens over time to the position of bodies in space. The mutual mechanical effect between bodies is the one by which these bodies change their motions according to the effect of different forces on them. So, the main issue in mechanics is the study of the general laws of the motion and balance of bodies subjected to the action of forces. Mechanics is divided into two branches:

Statics[1]

(**The science of the equilibrium of bodies**) It is concerned with the forces that produce a state of rest in a system of bodies. These forces are known as equilibrium if they don't change the state of the body which is said to be equilibrium under the effect of these forces.

Dynamics[2]

(**The science of the motion of bodies**) It is concerned with the study of forces and their effect on motion of bodies. Dynamics is divided into **Kinematics** which studies motion geometrically (describing motion without reference to the forces causing it), and **Kinetics** which studies the relationship between the motion of bodies and its causes, namely forces.

There are:

Mechanics of Particles (You can ignore the dimensions of the body on studying its motion or equilibrium.)

Mechanics of Rigid Bodies (the body which is consisted of a very large number of connected particles, so close to each other and the distance between any two particles of them is fixed and not affected by any external effect).

Mechanics of Bodies of Variable Mass (Some systems and bodies have varies in which the mass varies as time due to separating out or joining up of particles which decrease or increase during motion. As examples for these bodies, there are the jet rockets and the mining cars; their masses vary as a result of the consumption of fuel and other different systems).

Mechanics of Elastic Bodies (**Elasticity**) It is the property of bodies that are able to return to their original shape and dimensions after being formed; whereas in **plasticity**, if bodies are

1 In this unit, we'll study the concept of force and its properties, its measuring units, resolutions of force into two components and finding the resultant of a set of coplanar forces which act at a point and some applications on that.

2 In this unit, we will study Kinematics which is concerned with the description of the motion of bodies without reference to the forces causing it. This study deals with the motion of bodies and the phenomena associated with this motion and its causes and laws as well as applications on the vertical and horizontal with a uniform acceleration and the general gravitational law of Newton.

exposed to external effects, they don't return to their normal shape on dismissing these effects.

The revelation of mechanics

Classical machanics

It is the oldest branch of mechanics which concerns with the study of forces that act on bodies, It also concerns with the motion of the planets. It helps in many modern technics (constitutive engineering, civil engineering and space's remarks..)

Quantum mechanics

It is a set of physical theories that emerged in the twentieth century, to explain the phenomena at the level of the atom and the particles. It combines between the particle property and the wave length property to show the term of dual wave-particle. Thus, quantum mechanics is responsible for the physical interpretation at the atomic level. Moreover, it is applied on classical mechanics, but its effect doesn't appear at this level. So, quantum mechanics is the generalization of classical physics to be applied on both atomic and normal levels. It is called quantum mechanics due to the importance of quantity in its structure (it is a physical term used to describe the smallest quantity of energy which could be exchanged between particles, and used also to refer to the finite quantity of energy which emits in a discontinuous state.)

Fluid Mechanics

It is a branch of quantum mechanics and it studies mainly fluid (liquids, gases). This branch studies the physical behavior for the fluids and is divided into fluid statics (studying its rest state), and fluid dynamics(studying its motion state).

Biomechanics:

It is the application of the mechanical principles on the living organisms; this includes the study and analysis of the mechanism of living organisms physically, mechanically , physiologically systems. Some simple examples of biomechanics researches include the study of the forces that act on limbs (organs) in its rest or movement status. Some simple examples of that "the movement of the intestine, the blood flowed, the movement of the nucleus in fallopian tube, the transfer of the liquids in the ureter to the kidney and the digested operation of the food and its movement. The Applied Mechanics plays key roles in the study of biomechanics by which we can discover new cases, suitable to improve the applied state.

General relativity theory

The theory of relativity by Einstein changed a lot of concepts with respect to the basic terms in physics, time, place, mass, and energy which brought about a quantum leap in theoretical physics and space physics in the twentieth century. When first published, it modified Newton's mechanical theory that existed for two hundred years. The theory of relativity converted Newton's concept of motion; it stipulates that every motion is relative. The concept of time has been changed from being fixed and definite to yet another non spatial. Time and place has become one thing after being dealt with as two different things. The concept of time is made to depend on the speed of bodies. The dilation and contraction of time has become a key concept for understanding the universe; and so all the Newtonian classical physics have been changed.

Activity

1 - The international web for information (internet) is used to search for the role of mathematicians in improving the science of mechanics. There are some of the searching results:

Thanks to the English scientist **Isaac Newton**, the route of classic mechanics has been prefaced through the laws of motion which illustrated the most of a strological and natural phenomena. The German scientist **Johannes Kepler** and the Italian **Galileo Galilei** have had a great role in putting laws which describe the planets movements.

Kepler's laws show that, there is an attraction power among each of them, and also it shows the movement of planets around the sun according to the new perspective which depends on Heliocentric in a form that calculation in it is matching the astronomical observations substantially. All these rules have been used since the seventeenth century and led to the appearance of the theory of relativity composed by **Einstein** through the years 1905-1916 and the quantum mechanics that composed by the help of **Max plank** , **Heisenberg** , **Schrodinger** and **Dirac** at the beginning of the twentieth century.

Dr. Ahmed Zewail invented a very fast photographic system using laser. It has the ability to determine the motion of the particles when they are formed and when they are connected to each other.

Ahmed Zewail is recorded in the honor list in the United States of America which included **Albert Einstein** and **Alexander Graham Bel**.

For more information search in the Wikipedia using the site http: //ar.wikipedia.org

Measuring Units

When students apply to join the military faculties, some medical tests must be done as height measurment, weight, blood pressure, and average of the beats of the hearts, ...etc.

Measurement operation compares a quantity to another quantity from the same type, to know the number of times the first quantity included into the second quantity.

The system used in most of the parts of the world is called 'International system of units (SI)".

The 'International system of units (SI) include seven basic units. The units of these basic quantities are determined by the direct measurements that depends on the standard units for each of the length, the time, and the weight that was saved in the department of weights and measurements in France. The other units are derivative from the basic units, We are concerned with the following quantities in our study:

First: Fundamental quantities and its measurements units in (SI)

Basic quantities	basic unites	symbols
length	metre	(m)
mass	kilogram	(kg)
time	second	(s)

One of the benefits of using the international system is that: it's too easy to transfer among the units.

Add to your information:

1- Femtosecond:

is a part of a million billion of a second, i.e. (ten raised to the power of -15) and the ratio between the second and femtosecond is as that between the second and thirty two million years. In 1990, the Egyptian scientist, Ahmed Zweil, was able to install his invention which is known as femto-chemistry, after painstaking effort with his research at California Institute of Technology since 1979. His invention can be summed up in inventing of the unit time surpassed the normal time and access to the femtosecond unit of time. He achieved his scientific discovery using ultra short laser flashes and a molecular beam inside a vacuum chamber, a digital camera with unique specifications so as to photo the motion of the particle since birth and before joining the rest of the other particles. It was then possible to intervene rapidly and surprise the chemical reactions as they occur using the laser flashes as a telescope to watch and follow the destruction and construction processes in the cell. This giant Arab scientist has left the door open to the use of this scientific discovery in the field of medicine, physics, spaces researches and others fields; a new scientific school has been recorded by his name and known as femto-chemistry.

2- The multiples of units:

Unit	symbol	measure
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	K	10^{3}

Fractions of units:

units	symbol	measure
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	u	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

According to that we can transfer each of the following units to their corresponding units:

1. 2.75 Km into m.
2. 635 mm. into dm
3. 750 k.Hertz into M.Hertz.
4. 1970 gm into K.gm.

As follow:

1. 2.75 Km $=2.75 \times 1000 = 2750$ m
2. 635 mm. $=635 \times 10^{-2} = 6.35$ dm.
3. 750 k.Hertz$= 750 \times 10^{-3} = 0.75$ M.Hertz
4. 1970 gm $= 1970 \times 10^{-3} = 1.97$ kgm

Second: Derived quantities:

1 Unit of measuring the velocity

Velocity is known as the rate of changing of displacement according to time.

Unit of measuring the velocity = unit of measuring the distance÷unit of measuring the time.

So that, velocity is measured by the unit: m/sec. (m/s)

2 Acceleration

Acceleration is known as the rate of changing of velocity according to time. So that, acceleration is measured by the unit: m/sec. square (m/s^2)

According to that we can transfer each of the following units to their corresponding units:

1. 1 km/h into m/sec.
2. 1 Km/h into cm/sec.
3. 1 km/h/sec into m/sec^2
4. 1 km/h/sec into cm/sec^2

As follow:

1. $1\text{Km/h} = \dfrac{1 \times 1000 \text{ sec}}{60 \times 60 \text{ sec}} = \dfrac{5}{18}$ m/sec

Remember that

Km =1000m

M =10 dm

dm = 10 cm

cm = 10 mm

Do you Know

Normative second : is the time interval in which the cesium atom use to ascillate by one complete cycle

Note that

The units used for the vector quantities (velocity - Acceleration - force) concerned on their magnitudes regardless of its direction.

Remember that

The average sun day= 24 hour

Hour = 60 min.

Min. = 60 sec.

2. 1 Km/h $= \dfrac{1 \times 1000 \times 100 \text{ cm}}{60 \times 60 \text{ sec}} = \dfrac{250}{9}$ cm/sec

3. km/h/sec $= \dfrac{1000 \text{ m}}{60 \times 60 \text{ sec} \times \text{sec}} = \dfrac{5}{18}$ m/sec^2

4. Km/h/sec $= \dfrac{1000 \times 100 \text{ cm}}{60 \times 60 \text{ sec} \times \text{sec}} = \dfrac{250}{9}$ cm/sec^2

Activity

1. Transfer each of the following units into their corresponding units:
   a) 72 km/h into m/sec   b) 1000 cm/sec into km/h   c) 36 km/h/sec into cm/sec^2

3 Force

Force is defined as the product of the mass(m) with the acceleration (a)
If we denoted by (F) to the force ,then $F = m \times a$

Units of measuring the magnitude of the force

Absolute units:

As: Dyne and Newton, where: 1 Newton = 10^5 Dyne .and they will define as follow:

Newton: is the magnitude of the force that if it is acts on a mass equals 1 kilogram it gains an acceleration of magnitude 1 m /sec^2

Dyne: is the magnitude of the force that if it is acts on a mass equals 1 gram it gains an acceleration of magnitude 1 cm /sec^2

Gravitational units:

As: Gram weight (gm.wt) and kilogram weight (Kg.wt), where: 1 Kg.wt = 10^3 gm.wt.

and they will define as follow:

Kilogram weight: is the magnitude of the force that if it is acts on a mass equals 1 Kilogram it gains an acceleration of magnitude 9.8 m /sec^2

Gram weight: is the magnitude of the force that if it is acts on a mass equals 1 gram it gains an acceleration of magnitude 980 cm /sec^2

The Gravitational units joined with **the Absolute units** by the relations: 1 Kg.wt = 9.8 Newton and 1 gm.wt = 980 Dyne

Add to your knowlage

All bodies fell to the ground with uniform acceleration between 9.78. 9.82 m/sec^2 ragardless of their masses, Counting on latitude we will consider the acceleration equals 9.8 m/sec^2 for case of use if there is no other values of it are set.

According to that we can transfer each of the following units into their corresponding units:

1. 3.14 Newton into Dyne
2. 6.75×10^7 Dyne into Newton

As follow:

1. 3.14 Newton $= 3.14 \times 105 = 314000$ Dyne
2. 6.75×10^7 Dyne $= 6.75 \times 10^7 \times 105 = 675$ Newton

Activity

2. Transfer each of the following units into their corresponding units:
   - a. $\frac{1}{7}$ gm.wt into Dyne
   - b. 5.36×1250 Dyne into Newton
   - c. 2.50 Newton into Dyne

You can put the derived quantities in the following table as follow:

Derived quantity	Relation with other quantities	measurement unit
Velocity (V)	Displacement ÷ time	m/s
Acceleration (a)	velocity ÷ time	m/s^2
Force (F)	mass × acceleration	N

Check your understanding

Choose the correct answer from those given:

1. Mass is measured by:

   a. Dyne b. Newton c. kilogram d. kilogram weight

2. From the basic quantities in the international system:

   a. Mass b. Velocity c. Acceleration d. Force

3. Millimeter unit is equivalent to:

   a. 10^{-3} meter b. 10^{-3} meter cube c. 10^{-2} centimeter d. 10^{-4} decimeter

Answer the following questions

4. Mention the name of the following values

   a. 10^{-2} meter b. 10^{-3} meter c. 1000 meter

5. Transfer each of the following into meter:

   a. 63.4 centimeter b. 512.6 millimeter c. 0.534 decimeter

6. **Critical thinking:** Calculate in kilogram unit the mass of water that must fill in a container in a form of cuboid with length 1.6 m ,width 0.650 m and height 36 cm ,known that the density of the water equals 1 gm/cm^3 approximating the result to the nearest integer number.

[Hint: mass = volume × density]

Unit one

Statics

Introduction of the unit

The science of the statics concern with on solving all geometrical problems related to the equilibrium of bodies, the operations of resolving and resultant of the forces acting on a body, and the reaction of the bodies according to the forces act on it, the life applications in houses, buildings, bridges and the designs of engines & machines.

Newton has had more researches and books in this field such as the book , athematical Principles of natural Philosophy which is consisted of three parts and it is the foundation of classical mechanics. One of his famous sayings about himself." I don't know what I may appear to the world but to myself I seem to have been only like a boy playing on the sea-shore and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me".

Unit Objectives:

By the end of the unit the students should be able to:

- Find the magnitude, the direction of the resultant of two forces act at a point
- Recognize resolution of a given force into two components in a given directions.
- Recognize resolution of a given force into two perpendicular components.
- Find the magnitude and the direction of the resultant of a set forces meeting at a point.
- Investigate equilibrium of a particle under the effect of a set of coplanar forces meeting at a point.
- Distinguishes between smooth and rough surfaces.
- Identify the concept and properties of friction
- Identify the friction force and the limiting friction force
- Determine the coefficient of the friction, angle of friction and the relation between them.
- Determine the conditions of equilibrium of a body on a rough horizontal plane.
- Determine the conditions of equilibrium of a body on a rough inclined plane.
- Deduce the relation between the measure of the angle of fraction and the measure of the angle of inclination of the plane on the horizontal as a body is placed on a rough inclined plane on a condition that the body is about to slide under the effect of its weight only.
- Solve life applications on the friction.

Key - terms

- Statics
- Force
- Rigid body
- Gravitation force
- Acceleration of gravity
- Newton
- Dyne
- Kilogram weight
- Gram weight
- Line of action of the force
- Resolving force
- Force Component
- Equilibrium of a body
- Triangle of forces
- Lami's rule
- Equilibrium of rigid body
- Smooth plane
- Inclined smooth plane
- Centre of gravity
- Friction
- Smooth Surface
- Rough Surface
- Reaction Normal
- Static Frictional force al force
- Kinetic Frictional force al force
- Limiting Static Friction
- Resultant Reaction
- Angle of Friction
- Rough horizontal plane
- Rough inclined plane

Unit Lessons

lesson (1 - 1):Forces.

lesson (1 - 2):Forces resolution into two components.

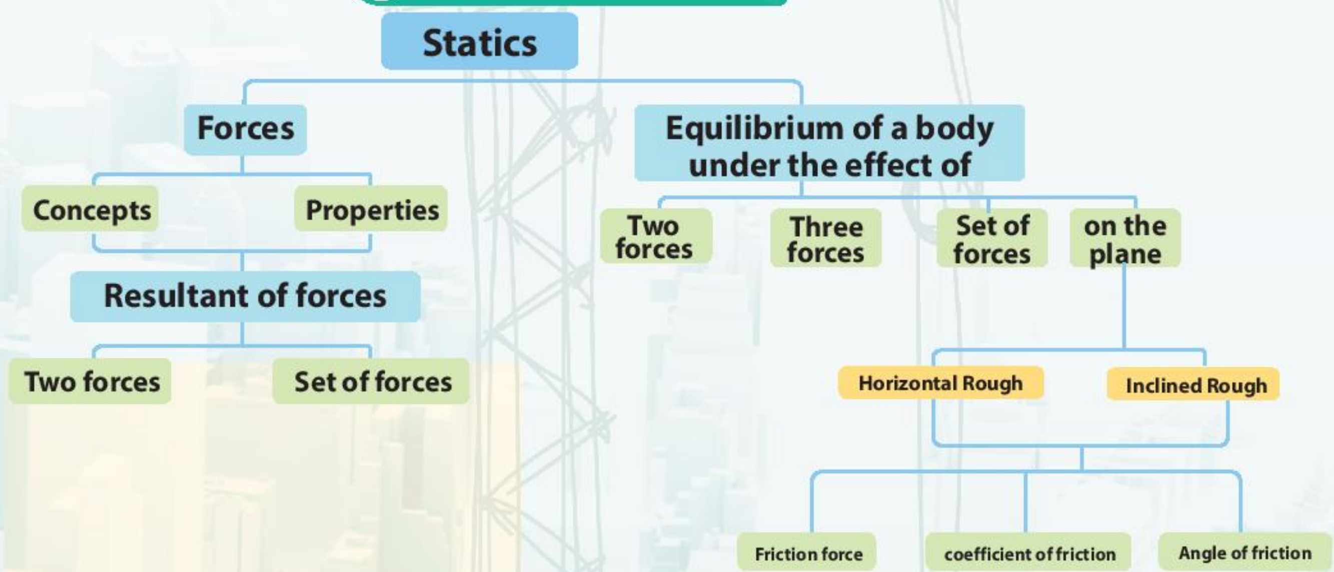
lesson (1 - 3):The Resultant of caplanar forces meeting at a point.

lesson (1 - 4): Equilibrium of a rigid body under the effect of coplanar forces meeting at a point.

Materials

- Scientific calculator
- Graphical computer programs

Charts of the unit



Forces

We will learn

- Some basic concepts in statics
- Properties of a force
- Resultant of two forces acting at a point.
- Finding the resultant of two forces acting at a point analytically

Key - term

- Force
- Resultant
- Rigid body
- Gravitation force
- Acceleration of gravity
- Newton
- Dyne
- Kilogram weight
- Gram weight

Matrials

- Scientific calculator
- Graphical programs

Preface:

You knew that statics is one of the branches of mechanics that studies forces and conditions of equilibrium of material bodies subjected to acting forces. We will study in this unit only the equilibrium of rigid bodies[1].

Remember that

Scalar quantity is determined completely by a real number (Magnitude) such as distance, time, mass, area, volume.
Vector quantity is determined by its direction in addition to its magnitude
Such as : Force, displacement, velocity, weight, ...

Force

Equilibrium or movement of body depends on the nature of the mechanical mutual influence between it and other objects, i.e on cases of pressure or tension or attraction or repulsion of the body that occur as a result of this influence.

1 Def.

➢ Force : is defined as the effect of a natural body upon another one.

Properties of forces:

The effect of any force depends on the following factors:

First: magnitude of a force.

The magnitude of a force is determined by comparing it by a unit of force, the main units of measuring the magnitude of a force in mechanics are **Newton (N)** or kilogram weight **(kg.wt) where:**

Add to your knowlage

The natural bodies are divided into:
- The rigid bodies whose shape does not change under the effect of any force
- The elastic bodies whose shape can be reformed under the effect of forces as liquids, gazes, rubber, clay, ...

➢ 1 Kg.wt = 1000 gm.wt , 1 Newton = 10^5 dyne

➢ 1 Kg.wt = 9.8 newton , 1 gm.wt = 980 dyne

(unless something else is mentioned) [2]

1- Rigid body: A body whose deformation is neglected whatever the action of the external effect.

2- Weight force (weight): it is the gravitational force of attraction between the Earth and the body with acceleration of changes from position to other on the Earth and its approximate magnitude = $9.8 m/sec^2$ unless something else is mentioned. This topic will be shown in detail elsewhere in mechanics.

Secondly: Direction of a force

Figure (1) represents the force vector $\overrightarrow{F}$ which could be represented by a directed line segment $\overrightarrow{AB}$ where A is its initial point, B is its terminal point. The magnitude of the force is determined by $\|\overrightarrow{AB}\|$ (its length with a suitable drawing scale). The direction of the arrow is corresponding to the direction of the force $\overrightarrow{F}$, where θ is called the polar angle in the plane of the force $\overrightarrow{F}$. Force is written in the polar form as(f, θ)

Add to your knowlage

Polar angle is the positive angle that the vector makes with the positive direction of x-axis.

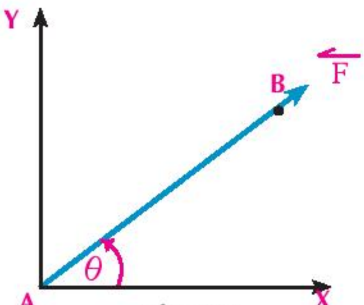


Fig (1)

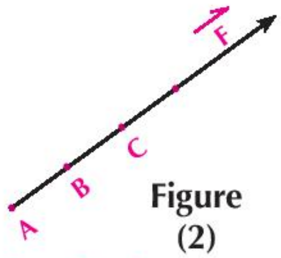


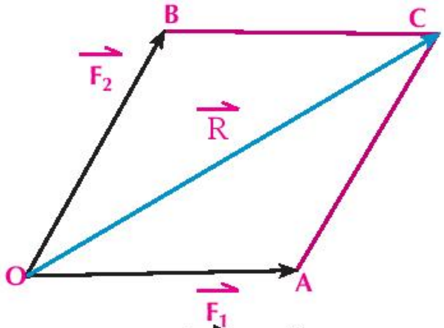
Figure (2)

Thirdly: Point of action of the force and its line of action

In figure (1) : Point A is always coincide on the point of action of the force $\overrightarrow{F}$, it is possible to transfer the point of action of the force to any other point lying on the line of action of the force without changing in its effect on the body as in **figure (2),** The line of action of the force $\overrightarrow{F}$ in **figure (1)** is denoted by $\overleftrightarrow{AB}$, i.e. the line of action of a force, is the straight line which passes through its point of action and is parallel to its direction.

Resultant of two forces acting on one point:

for any two forces acting on a body at the same point there is a resultant force $\overrightarrow{R}$ that acts at the same point and has the same effect of the two forces, it is represented geometrically by the diagonal of the parallelogram that represent the two forces by two adjacent sides.



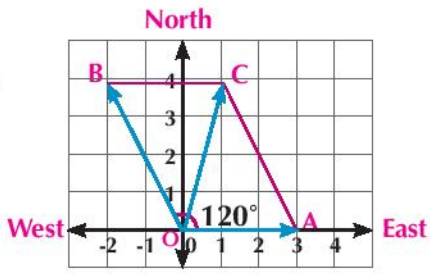
In the opposite figure: $\overrightarrow{R}$ is repreented by the diagonal of the parallelogran ($\overrightarrow{OC}$), $\overrightarrow{R}$ represents the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$. **i.e.:** $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

Activity

Using (GeoGebra) program

$\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces at a point on a rigid body where $\overrightarrow{F_1}$ = 300 Newton, acts in direction of east, $\overrightarrow{F_2}$ = 400 Newton acts in direction 60° Northwest. Find their resultant.

- Choose a suitable drawing scale (1 cm for each 100 Newton).
- Draw $\overrightarrow{OA}$ to represent the force $\overrightarrow{F_1}$ such that: $\|\overrightarrow{OA}\|$ = 3cm in the positive direction of the x-axis.
- Draw $\angle$ AOB is the polar angle where m ($\angle$ AOB) = 120°
- Draw $\overrightarrow{OB}$ to represent the force $\overrightarrow{F_2}$ such that $\|\overrightarrow{OB}\|$ = 4 cm.
- Draw the parallelogram OACB,

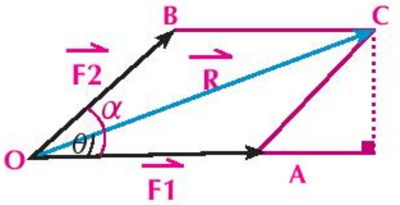


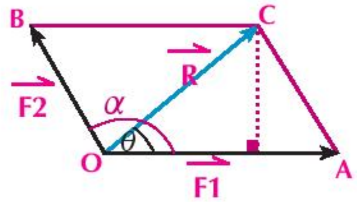
- Notice that the resultant of the two forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ is represented by the directed line segment $\overrightarrow{OC}$
- by using the program we can determine $\| \overrightarrow{OC} \| \simeq$ 3.6 cm. **i.e.** $R = 3.6 \times 100 \simeq 360$ N
- Notice that: $\overrightarrow{OC}$ inclined by an angle of measure 73° 53` 53`` with $\overrightarrow{OA}$, $\overrightarrow{R}$ makes an angle of inclination of measure 73° 53` 53`` with the direction of $\overrightarrow{F_1}$.

Application on the Activity

1. Use program (*GeoGebra*) to find the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which act on a point an a rigid body where $\overrightarrow{F_1}$ = 400 N acts in the east direction, $\overrightarrow{F_2}$ = 500 N acts in direction 80° north of east.

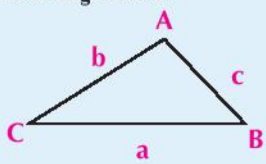
The resultant of two forces meeting at a point analytically





Remember that

Cosine rule:
in triangle ABC:



$a^2 = b^2 + c^2 - 2bc \cos A$

let $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ be two forces acting at O, θ is the angle between their directions. $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are represented by $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{R}$ is represented by $\overrightarrow{OC}$ .let θ be the angle between $\overrightarrow{R}$ and $\overrightarrow{F_1}$, using the cosine rule, we can deduce the magnitude, direction of $\overrightarrow{R}$ as follows:

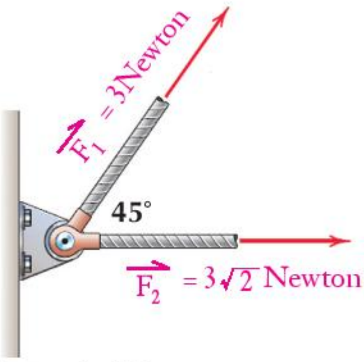
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha} \quad , \quad \tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

where: F_1 , F_2 , R are the magnitudes of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{R}$ respectively.

Think: How can you investigate the truth of the previous relations ?

Example

1. Two forces of magnitudes 3, $3\sqrt{2}$ newton act on a particle and the measure of angle between their lines of action is 45°. find the magnitude of their resultant and the measure of its inclination angle with the first force.



Solution

let: $F_1 = 3$, $F_2 = 3\sqrt{2}$, $\alpha = 45°$

$\because \quad R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha}$

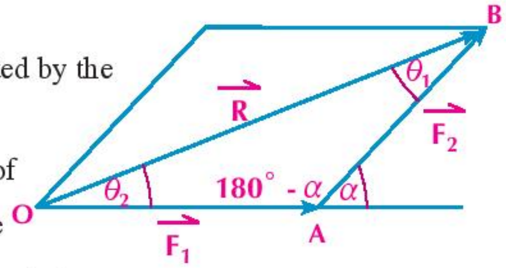
$\therefore \quad R = \sqrt{(3)^2 + (3\sqrt{2})^2 + 2 \times 3 \times 3\sqrt{2} \cos 45°}$

$= \sqrt{9 + 18 + 18\sqrt{2} \times \frac{1}{\sqrt{2}}} = \sqrt{45} = 3\sqrt{5}$ Newton

$\because \tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$ $\quad \therefore \tan \theta = \frac{3\sqrt{2} \times \sin 45°}{3 + 3\sqrt{2} \cos 45°} = \frac{1}{2}$

Using the calculator : $m(\angle \theta) = 26° 33` 54``$

Another solution for the second part of the example :



Notice that: the opposite figure $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are represented by the triangle OAB

where $\angle\theta_1$ is the angle of inclination of line of action of $\overrightarrow{F_2}$ with the resultant $\overrightarrow{R}$, $\angle\theta_2$ is the inclination angle of line of action of $\overrightarrow{F_1}$ with the resultant $\overrightarrow{R}$. using the sin law

Remember that: $\sin(180° - \alpha) = \sin\alpha$;

then: $\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{R}{\sin\alpha}$ where $\alpha = \theta_1 + \theta_2$

This rule is used to find the measure of the inclination angle of the resultant $\overrightarrow{R}$ with each of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$

from the previous example:

To find the measure of the inclination angle of $\overrightarrow{R}$ with $\overrightarrow{F_1}$ we use the rule: $\frac{F_2}{\sin\theta_2} = \frac{R}{\sin\alpha}$

$\therefore \frac{3\sqrt{2}}{\sin\theta_2} = \frac{3\sqrt{5}}{\sin 45}$

$\therefore \sin\theta_2 = \frac{3\sqrt{2} \times \sin 45°}{3\sqrt{5}}$

The measure of the inclination angle of $\overrightarrow{R}$ with $\overrightarrow{F_1}$ equals 26° 33` 54`` as the same result as we get before.

Notice : we can use this method in solving exercices.

Try To Solve

(2) Two forces of magnitudes 10, 6 newton act on a particle and the measure of the angle between their directions is 60°. Find the magnitude of their resultant, and its angle of inclination with the first force.

Critical thinking: Find the magnitude and the direction of the resultant of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in the following cases:

1- The two forces are perpendicular.

2- The two forces have the same magnitude.

Example

(2) Find the magnitude, and the direction of the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in each of the following cases:

(A) $F_1 = 5$ newton , $F_2 = 12$ Newton and the angle between their lines of action is 90°

(B) $F_1 = F_2 = 16$ newton, the measure of angle between their directions equals 120°

Remember that

If $\overrightarrow{F_1} \perp \overrightarrow{F_2}$ then :

$R = \sqrt{F_1^2 + F_2^2}$

$\tan\theta = \frac{F_2}{F_1}$

Solution

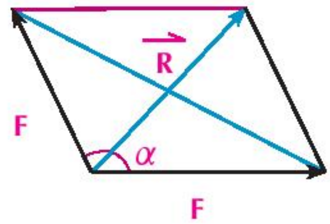
(A) $\because$ $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are perpendicular, then $m(\angle\alpha) = 90°$, then: $\sin(\alpha) = 1$, $\cos(\alpha) = 0$

$\therefore R = \sqrt{F_1^2 + F_2^2}$, $\therefore R = \sqrt{(5)^2 + (12)^2} = 13$ Newton let θ be the measure of the angle between $\overrightarrow{R}$, $\overrightarrow{F_1}$: $\tan\theta = \frac{F_2}{F_1}$ $\therefore \tan\theta = \frac{12}{5}$

$\therefore \theta = \tan^{-1}(\frac{12}{5}) = 67° 22` 49``$ $\therefore$ the inclination angle of $\overrightarrow{R}$ on $\overrightarrow{F}$ equals 67° 22` 49``

B $\because R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\alpha}$ **substitute by** $F_1 = F_2 = 16$

$\therefore R = \sqrt{(16)^2 + (16)^2 + 2 \times 16 \times 16 \cos 120} = 16$ N



we notice that: $F_1 = F_2 = R = 16$ N and the resultant force bisects the angle between the two equal forces i.e. $\overrightarrow{R}$ inclines by an angle of measure 60° with the line of action of each force.

Notice that: From the geometry of the figure : $\cos\frac{\alpha}{2} = \frac{\frac{1}{2}R}{F}$ $R = 2F\cos\frac{\alpha}{2}$

Try To Solve

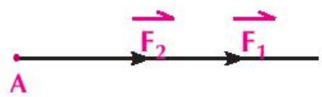
(3) Find the magnitude and the direction of the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in each of the following cases:

A $F_1 = 4.5$ newton, $F_2 = 6$ newton and the measure of the angle between them equals 90°

B $F_1 = F_2 = 12$ newton, the measure of the angle between their lines of action equals 60°

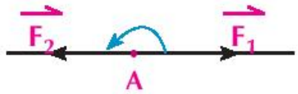
Special cases:

1- If the two forces have the same line of action and the same direction:



- In this case: $m(\angle\alpha) = 0$; $\cos\alpha = 1$ and by substitution in the resultant rule, we will find that: $R = F_1 + F_2$, the direction of the resultant is the same direction as the two forces, In this case R is called the maximum magnitude of the resultant.

2- If the two forces have the same line of action and opposite directions:



- In this case: $m(\angle\alpha) = 180°$; $\cos\alpha = -1$ and by substitution in the resultant rule, we will find that: $R = |F_1 - F_2|$ and $\overrightarrow{R}$ acts in direction of the force or large magnitude, R is called the minimum magnitude of the resultant.

Example: Find the magnitudes of the minimum and the maximum magnitude of the resultant of the two forces of magnitudes 4 , 7 newtons.

- Maximum magnitude = $R_{Max} = 4 + 7 = 11$ newton acts in the direction of the two forces.
- Minimum magnitude = $R_{Min} = |4 - 7| = 3$ newton acts in direction of force of magnitude 7 newton.

Example

(3) Two forces of magnitudes F , 4 newton act on a particle and the measure of the angle between their directions is 120°, the magnitude of their resultant equals $4\sqrt{3}$ newton. Find the magnitude of $\overrightarrow{F}$ and the measure of the angle that $\overrightarrow{R}$ form with $\overrightarrow{F}$.

Solution

Substituting by: $F_1 = F$, $F_2 = 4$, $R = 4\sqrt{3}$, $\alpha = 120°$

in the rule : $R^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos\alpha$

$\therefore (4\sqrt{3})^2 = F^2 + (4)^2 + 2 \times F \times 4 \cos 120°$

$\therefore F^2 - 4F - 32 = 0$ **i.e.:** $(F + 4)(F - 8) = 0$, **then** $F = 8$ newton or $F = -4$ (refused)

To find the angle between $\overrightarrow{F}$, $\overrightarrow{R}$ we will use the rule: $\tan\theta = \dfrac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$

$\therefore \tan\theta = \dfrac{4 \times \sin 120}{8 + 4 \times \cos 120} = \dfrac{1}{\sqrt{3}}$

$\therefore \overrightarrow{R}$ make an angle of measure 30° with $\overrightarrow{F_1}$

Another solution for the second part:

To find the angle between $\overrightarrow{F}$, $\overrightarrow{R}$ we will use the sine rule: $\dfrac{F_2}{\sin\theta_2} = \dfrac{R}{\sin\theta}$

$\therefore \dfrac{4}{\sin\theta_2} = \dfrac{4\sqrt{3}}{\sin 120}$

$\sin\theta_2 = \dfrac{1}{2}$ by reducing and simplifying

$\therefore \overrightarrow{R}$ makes an angle of measure 30° with $\overrightarrow{F}$

Try To Solve

4. Two forces of magnitudes 6, F Kg.wt act on a particle and the measure of the angle between them is 135°. Find the magnitude of the resultant if the line of action of the resultant make an angle of measure 45° with the line of action of the force whose magnitude is F.

Verbal Expression: Find the resultant of two forces of equal magnitudes if they have the same line of action and act in opposite directions.

Exercises (1 - 1)

Complete the following:

1. The effect of a force on a body is determined by the following: ....
2. The vector of the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ is equal to: ....
3. The maximum value of the resultant of two forces of magnitudes 4 , 6 Newton meeting at a point equals ....
4. The minimum value of the resultant of two forces of magnitudes 5 , 9 Newton meeting at a point equals ....
5. 2 , 3 Newton are two forces, if the angle between them is 60°, then the magnitude of their resultant equals ....

Choose the correct answer from those given:

6. The magnitude of the resultant of the two forces of magnitudes 3 , 5 newton and the measure of the angle between them is 60° equals

   A) 2 N B) 6 N a) 7 N D) 8 N

7. Two forces of magnitudes 3 , 4 N act on a particle and the magnitude of their resultant is 5 N , then the measure of the angle between them equals

A 30°   B 45°   a 60°   D 90°

8. Two equal forces, the magnitude of each of them is 6 N, the magnitude of their resultant is 6N, then the angle between them equals:

A 30°   B 60°   a 120°   D 150°

9. Two forces of magnitudes 3 , F Newton and the measure of the angle between them is 120°. If their resultant is perpendicular to the first force, so the value of F in Newton is

A 1.5   B 3   a $3\sqrt{3}$ D 6

10. If the two forces 6 , 8 N are perpendicular then the sine of the angle of inclination of their resultant with the first force equals:

A $\frac{3}{5}$ B $\frac{4}{5}$ a $\frac{3}{4}$ D $\frac{4}{3}$

Answer the following questions:

11. Two forces of magnitudes 5 , 10 Newton act on a particle and the measure of the angle between them is 120°. Find the magnitude of their resultant and the measure of the angle made by the resultant with the first force.

12. Two forces of magnitudes 3, $3\sqrt{2}$ kg.wt act on a particle and the measure of the angle between them is 45°. Find the magnitude and the direction of their resultant.

13. Two forces of magnitudes 15 , 8 kg.wt act on a particle. If their resultant equals 13 kg.wt, find the angle between the two forces.

14. Two forces of magnitudes 8 , F Newton act on a particle and measure of the angle between them is 120° . If their resultant is F $\sqrt{3}$ N , find the magnitude of F.

15. Two forces of magnitudes 4 , F Newton act on a particle and the measure of angle between them is 135°, If the direction of their resultant is inclined by an angle of measure 45° on F. Find the magnitude of F.

16. Two forces of magnitudes 4 , F Newton act on a particle and the angle between them is 120°. If their resultant is perpendicular to the first force, find the magnitude of F.

17. Two forces of magnitudes F , F $\sqrt{3}$ Newton act on a particle. If the magnitude of their resultant is 2F Newton. Find the measure of the angle between the two forces.

18. Two forces of magnitudes 12 , 15 Newton act on a particle and the (cosine) of the angle between them equals $\frac{-4}{5}$. Find the magnitude of their resultant and the measure of the angle of inclination of the resultant to the first force.

19. Two forces of same magnitude F kg.wt enclose between them an angle of measure 120°. If the two forces are doubled and the measure of the angle between them became 60°, then the

magnitude of their resultant increases by 11 k.g.wt than the first case . Find the magnitude of F.

20 Two forces of magnitudes 12 , F kg.wt act on a point. The first force acts in direction of east and the second force acts in direction 60° south of the west. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° south of the east.

21 F_1 , F_2 are two forces act on a particle and enclose between them an angle of measure 120° and the magnitude of their resultant is $\sqrt{19}$ N , if the angle between them becomes 60° ,then the magnitude of their resultant becomes 7 newton. Find the value of each of F_1 , F_2.

22 Two forces of magnitudes F , 2F kg.wt act on a point, If the magnitude of the second force is doubled, the magnitude of the first force is increased by 15 kg.wt and the direction of their resultant doesn't change. Find the magnitude of F.

1 - 2 Forces resolution

We will learn

- Resolution of a force into two given directions.
- Resolution of a force into two perpendicular directions.

Key - term

- Force Component
- Triangle of forces
- Centre of gravity

Matrials

- Scientific calculator .
- Computer - graph programs.

Preface:

Resolution of a given force into components, generally means finding a group of forces where the known force represents their resultant. We will only study the resolution of a force into two known directions.

Resolution of a force into two given directions

Figure (1): shows resultant vector $\overrightarrow{R}$ which is required to be resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in the two directions $\overrightarrow{OA}$, $\overrightarrow{OB}$ which make angles of measure θ_1 , θ_2 respectively with $\overrightarrow{R}$

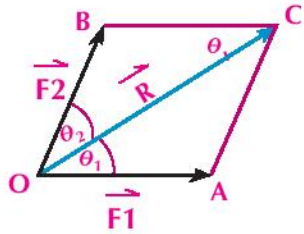


figure (1)

Figure (2): shows triangle of forces where $\overrightarrow{AC} = \overrightarrow{OB}$

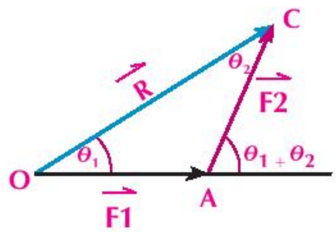


figure (2)

(From the properties of parallelogram) and by applying Sine rule:

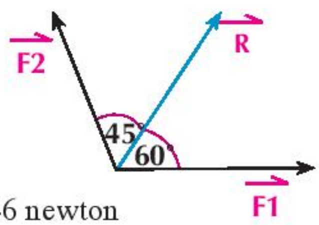
➢ $\frac{F_1}{\sin\theta_2} = \frac{F_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1 + \theta_2)}$

➢ Notice that : $\sin(180 - (\theta_1 + \theta_2)) = \sin(\theta_1 + \theta_2)$

Example

(1) Resolve a force of magnitute 12 newton into two components: inclined to the force by angles of measures 60° , 45° in two different sides of it. (Approximate the result into 4 decimal places)

Solution



by applying sine rule:

$$\frac{F_1}{\sin 45^\circ} = \frac{F_2}{\sin 60^\circ} = \frac{12}{\sin 105^\circ}$$

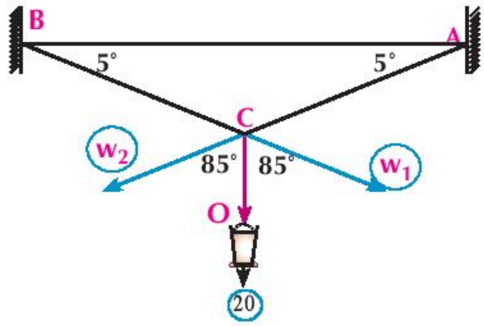
$$\therefore F_1 = \sin 45^\circ \times \frac{12}{\sin 105^\circ} \simeq 8.7846 \text{ newton}$$

$$F_2 = \sin 60^\circ \times \frac{12}{\sin 105^\circ} \simeq 10.7589 \text{ newton}$$

Try To Solve

1. Resolve a force of magnitude 36 newton into two component inclined to the force by angles of measures 30° , 45° in two opposite sides of the force.

Example life applications



A lamp of weight 20 newtons suspended by two metal rods $\overline{AC}$, $\overline{BC}$ inclined to the horizontal by two equal angles, the measure of each is 5°.

- Resolve the weight of the lamp into two components in the direction $\overrightarrow{AC}$, $\overrightarrow{BC}$ approximating the result to the nearest newton.

Solution

The force of the weight (**20 newton**) is represented by a vector **act** vertically downwards, starting from the point c, resolve the weight vector into two components as follow:

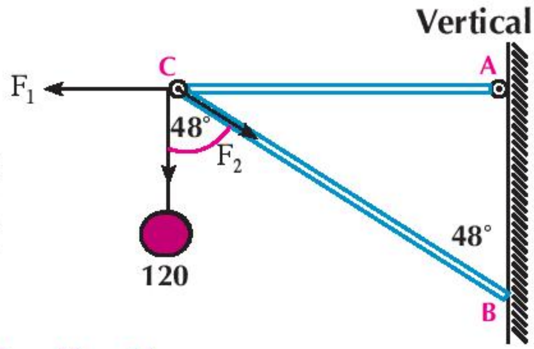
$$\frac{W_1}{\sin 85^\circ} = \frac{W_2}{\sin 85^\circ} = \frac{20}{\sin 170^\circ} \quad \textbf{then:}$$

$$W_1 = W_2 = 20 \times \frac{\sin 85^\circ}{\sin 170^\circ} \quad \text{From that we get:}$$

$$W_1 = W_2 = 114.73713 \simeq 115 \text{ newton.}$$

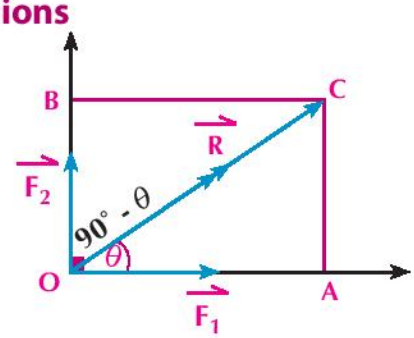
Critical thinking: What happens to the magnitude of the components of the weight in the directions of the two metal rods if the measure of the inclination angle to the horizontal decreated to be smaller than 5°? And what do you expect to the components when the rods become horizontal? Justify your answer.

Try To Solve



2. **In the opposite figure:**
Resolve the vertical force of magnitude 120gm.wt into two components one of them is the horizontal and the other inclined by an angle of measure 48° to the line of action of the force.

Resolution of a force into two perpendicular directions



If $\overrightarrow{R}$ acts on a particle (O) as in the opposite figure and their perpendicular components are $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ where $\overrightarrow{F_1}$ is inclined by an angle of measure θ with $\overrightarrow{R}$, then the parallelogram will be a rectangle ACBO, by applying sine rule on triangle OAC then:

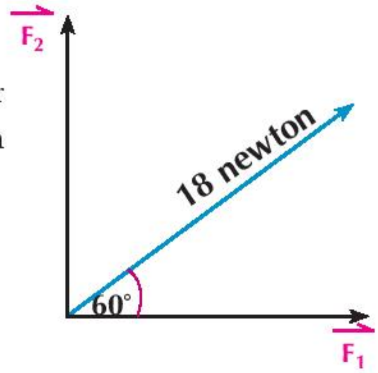
- $\dfrac{F_1}{\sin(90^\circ - \theta)} = \dfrac{F_2}{\sin \theta} = \dfrac{R}{\sin 90^\circ}$ $\quad \therefore \dfrac{F_1}{\cos \theta} = \dfrac{F_2}{\sin \theta} = R$

and hence we deduce that:

- F_1 (magnitude of the component in a given direction) = R cos θ
- F_2 (magnitude of the component in the direction perpendicular to the given direction) = R sin θ

Example

3 Resolve a force of magnitude 18 newton into two perpendicular components where one of them inclines to the force by an angle of measure 60°.



Solution

$F_1 = 18 \cos 60° = 18 \times \frac{1}{2} = 9$ newton

$F_2 = 18 \sin 60 = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$ newton.

Try To Solve

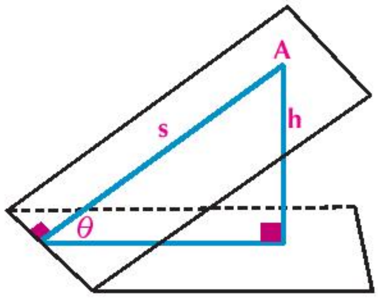
3 Resolve a force of magnitude $6\sqrt{2}$ newton that acts in the north-east direction into two components one of them acts in the eastern direction, the other in the northern direction.

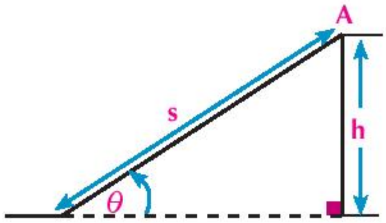
Inclined Plane

It is a surface that inclines to the horizontal plane by and angle whose measure is θ, $0<\theta<\frac{\pi}{2}$ as shown in the opposite figure

The line of the greatest slope is the line lie in the inclined plane orthogonal to the line of intersection of the inclined plane and the horizontal plane the blue line in the given figures, if we denote to its length by (**s**), height of the inclined plane by (**h**), angle of inclination to the horizon by (θ), then $\sin\theta = \frac{h}{s}$.

Such that : (h) represents the distance between the Point A and the horizontal, (S) represents the distance between the point A and the line of intersection between the inclined plane and the horizontal plane.





Example

4 A body of weight 6 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30°. Find the components of the weight in the direction of the line of the greatest slope and the direction normal to it.

Solution

Figure (1): shows the force of the weight of magnitude 6 newton which acts vertically downwards, $\overrightarrow{F_1}$ is the component of the weight in the direction of the line of the greatest slope of the inclined plane downwards, $\overrightarrow{F_2}$ is the

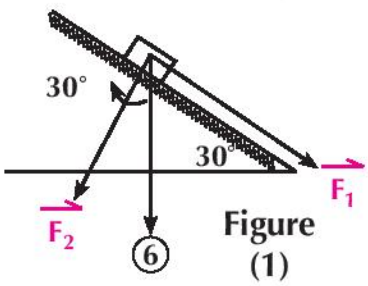


Figure (1)

second component which acts normal to the plane downwards.

($\overrightarrow{F_1}$) : the component of the weight on the direction of the line of the greatest slope.

i.e.: $F_1 = 6 \sin \theta$,

$= 6 \text{ Sin } 30° = 6 \times \frac{1}{2} = 3$ Newton

($\overrightarrow{F_2}$) :the component of the weight acts normal to the plane downwards:

i.e.: $F_2 = 6 \text{ Cos } \theta$

$= 6 \text{ Cos} 30° = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$ Newton

Verbal Expression: Are the 2 components of force smaller than the force $\overrightarrow{F}$ Itself? Explain your Answer?.

Try To Solve

(4) A rigid body, the magnitude of its weight is 36 newton is placed on a plane inclined to the horizontal at an angle of measure 60°. Find the two components of the weight in a direction parallel to the plane downwards and the direction normal to it.

Add to your knowledge

Rigid body's center of gravity

Is the point where it always the vertical line passing through the suspension point when the body is hanged from any point of it , for example:

(1) The center of gravity of a spherical regular homoginious body is the point where the center of body is located.

(2) The center of gravity of a rod of regular thickness and density is the midpoint of that rod.

Exercises (1-2)

Complete the following:

(1) A force of magnitude 6 Newton acts in direction of North. It is resolved into two perpendicular components, so its component in direction of the East equals .... Newton.

(2) A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components, so its component in the direction of Northern East equals .... Newton.

(3) **In figure (1):**

If the force $\overrightarrow{R}$ is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which make with the force $\overrightarrow{R}$ two angles of measures 30°, 45° from different directions of its line of action, $\| \overrightarrow{R} \| = 12$ newton,

So: $F_1 =$ .... Newton , $F_2 =$ .... Newton.

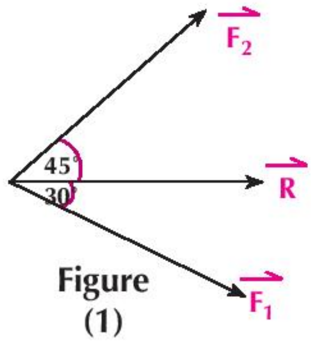


Figure (1)

(4) **In figure (2):**

If the force $\overrightarrow{R}$ is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which make with the force $\overrightarrow{R}$ two angles of measure 45°, 90° from different directions of its line of action and $\| \overrightarrow{R} \| = 18$ Newton, So: $F_1 =$ .... Newton, $F_2 =$ .... Newton

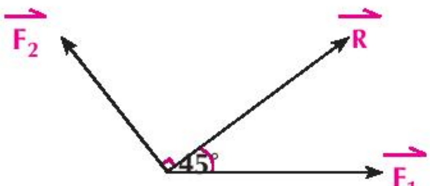


Figure (2)

(5) In figure (3):

If the force $\overrightarrow{F}$ is resolved into two perpendicular components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and the force vector $\overrightarrow{F}$ bisects the angle between the directions of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\|\overrightarrow{F}\| = 6\sqrt{2}$ kg. wt

so: $\|\overrightarrow{F_1}\|$ = ... kg wt ,

$\|\overrightarrow{F_2}\|$ = ... kg wt.

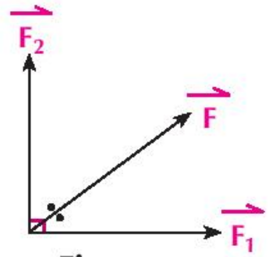


Figure (3)

(6) In figure (4):

Force of magnitude $12\sqrt{2}$ newton acts in direction 30° North of the west.

- Magnitude of the component of the force in the western direction = ... Newton.
- Magnitude of the component of the force in the northern direction = ... Newton.

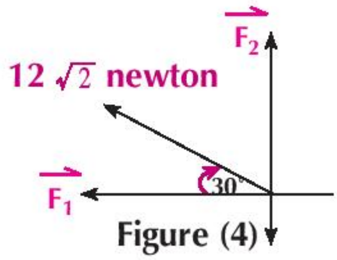


Figure (4)

(7) A force of magnitude 600 gm.wt acts on a particle. Find its two components in two directions making with the force two angles of measures 30° , 45°.

(8) A force of magnitude 120 newton acts in direction of the North of east. Find its two components in the direction of East and in the direction of North.

(9) Resolve a horizontal force of magnitude 160 gm.wt in two perpendicular directions one of them inclined to the horizontal with an angle of measure 30° upwards.

(10) A force of magnitude 18 newton acts in the direction of South. Find its two components in the 2 directions , 60° East of the South and the other direction towards 30° West of the South.

(11) A rigid body of weight 42 newton is placed on a plane inclined to the horizontal with a angle of measure 60°. Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it.

Creative thinking:

(12) An inclined plane of length 130 cm and height 50 cm, a rigid body of weight 390 gm wt. is placed on it. Find the two components of the weight in the direction of the line of the greatest slope of the plane and the direction normal to it.

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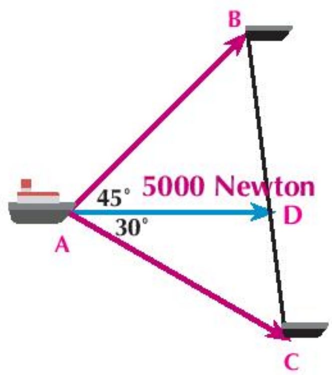
(13) A cruiser is pulled by two ships B and C using two strings hanged to a point A on the cruiser ,the angle between the two strings equals 75° , if the angle between one of the strings and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 Newton and acts on $\overrightarrow{AD}$ Find the tension in the two strands.



The resultant of coplanar forces meeting at a point

Think and discuss

We have studied how to find the resultant of two forces acting on a rigid body in a specific point. The resultant is represented geometrically by the diagonal of the parallelogram drawn by the two forces as two adjacent sides on it.

Can you find the resultant of a set of forces meeting at a point geometrically?

We will learn

- The resultant of a set of coplanar forces meeting at a geometrical point .
- The resultant of a set of coplanar forces meeting a point analytically.

Key - term

- Resultant
- Algebraic component
- Unit vector

Matrials

- Scientific calculator
- Computer graphics program

Learn

Resultant of a set of coplanar forces act at a point geometrically:

If set of forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$, ..., $\overrightarrow{F_n}$ act on **a particle as shown in figure (1).**

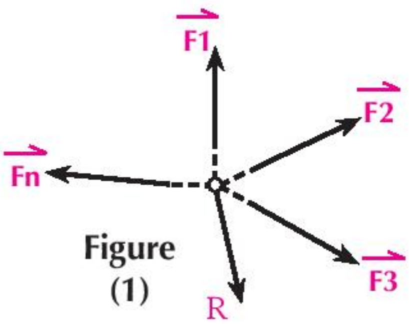


Figure (1)

Using a suitable drawing scale.

draw $\overrightarrow{OA}$ represents $\overrightarrow{F_1}$, $\overrightarrow{AB}$ represents $\overrightarrow{F_2}$, $\overrightarrow{BC}$ represents $\overrightarrow{F_3}$ and so on, ... till $\overrightarrow{F_n}$ is represented by $\overrightarrow{DH}$.

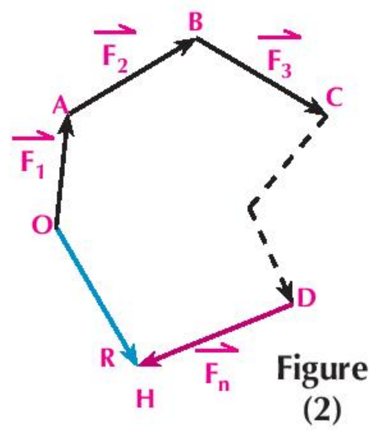


Figure (2)

Then the vector $\overrightarrow{OH}$ in the opposite cyclic order represents the resultant of the forces, where:

$$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + ... + \overrightarrow{F_n}$$

and the polygon is called polygon of forces, it is easy to notice that forming a polygon of forces is the result of applying the triangle of forces several consecutive times.

The resultant of coplanar forces meeting at apoint analytically

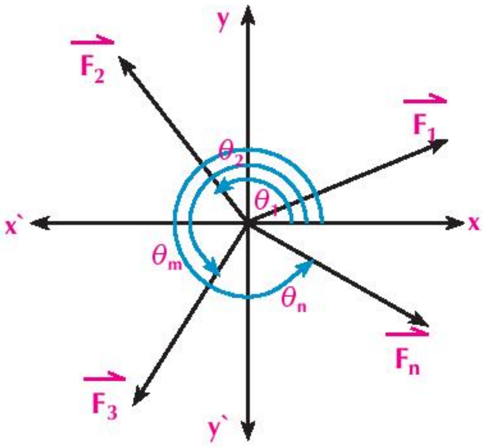
If the coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$,..... , $\overrightarrow{F_n}$ act at a point in the coordinate plane system, to make the polar angles θ_1 , θ_2 , θ_3 , ... , θ_n respectively and $\overrightarrow{i}$, $\overrightarrow{j}$ are two fundamental unit vectors in directions $\overrightarrow{OX}$, $\overrightarrow{OY}$ then: $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + ... + \overrightarrow{F_n}$

Resolving each force in the perpendicular directions $\overrightarrow{OX}$, $\overrightarrow{OY}$ then:

$$\overrightarrow{R} = (F_1 \cos\theta_1 \overrightarrow{i} , F_1 \cos\theta_1 \overrightarrow{j})$$
$$+ (F_2 \cos\theta_2 \overrightarrow{i} , F_2 \cos\theta_2 \overrightarrow{j})$$
$$+ .... + (F_n \cos\theta_n \overrightarrow{i} , F_n \sin\theta_n \overrightarrow{j})$$

$$\overrightarrow{R} = (F_1 \cos\theta_1 + F_2 \cos {}_2\theta + ... + F_n \cos\theta_n) \overrightarrow{i}$$
$$+ (F_1 \sin\theta_1 + F_2 \sin {}_2\theta + ... + F_n \sin\theta_n) \overrightarrow{j}$$

$$\overrightarrow{R} = (\sum_{r=1}^{n} F_r \cos\theta_r) \overrightarrow{i} + (\sum_{r=1}^{n} F_r \sin\theta_r) \overrightarrow{j}$$



- **Where the Expression:** $\sum_{r=1}^{n} F_r\cos\theta_r$ is called the sum of algebraic components of the forces in direction $\overrightarrow{OX}$ and denoted by x.
- **The Expression:** $\sum_{r=1}^{n} F_r\sin\theta_r$ is called the algebraic sum of the components of the forces in direction $\overrightarrow{OY}$ and denoted by y.

Then $\overrightarrow{R} = x \overrightarrow{i} + y \overrightarrow{j}$

and if R is the norm of the resultant , θ is its polar angle then,

$R = \sqrt{x^2 + y^2}$, $\tan\theta = \frac{y}{x}$

Add to your knowlage

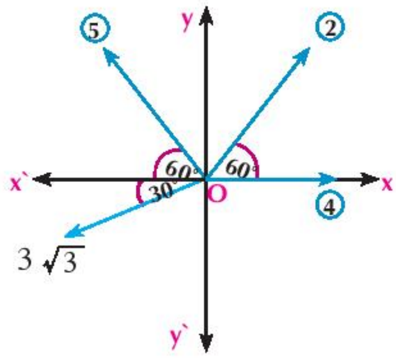
The symbol is called segma, Σ denoted for summation and the expression $\sum_{r=1}^{n}$ means the summation of n elements starting from the first element.

Example

1. Four coplanar forces act on a particle, the first of magnitude 4 newton, acts in the East direction, The second of magnitude 2 newton, acts in direction 60° North of the East, the third of magnitude 5 newton, acts in direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° west of the south. Find the magnitude and the direction of their resultant.

Solution

Forces of magnitude 4 , 2 , 5 , 3 $\sqrt{3}$ newton, the measures of their polar angles are zero° , 60° , 120° , 210° respectively. Then we will find the algebraic sum of the components of the forces in direction of $\overrightarrow{OX}$, $\overrightarrow{OY}$



$x = 4\cos 0° + 2\cos 60° + 5\cos 120° + 3\sqrt{3}\cos 210°$

$= 4 + 2\times\frac{1}{2} - 5\times\frac{1}{2} - 3\sqrt{3}\times\frac{\sqrt{3}}{2} = 4 + 1 - \frac{5}{2} - \frac{9}{2} = -2$

$y = 4\sin 0° + 2\sin 60° + 5\sin 120° + 3\sqrt{3}\sin 210°$

$= 0 + 2\times\frac{\sqrt{3}}{2} + 5\times\frac{\sqrt{3}}{2} - 3\sqrt{3}\times\frac{1}{2}$

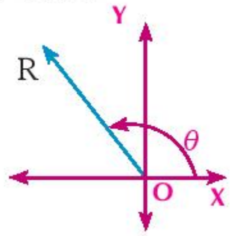
$= \sqrt{3} + \frac{5}{2}\sqrt{3} - \frac{3}{2}\sqrt{3} = 2\sqrt{3}$

$\therefore \overrightarrow{R} = 2\sqrt{3}\,\overrightarrow{j} - 2\,\overrightarrow{i}$ $\quad\therefore R = \sqrt{x^2+y^2} = \sqrt{4+12} = \sqrt{16}$ $\therefore R = 4$ newton

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\because x < 0,\ y > 0$ $\qquad\therefore \theta = 120°$

meaning that the magnitude of the resultant = 4 newtons, its polar angle is of measure 120°



Try To Solve

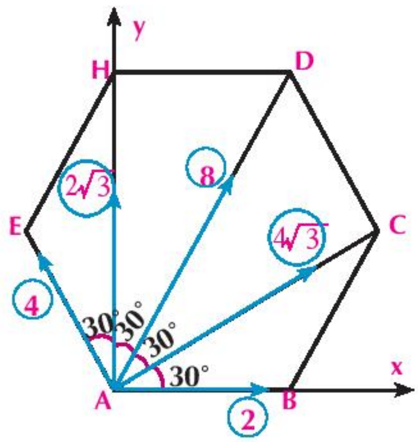
1) Coplanar forces of magnitudes 10 , 20 , 30 $\sqrt{3}$ and 40 newton act at a point where the measure of the angle between the directions of the first and the second forces= 60° , between the directions of the second and the third = 90° and between the directions of the third and the fourth = 150°. Find the magnitude and the direction of their resultant

Example

2) A B C D H E is a regular hexagon. Forces of magnitudes 2 , 4 $\sqrt{3}$, 8 , 2 $\sqrt{3}$ and 4 kg.wt act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AH}$, $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant.

Solution

By considering $\overrightarrow{AB}$ is the direction of the first force so the polar angles to the forces are : 0°, 30°, 60°, 90°, 120° respectively.



$\therefore x = 2\cos 0° + 4\sqrt{3}\cos 30° + 8\cos 60° + 2\sqrt{3}\cos 90° + 4\cos 120°$

$x = 2 + 4\sqrt{3}\times\frac{\sqrt{3}}{2} + 8\times\frac{1}{2} + 2\sqrt{3}\times 0 - 4\times\frac{1}{2}$

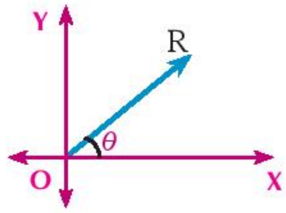
$x = 2 + 6 + 4 - 2 = 10$ newton

$y = 2\sin 0^\circ + 4\sqrt{3}\sin 30^\circ + 8\sin 60^\circ$

$+ 2\sqrt{3}\sin 90^\circ + 4\sin 120^\circ$

$y = 0 + 4\sqrt{3}\times\frac{1}{2} + 8\times\frac{\sqrt{3}}{2} + 2\sqrt{3} + 4\times\frac{\sqrt{3}}{2}$

$y = 2\sqrt{3} + 4\sqrt{3} + 2\sqrt{3} + 2\sqrt{3} = 10\sqrt{3}$ newton



$\overrightarrow{R} = 10\,\overrightarrow{i} + 10\sqrt{3}\,\overrightarrow{j}$

$\therefore R = \sqrt{x^2 + y^2} = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20$ newton

$\tan\theta = \frac{y}{x} = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\because x > 0,\ y > 0 \qquad \therefore m(\angle\theta) = 60^\circ$

i.e. The resultant acts in the direction of $\overrightarrow{AD}$

Exercises (1 - 3)

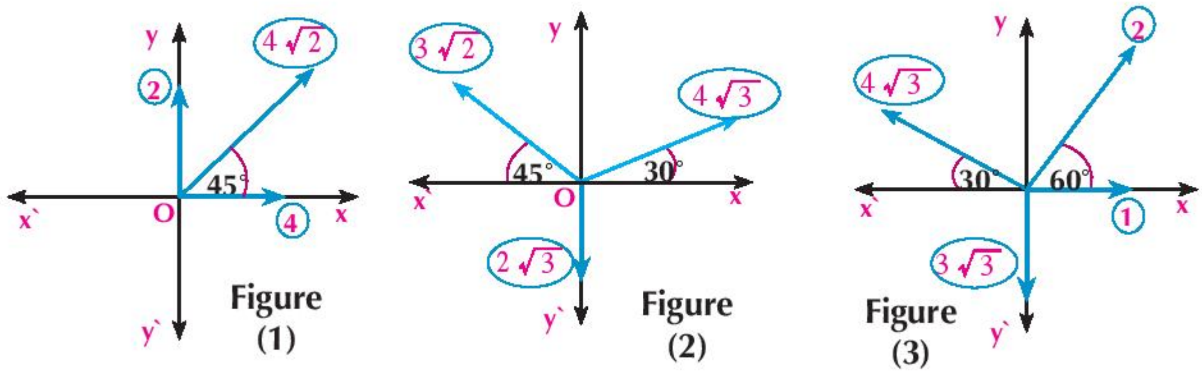
Complete the following:

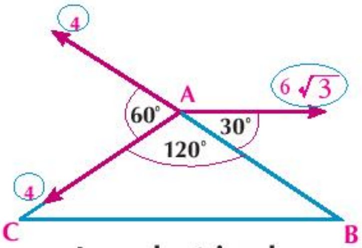
1. If the forces $\overrightarrow{F_1} = 2\overrightarrow{i}$, $\overrightarrow{F_2} = \overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_3} = 6\overrightarrow{j}$ then:
the magnitude of the resultant of the forces = ... and its direction = ...

2. If the forces $\overrightarrow{F_1} = 2\overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_2} = 4\overrightarrow{i} - 8\overrightarrow{j}$, $\overrightarrow{R} = 2a\overrightarrow{i} - 3b\overrightarrow{j}$
then: a = ... , b = ...

3. If $\overrightarrow{F_1} = 3\overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_2} = a\overrightarrow{i} - \overrightarrow{j}$, $\overrightarrow{F_3} = 4\overrightarrow{i} - b\overrightarrow{j}$, $\overrightarrow{R} = 6\overrightarrow{i} - 4\overrightarrow{j}$
then: a = ... , b = ...

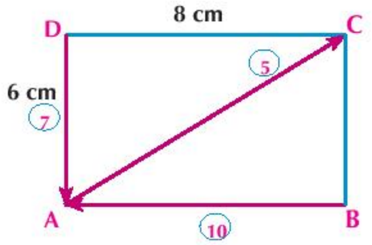
4. Find the magnitude and the direction of resultant of the forces shown in each of the following figures:





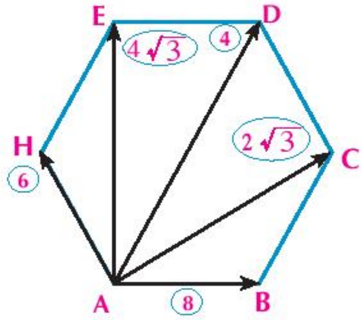
Isosceles triangle

Figure (4)



Rectangle it's dimensions are 6cm , 8cm

Figure (5)



Regular hexagon

Figure (6)

5) The forces 3 , 6 , $9\sqrt{3}$ and 12 kg.wt act on a particle and the measure of the angle between the first and the second is 60° , between the second and the third is 90° and between the third and the fourth is 150°. Find the magnitude and the direction of resultant of these forces.

6) Three forces of magnitudes 10 , 20 , 30 newton act at a particle. The first acts towards the east and the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° South of the west. Find the magnitude and the direction of resultant of these forces.

7) Four forces of magnitudes 10 , 20 , $30\sqrt{3}$ and 40 gm.wt act on a particle, the first acts in the east direction and the second acts in the direction 60° north of the east and the third acts in the direction 30° north of the west and the fourth acts in the direction making an angle of 60° South of the east. Find the magnitude and direction of resultant of these forces.

8) A B C is an equilateral triangle , M is the point of intersection of its medians. The forces of magnitudes 15 , 20 , 25 newton act on a particle in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$. Find the magnitude and the direction of the resultant of these forces.

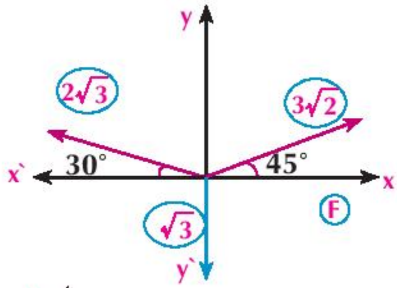
9) ABCD is a square of side length 12cm , H ∈ $\overline{BC}$ so BH = 5cm. Forces of magnitudes 2 , 13 , $4\sqrt{2}$ and 9 gm.wt act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$, $\overrightarrow{AD}$ respectively. Find the magnitude of the resultant of these forces.

10) If $\overrightarrow{F_1} = 5\overrightarrow{i} + 3\overrightarrow{j}$, $\overrightarrow{F_2} = a\overrightarrow{i} + 6\overrightarrow{j}$ and $\overrightarrow{F_3} = 14\overrightarrow{i} + b\overrightarrow{j}$ are three coplanar forces meeting at a point and their resultant $\overrightarrow{R} = (10\sqrt{2}\ , 135°)$ Find the values of a , b

.

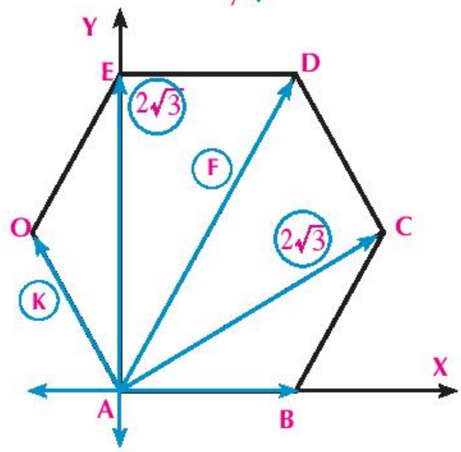
(11) In the opposite figure :

If the magnitude of the resultant of the forces equals $3\sqrt{2}$ Newton, then find the value of F and the measure of the angle between the line of action of the resultant and the first force



(12) In the opposite figure :

If the magnitude of the resultant of the forces equals 20 Kg.wt and acts in the direction of $\overrightarrow{AD}$ Find the values of F and K.



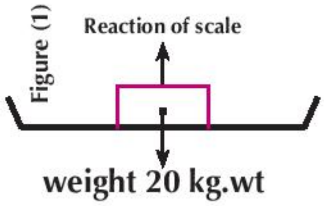
Equilibrium of a rigid body under the effect of coplanar forces meeting at a point

If two forces or more act on a rigid body and the status of the body does not change, it is said that the two forces or the forces are equilibrium and the body is in an equilibrium state. The simplest type of equilibrium resulted by the effect of two forces on a rigid body.

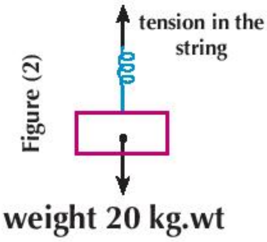
Equilibrium of a rigid body under the effect of two forces

Co-operative work

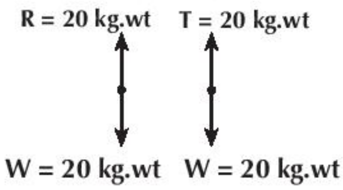
1- Put a body of weight 20 kg.wt on the scale pan of a horizontal pressure balance and notice the reading of the balance. figure (1)



2- Ask your classmate to suspend the same body by a light smooth string and the end of the string by a hook of spring balance then to notice the reading of the spring balance in the of case of rest . figure (2)



3- Compare the results in the two experiments. What do you notice?



Notice:

➢ Each of the reaction r in the first and tension force T in the second experiment equals 20 kg.wt which is the weight of the body.

Learn

Terms of balancing a rigid body under the effect of two forces

A rigid body is balanced under the effect of two forces only if these two forces are:

1- equal in magnitude.

2- opposite in direction.

3- their lines of action are on the same straight line.

We will learn

- Equilibrium of a rigid body under the effect of two forces.
- Equilibrium of a rigid body under the effect of three forces meeting at a point.
- Triangle of forces rule
- Lami`s rule
- The three forces theorem
- Equilibrium of a set of coplanar forces act at a point.

Key - term

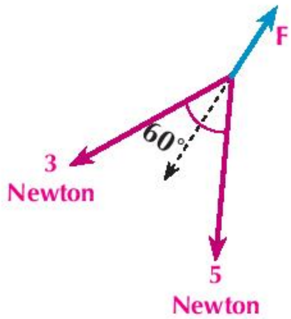
- Triangle of forces rule
- Lami`s rule
- Polygon of forces

Matrials

- Scientific calculator
- Computer graphics programs.

Example

1. If a force of magnitude F is in equilibrium with two forces of magnitudes 5 Newton and 3 Newton act at a point and enclosed between them an angle of measure 60°. Find the value of F.



Solution

We can find the resultant of the two forces 5,3 N from the rule:

$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$ $\therefore R = \sqrt{25 + 9 + 2 \times 5 \times 3 \cos 60^\circ}$

$\therefore R = \sqrt{25 + 9 + 15} = \sqrt{49} = 7$ newton

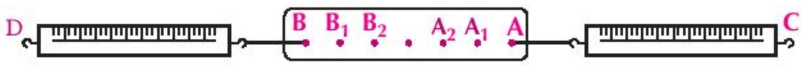
$\because$ the force (F) and the resultant of the two forces (R) are in equilibrium $\therefore F = 7$ newton

Try To Solve

1. If a force of magnitude F is in equilibrium with two perpendicular forces of magnitudes 5 Newton and 12 Newton. Find the magnitude of the force F.

Transfer the point of action of a force to any point on its line of action:

Activity



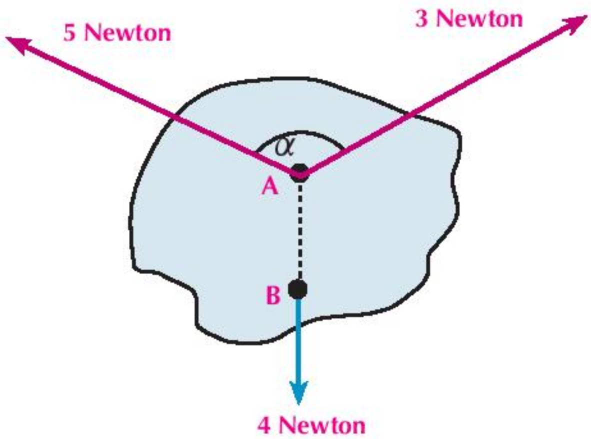
1. Use the following tools: spring balance – thin metal disk -water balance – ruler
2. Set the table horizontally using the water balance.
3. Join the disk by two light strings at the two holes A , B then join the other two ends of the strings by the spring balance.
4. Set the ring of one of the two balances in a nail fixed on the table at (C) ,then pull the other balance and set it at (D) to another nail far from the other nail so that the two strings are tensioned as in the figure.
5. Find the magnitude of the tension acting in the string and record the results.
6. Change the position of the end of the string from point A to A_1 , A_2, A_3 ... also change the position of the end of the other string from point B to B_1 , B_2, B_3 ... recognize the reading of the spring balance in each case and record the results ... what do you notice ?

We note that: the two readings are equal in the state of equilibrium.

From the previous activity we deduce that :

If a rigid body is in equilibrium under the effect of two forces, then we can transfer the point of action for any force to another point on its line of action without any impact on the equilibrium of the body.

Example



(2) The forces 3, 4 and 5 Newton are in equilibrium as in the opposite figure Find the measure of the angle between the two forces 3N,5N

Solution

∵ The set of forces are in equilibrium

∴ The resultant of the two forces 3N , 5N is equilibrium with the force 4N.

If the measure of the angles between the two forces 3N , 5N is α, then :

$R^2 = F^2_1 + F_2^2 + 2F_1 F_2 \cos \alpha$

By substitution: $R = 4$, $F_1 = 3$, $F_2 = 5$

$16 = 9 + 25 + 2 \times 3 \times 5 \cos \alpha$ ∴ $30 \cos \alpha = -18$

So that $\cos \alpha = \frac{-3}{5}$

∴ $m(\angle \alpha) = 180° - 53° 7' 49'' = 126° 52' 11''$

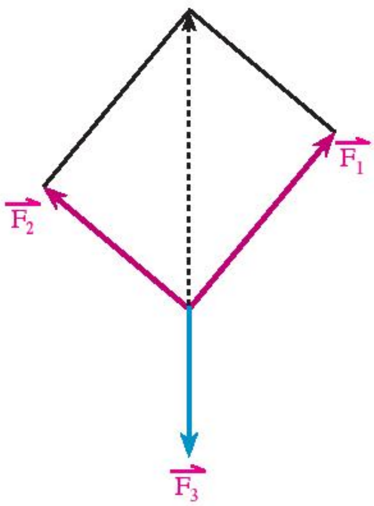
Try To Solve

(2) If the forces 7,8 and 13 Newton are in equilibrium ,Find the measure of the angle between the first and the second two forces .

Equilibrium of a rigid body under the action of three coplanar forces meeting at a point

You have studied the necessary and sufficient conditions of the equilibrium of a rigid body under the effect of two forces .Now we will study the equilibrium of three coplanar forces their lines of actions met at a point, these forces either act in a point (or a particle) or act on a body such that their lines of actions met at a point.

Learn



If it is possible to represent three coplanar forces meeting at a point by the sides of triangle of directions taken in the same cyclic order, then the forces are equilibrium.

So, in the opposite figure: The three forces will be in equilibrium if their magnitudes represent the sides length of a triangle taken in the same cyclic order.

Verbal Expression

Show which of the forces whose magnitudes are listed below are equilibrium? Explain your answer.

Consider that the forces are in different direction and act on one point:

(a) 3,5,9 N (b) 3,5,7 N (c) 4,10,6 N

Triangle of forces rule

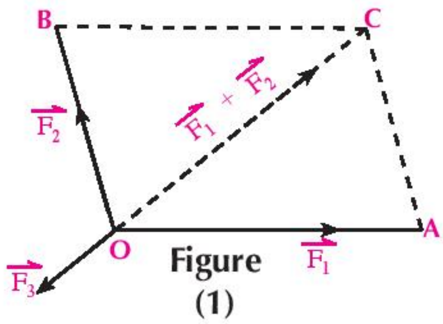


Figure (1)

Figure (1): represents the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ that act at a rigid body in directions $\overrightarrow{OA}$, $\overrightarrow{OB}$.

The resultant of the two forces is ($\overrightarrow{F_1} + \overrightarrow{F_2}$) which act on the diagonal $\overrightarrow{OC}$ of the parallelogram OACB.

But $\overrightarrow{F_3}$ equals in magnitude and opposite in direction to ($\overrightarrow{F_1} + \overrightarrow{F_2}$)

$\therefore \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \overrightarrow{0}$ $\therefore \overrightarrow{F_1}$ ، $\overrightarrow{F_2}$ ، $\overrightarrow{F_3}$ are equilibrium forces.

Check your understanding:

Show that the forces $\overrightarrow{F_1}$ ، $\overrightarrow{F_2}$ ، $\overrightarrow{F_3}$ are equilibrium such that:

$F_1 = 2\overrightarrow{i} - \overrightarrow{j}$, $F_2 = \overrightarrow{i} + 3\overrightarrow{j}$, $F_3 = -3\overrightarrow{i} - 2\overrightarrow{j}$

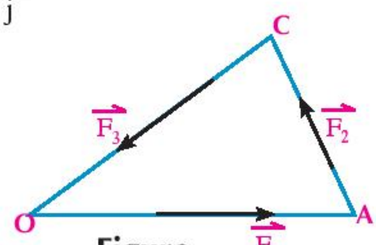


Figure (2)

Figure (2): represents the triangle of forces of three equilibrium coplanar forces $\overrightarrow{F_1}$ ، $\overrightarrow{F_2}$ ، $\overrightarrow{F_3}$, where the lengths of sides of the triangle are in a proportion with the magnitudes of the corresponding forces.

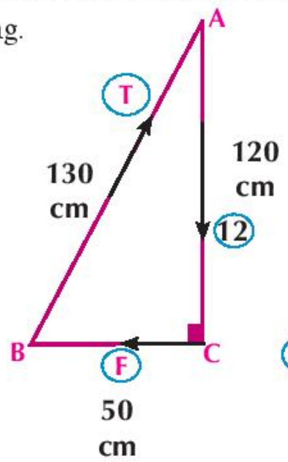
i.e.: $\frac{F_1}{OA} = \frac{F_2}{AC} = \frac{F_3}{CO}$

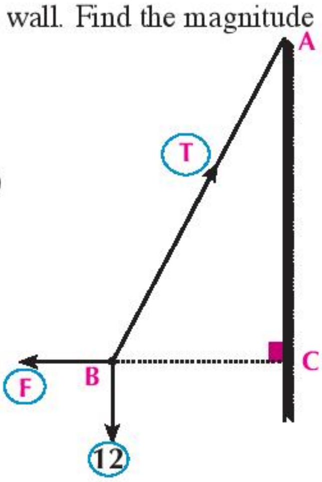
So that: If three coplanar forces met at a point are equilibrium and a triangle is drawn such that its sides are parallel to the lines of action of the forces and taken the same cyclic order, then the lengths of the sides of the triangle are proportional to the magnitudes of their corresponding forces

Think: Use the sin rule to prove "Triangle of forces rule".

Example

(3) A body of weight 12 Newton is hanged from one end of a light string whose length 130 cm. The other end is fixed at a point in the vertical wall, the body is pulled by a horizontal force which makes the body in equilibrium when it is at distance 50 cm from the wall. Find the magnitude of each of the force and the tension in the string.





Solution

The weight is in equilibrium under the effect of three forces:

- The weight of magnitude 12 N that acts vertically downwards.
- The horizontal force F.
- The tension in the string which acts in the direction $\overrightarrow{BA}$

In $\triangle$ BAC We can find the length of $\overline{AC}$ from pythagorus rule.

$AC = \sqrt{(130)^2 - (50)^2} = 120$ cm

$\triangle$ BAC is the a triangle of forces: $\frac{T}{130} = \frac{12}{120} = \frac{F}{50}$ T = 13 newton , F = 5 newton

Try To Solve

3) A weight of 16 Newton is hanged by a string of length 50 cm and the other end of the string is fixed on a point in the ceiling of a room . The weight is pulled by a horizontal force till it becomes equilibrium when it is at a distance 40 cm from the ceiling , find the magnitude of the force and the tension in the string.

lami's rule:

If the coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ act at a point and equilibrium as in figure (1) ,then it could be represented by the sides of a triangle taken in the same cyclic order as in figure (2)

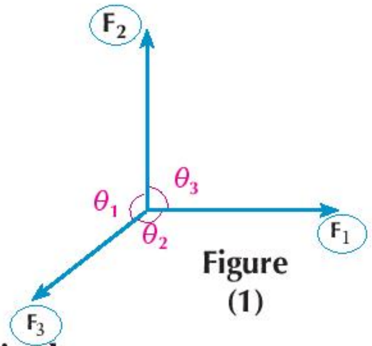


Figure (1)

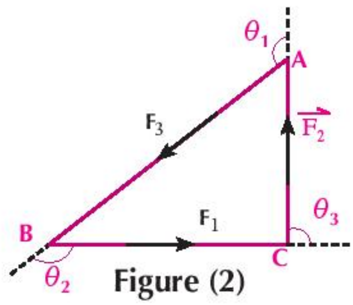


Figure (2)

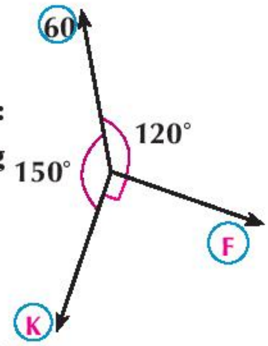
Using the sine law:

$$\frac{BC}{\sin(180 - \theta_1)} = \frac{CA}{\sin(180 - \theta_2)} = \frac{AB}{\sin(180 - \theta_3)} \quad \textbf{i.e.} \quad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

If a body is in equilibrium under the effect of three forces meeting at a point, then the magnitude of each force is proportional to the sine of the angle between the two other forces

Example

4) Three coplanar forces of magnitudes 60, f and k Newton meeting at a point are in equilibrium. If the measure of the angle between the line of action of 1st and 2nd forces is 120° and between the 2nd and the 3rd is 90° Find the value of each of f and k.



Solution

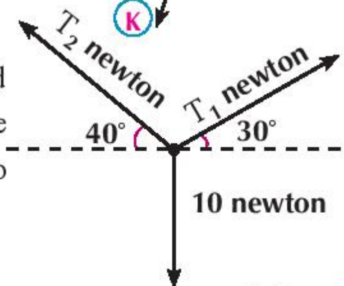
The system is in equilibrium under the effect of the following three forces:

a force of magnitude 60 N , forces of magnitudes F , k N By applying lami's rule we get: $\frac{60}{\sin 90^\circ} = \frac{F}{\sin 150^\circ} = \frac{K}{\sin 120^\circ}$

$\frac{60}{1} = 2F = \frac{2K}{\sqrt{3}}$ **i.e.:** F = 30 newton , $K = 30\sqrt{3}$ newton

Try To Solve

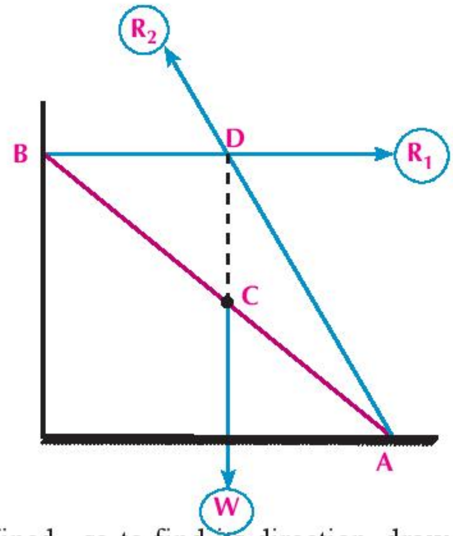
4) In the opposite figure: A weight of magnitude 10 Newton is suspended by two strings, the first is inclined by an angle of measure 30° to the horizontal, and the second is inclined by an angle of measure 40° to the horizontal. Find T_1 , T_2 in case of equilibrium.



1 - 4 Equilibrium of a rigid body under the effect of coplanar forces meeting at a point

Rule:

If a rigid body is in equilibrium under the effect of three coplanar non parallel forces, then the lines of action of the three forces meet at one point.



For example: If a uniform rod of weight w rests with its end B on a smooth vertical wall and with its end A on rough horizontal ground, then:

- The weight of the rod acts at the midpoint of the rod vertically downwards.
- Reaction of the wall R_1 is perpendicular to the smooth wall in the direction $\overrightarrow{BD}$.
- Reaction of the rough ground R_2, its direction is not defined , so to find its direction, draw $\overrightarrow{AD}$ which passes through the point D (point of intersection of the two lines of action of $\overrightarrow{W}$ and $\overrightarrow{R_1}$) as in the figure.

Example

(5) A metallic smooth regular sphere of weight 1.5 kg .wt. and radius length 25 cm is suspended from a point B on its surface by a string of length 25 cm. Its other end A is fixed at a point in a smooth vertical wall to be in equilibrium as it rests on the wall. Find the magnitude of the tension in the string and the reaction of the wall.

Remember that

The center of graphity of a homaginious sphere is its graphical center

Solution

The sphere is in equilibrium under the effect of three forces:

- The weight of the sphere whose magnitude is 1.5 kg wt. acts vertically downwards .
- The reaction of the wall of magnitude R, acts at the point of touch of the sphere with the wall in a perpendicular direction to the wall, hence it passes through the center of the sphere M.
- The tension in the string of magnitude T acts in the direction of $\overrightarrow{BA}$,

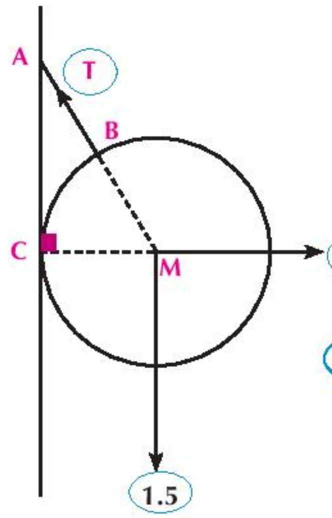
∵ The weight force and the reaction force meet at M.

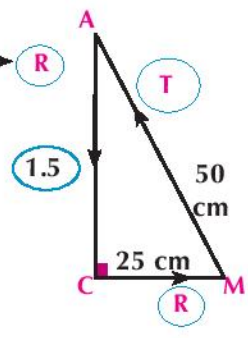
∴ The line action of the tension force in the string should pass through the point M.

i.e. $\triangle$ MAC is the triangle of forces where: MA = MB + BA, MA = 25 + 25 = 50cm. CM = 25 cm.

$\triangle$ AMC is a right-angled triangle

: $AC = \sqrt{(50)^2 - (25)^2} = 25\sqrt{3}$ cm





Apply triangle of force Rule

$$\frac{T}{50} = \frac{1.5}{25\sqrt{3}} = \frac{R}{25} \qquad \therefore T = \sqrt{3} \text{ kg.wt} , \quad R = \frac{\sqrt{3}}{2} \text{ kg.wt}$$

Think: Can you solve the previous example using another methods? mention these methods, then use one of them to solve the previous example.

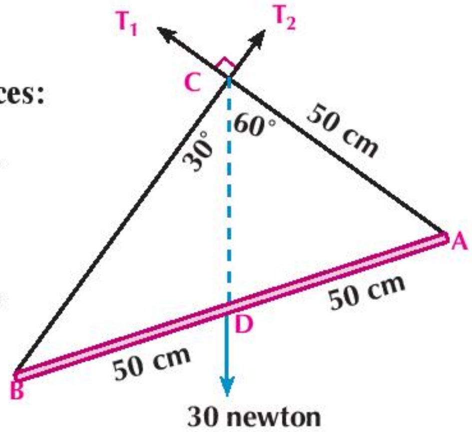
Try To Solve

(5) A metallic smooth regular sphere of weight 100 gm.wt and radius length 30 cm is suspended from a point on its surface by a string of length 20 cm. Its other end is fixed at a point in a smooth vertical wall to be in equilibrium. Find the magnitude of the tension in the string and the magnitude of the reaction of the wall.

Example

(6) A uniform rod of length 100 cm and weight 30 N is suspended at its ends freely by two perpendicular strings, their two ends are fixed at a hook. If the length of one string is 50 cm. Find the magnitude of the tension in each of the two strings.
When the rod is hanged freely and in equilibrium.

Solution



The rod is in equilibrium under the effect of three forces:

The weight 30 N, act vertically down in the middle of the rod.
The tension in the two strings T_1, T_2 that act on $\overrightarrow{AC}$, $\overrightarrow{BC}$ respectivly and intersected prependiculary at C.

$\because$ $\overrightarrow{CD}$ is drawn from the vertex of the right angle to the midpoint of the hypotenuse.

$\because$ D is the midpoint of the hypotenuse $\overline{AB}$

$\therefore CD = \frac{1}{2}AB = 50cm$ $\quad\therefore$ ACE is an equilateral triangle

$\therefore m(\angle ACD) = 60°$, $m(\angle BCD) = 30°$

Applying Lami's rule we get:

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{30}{\sin 90°} \qquad T_1 = 15 \text{ newton}, \; T_2 = 15\sqrt{3} \text{ newton}$$

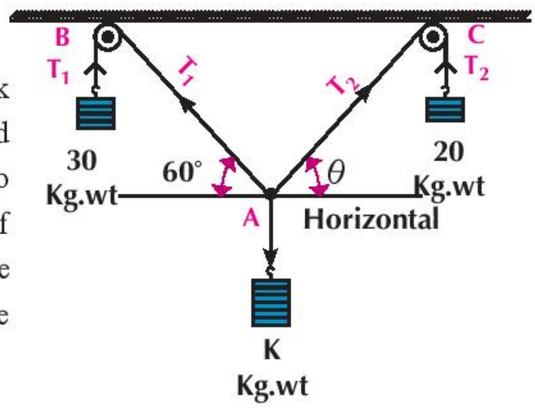
Add to your knowledge

If a string passes through a smooth pulley and stretched on it, then the two tensions in the two terminals of the string are equal "in magnitude.

Think: Use another method to solve the previous example.

Example

(7) **In the opposite figure:** A weight of magnitude k is suspended by an end of a string , the other end is suspended by two strings passing over two smooth pulleys at B, C and carries two weights of magnitudes 30 kg.wt and 20 kg.wt Find the value of the weight k and the measure of angle θ in state of equilibrium.



Solution

In the previous figure : Let the tensions in the strings be T_1 , T_2 and act in the directions $\overrightarrow{AB}$, $\overrightarrow{AC}$

$\because$ **the pulleys are smooth, then :** $T_1 = 30$ kg.wt , $T_2 = 20$ kg.wt

The body of weight K is in equilibrium under the effect of three forces: the weight of magnitude k and the tensions in the strings T_1 , T_2

Applying Lami's rule we get:

$$\frac{30}{\sin(90^\circ + \theta)} = \frac{20}{\sin(60^\circ + 90^\circ)} = \frac{K}{\sin[180^\circ - (60 + \theta^\circ)]}$$ **By simplifying**

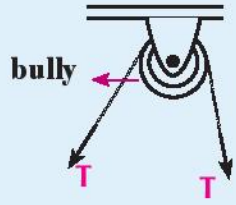
$$\frac{30}{\cos\theta} = 40 = \frac{K}{\sin(60 + \theta^\circ)}$$

$\therefore \cos\theta = \frac{3}{4}$ $\qquad \therefore m(\angle\theta) = 41^\circ\, 24`\, 35``$

$K = 40 \times \sin(41^\circ\, 24`\, 5`` + 60^\circ)$

$\therefore K \simeq 39.2107$ kg. wt

Notice that



The tension is equal in the two parts of the strings

Remember that

sin $(90^\circ + \theta)$ **= cos**θ

sin $(180^\circ + \theta)$ **= sin**θ

Try To Solve

6) The ball of a pendulum of weight 600 gm wt is displaced until the string makes an angle of measure 30° to the vertical under the action of a force perpendicular to the string. Find the magnitude of each of the force and the tension in the string.

Equilibrium of a body under the effect a set of coplanar forces meeting at a point

we can define the necessary condition of equilibrium of a set of coplanar forces meeting at a point as the following: If a body is in equilibrium under the effect of a set of coplanar forces meeting at a point ,then the algebraic sum of the algebraic components of the forces in each of two perpendicular directions equals zero

We conclude that in order that a set of coplanar forces meeting at a point to be in equilibrium, the following conditions must be satisfied:

- The sum of the algebraic components in the direction $\overrightarrow{OX}$ = zero
- The sum of the algebraic components in the direction $\overrightarrow{OY}$ = zero

then $x = 0$, $y = 0$

Example

(8) If $\overrightarrow{F_1} = 5\,\overrightarrow{i} - 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = -7\,\overrightarrow{i} + 2\,\overrightarrow{j}$, $\overrightarrow{F_3} = 2\,\overrightarrow{i} + \overrightarrow{j}$

Prove that the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ are in equilibrium.

Solution

$\because \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$

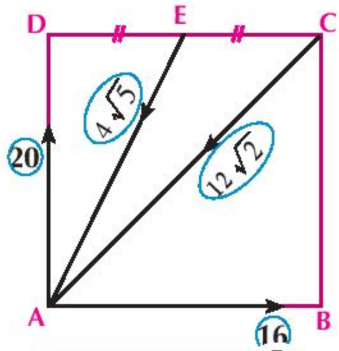
$\therefore \overrightarrow{R} = (5 - 7 + 2)\,\overrightarrow{i} + (-3 + 2 + 1)\,\overrightarrow{j} = \overrightarrow{0}$,**then the set of forces are in equilibrium.**

Try To Solve

(7) If the forces $\overrightarrow{F_1} = 4\,\overrightarrow{i} - 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = -a\,\overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_3} = -6\,\overrightarrow{i} + b\,\overrightarrow{j}$ are meeting at a point and they are in equilibrium, find the values of a , b.

Example

(9) **In the opposite figure:** ABCD is a square, The forces of magnitudes: 16 , 20 , $12\sqrt{2}$, $4\sqrt{5}$ newton act, in the directions $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$ respectively. where E is the midpoint of $\overline{CD}$. Prove that these forces are in equilibrium.



Solution

From the opposite figure, we notice that the forces 16, 20, $12\sqrt{2}$, $4\sqrt{5}$ have polar angles of measures: 0° , 90° , 225° , (180° + θ)

$\therefore x = 16\cos 0° + 20\cos 90° + 12\sqrt{2}\cos 225° + 4\sqrt{5}\cos(180° + \theta)$

$x = 16 + 0 - 12\sqrt{2} \times \frac{1}{\sqrt{2}} - 4\sqrt{5} \times \cos\theta$

$x = 16 - 12 - 4\sqrt{5} \times \frac{1}{\sqrt{5}} = 0$

$y = 16\sin 0° + 20\sin 90° + 12\sqrt{2}\sin 225°$

$+ 4\sqrt{5}\sin(180° + \theta)$

$y = 0 + 20 - 12\sqrt{2} \times \frac{1}{\sqrt{2}} - 4\sqrt{5}\sin\theta$

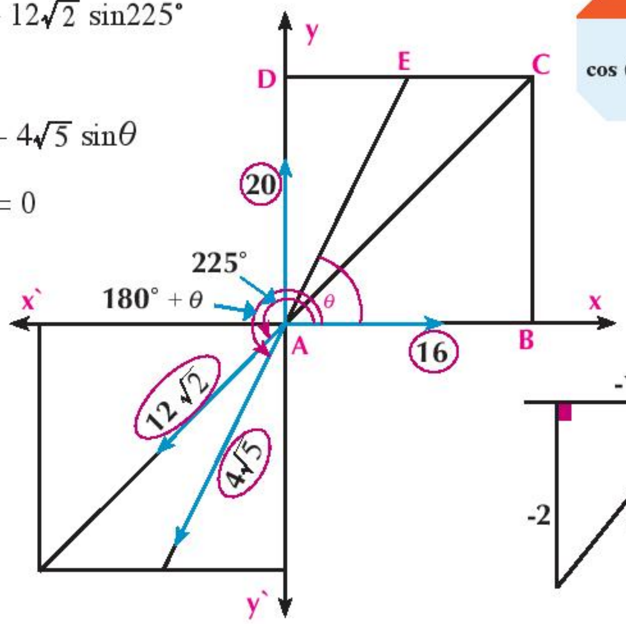
$y = 20 - 12 - 4\sqrt{5} \times \frac{2}{\sqrt{5}} = 0$

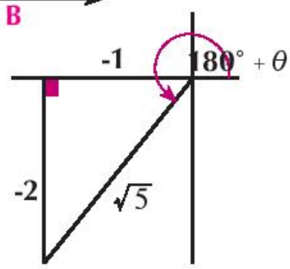
$\therefore x = 0$, $y = 0$

$\therefore$The set of forces are in equilibrium.

Note that

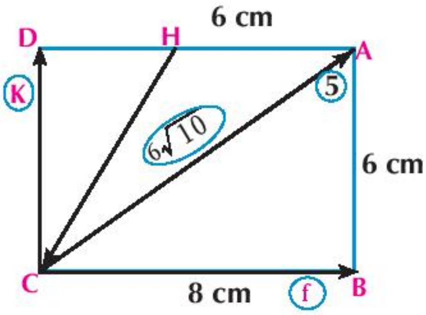
$\cos(180° + \theta) = -\cos\theta$





Try To Solve

8) **In the opposite figure:** The forces of magnitudes F, 5, K, and $6\sqrt{10}$ N are in equilibrium and they act in the rectangle ABCD in the directions $\overrightarrow{CB}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{HC}$ Such that: AB = 6cm, BC = 8cm, AH = 6 cm. Find the values of F, K.



Exercises (1 - 4)

Complete the following:

1) The necessary and sufficient condition for equilibrium of a set of coplanar forces meeting at a point is to be represented geometrically by ....

2) The condition for equilibrium of a set of coplanar forces, meeting at a point is to be .... , ...

3) If $\overrightarrow{F_1} = 4\overrightarrow{i} + b\overrightarrow{j}$, $\overrightarrow{F_2} = -7\overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_3} = a\overrightarrow{i} - 3\overrightarrow{j}$ are in equilibrium, so: a = ... , b = ...

4) If the force of magnitude F is in equilibrium with two perpendicular forces of magnitude 3, 4 newton so, the magnitude of F = ...

5) If three coplanar and equilibrium forces are completely represented by the sides of triangle taken in one cyclic order, then the lengths of the sides of the triangle are proportional with .. ..

6) Each figure from the following figures represents a set of coplanar equilibrium force meeting at a point. Find the value of the unknown either it is a force or a measure of angle .

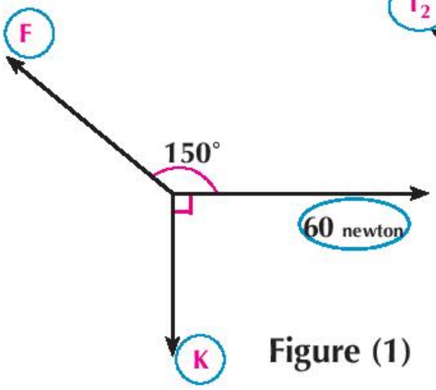


Figure (1)

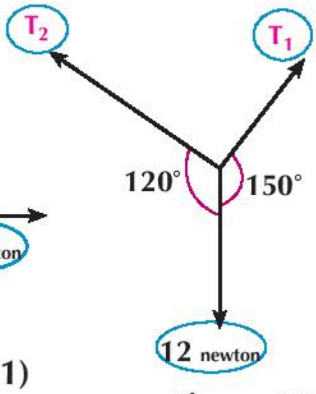


Figure (2)

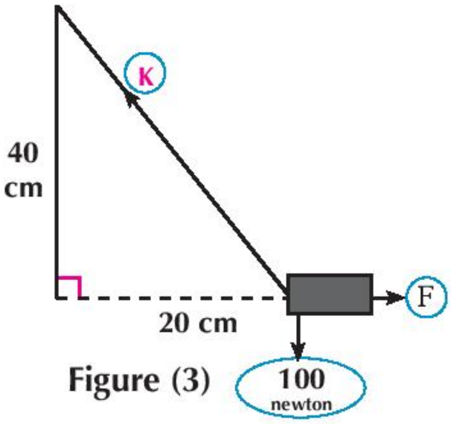


Figure (3)

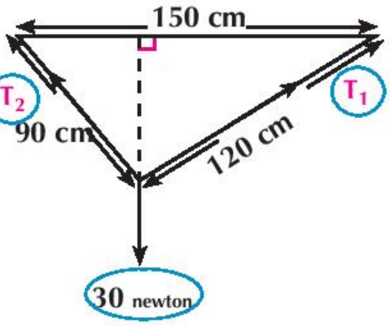


Figure (4)

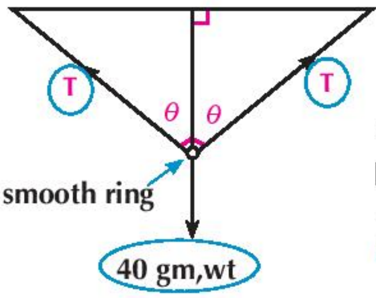


Figure (5)

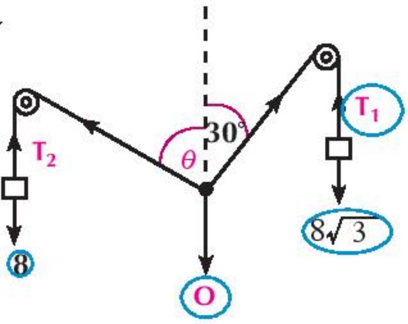


Figure (6)

7. AB is a uniform lader with weight 12 kg.wt rests with its upper end A on a smooth vertical wall and with its lower end B on a rought horizontal ground such that its upper end apart 4m from the ground and its lower end apart 3m from the vertical wall. Find in the case of equilibrium the presure on the wall and the ground

8. AB is uniform rod with length 60cm and weight 40 Newton conected to a hang on the vertical wall at A. If the rod keept in equilibrium by a light string connected to the rod at B and with point C on the wall just above A and at adistance 60 cm from A. Find the tension on the string and the reaction on the hang at A.

9. A homogeneous sphere rests on two parallel rods lie on the same horizontal plane and the distance between them equals the radius of the sphere. Find the pressure on the two rods if the weight of the sphere equals 60 Newton.

10. AB is a uniform rod whose weight is W kg. wt attached by its end A to a hang fixed on a vertical wall. If a horizontal force $\overrightarrow{F}$ acts on the rod at B and the rod get equilibrium when it inclined to the vertical by an angle of measure 60. Find the magnitude of $\overrightarrow{F}$ and the reaction of the hang.

11. A weight of magnitude 60 gm.wt is suspended from one end of a string of length 28 cm. The other end is fixed at a point in the ceiling of the room, A force acts on the body so that the body became in equilibrium when it is about 14 cm vertically down the ceiling, If the force is in equilibrium position when it is normal to the string . Find the magnitude of each of the

force and the tension in the string.

12) A weight of magnitude 200gm.wt is suspended by two strings of lengths 60 cm, 80cm from two points on one horizontal line. The distance between them is 100 cm. Find the magnitude of the tension in each of the two strings.

13) A particle of weight 200 gm.wt is suspended by two light strings. One of the them inclines to the vertical with an angle of measure θ and the other string inclines to the vertical with an angle of measure 30°. If the magnitude of the tension in the first string equals 100 gm.wt, find θ and magnitude of the tension of the second string.

14) A body of weight 800 gm.wt is placed on a smooth plane inclined to the horizontal by angle of measure θ so that $\sin\theta = 0.6$. The body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

15) A body of weight (W) newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and the body is kept in equilibrium by the effect of force of magnitude 36 newton acts in direction of the line of the greatest slope upwards. Find the magnitude of the weight of the body and the magnitude of the reaction of the plane.

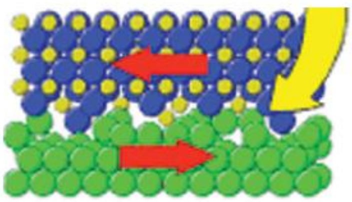
16) A smooth metal sphere of weight 3 Newton at rest (stable) between a smooth vertical wall and a smooth plane inclined to the vertical wall with angle of measure 30°. Find the pressure on each of the vertical wall and the inclined plane.

17) A rod of length 50 cm and weight 20 newton was suspended from its terminals with two strings such that the two ends are fixed in one point. If the length of the two strings are 30 cm , 40 cm respectively. Find the tension in each of the two strings.

Equilibrium of a body on a horizontal rough plane

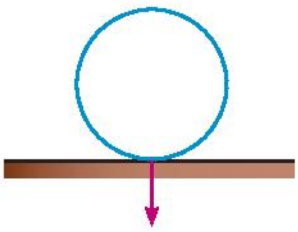
1 - 5

What would happen if the friction disappeared in a moment in the world? if the friction disappeared, we would find cars, trains and all the means of transportation could not move since they move relying on the friction between the ground and their wheels. If these machines moved, they would not stop since the brakes depend mainly up on the friction. Furthermore, people would not walk or stand properly because they seem to stand on icy land . People would not also catch the different objects since the objects would slide away from their hands. Without friction, mountain would break down and the soil cover would no longer cover them, building would tear down and tied ropes would go part. Briefly speaking, life would be impossible without friction. As a result, the friction has many benefits it makes car's wheels move on the roads and the train's wheels stick to the rail road it also allows the conveyor belt to rotate the pulley without sliding you also would not walk without the friction to keep your shoes from getting slided on the sidewalk in other words, it is extremely difficult to walk on the snow where the surface is smooth and cannot cause friction and the shoes slipped. Friction does not allow your shoes to slip on ice, but helps to fix the soil on the mountains and to fix and makes the plants straightly stand and keeps the tied ropes fixed in addition to other benefits.



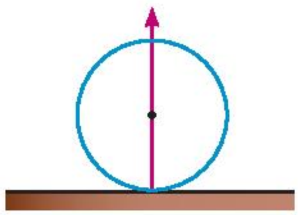
Reaction:

We have previously learned that there is a type of force generated when two bodies touch and it is called reaction . If you place a ball horizontal table, then the ball affects on the table by a force pressure (p) equal the ball weight in this case and the Newton's third law then the table affects the ball by force reaction (r) and equal the pressure ball on the table i.e r = p.



The pressure affecting the table

Figure (1)



The reaction affecting the ball

Figure (2)

We will learn

- Smooth surfaces and rough surfaces.
- The concept of the friction
- The force of the static friction
- The force of kinetic friction
- The relation between the coefficient of the friction and the tangent of the angle of the friction
- Properties of the friction
- The equilibrium of a body on a rough horizontal plane.

Key - term

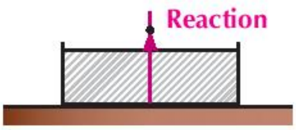
- Friction
- Smooth Surface
- Rough Surface
- Normal Reaction
- Static Friction
- Kinetic Friction
- Limiting Static Friction
- Resultant Reaction
- Angle of Friction
- Rough horizontal plane

Matrials

- Scientific calculator

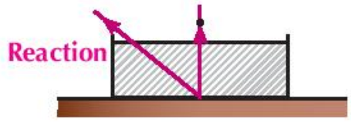
Smooth and Rough Surfaces

Scientists relate the friction forces among bodies to the presence of microscopic cavities and projections in the surfaces of the bodies whatever their smoothness is . The overlapping of such projections and cavities of the two surfaces in contact produces what is called the friction force. As a result, we can find resistance as we try to move one of the two surfaces against the other, The coefficient of the friction a good scale to measure the roughness degree of the surfaces. If the value of the coefficient of the friction increases, the roughness increases and vice versa. If the coefficient of the friction equals zero, the friction forces are not existed totally. The reaction between the two bodies in contact depends upon the nature of the two bodies and upon the other forces acting on the body in case of the smooth surfaces the reaction is normal to the common tangent plane to the surfaces of the two bodies in contact. On the contrary, when the two bodies are rough, the reaction would have a component in the direction of the tangent surface which is called the static friction. Besides the reaction has a normal component on the tangent surface which is called the normal reaction.



Reaction in case of smooth surfaces

figure (3)



Reaction in case of rough surfaces

figure (4)

The properties of the static friction force:

(1) The static friction force (F) acts in opposing the slide it is in the opposite direction to the direction which the body tends to slide

(2) The static friction force (F) is only equal to the tangential force which tends to move the body so that it can't be more than such a force and remain equal to the force as long as the body balanced.

(3) The static friction force (F) increases, whenever the tangential force which cause the motion increases until you arrive up to a certain limit which it doesnot exceed it . At such a limit, the body is about to slide in this case, the friction is called the limiting static friction and it is denoted by the symbol (F_s).

(4) The ratio between the limiting static friction and the normal reaction N is constant and this ratio depends up on the nature of the two bodies in contact but not up to their shape or mass. This ratio is called the coefficient of the static friction and is denoted by the symbol (μ_s) .

i.e. $\mu_s = \frac{F_s}{N}$ where F_s the limiting static friction

it is noticed that the static friction coefficient often have $0 \mu >_s 1 >$ but in some special cases it may be more than one.

Kinetic Friction force

if a body moves upon a rough surface , it is subjected to the kinetic friction force (F_k) and its direction is opposite to the direction of its motion and its value is given by the relation: $F_k = \mu_k N$:

where μ_k is the kinetic friction coefficient and R the normal reaction.

i.e.: the kinetic friction force equals the product of the kinetic friction coefficient multiplied by the normal reaction force

Hence, the kinetic friction coefficient can be defined as the ratio between the kinetic friction force and the normal reaction force .

i.e. : $\mu_k = \frac{F_k}{N}$ where F_k is the kinetic friction force

Resultant Reaction (R')

in case of the rough surfaces the resultant reaction is inclined on the tangent surface since it expresses the resultant of the normal reaction and the static friction force. It is called the resultant reaction.

Definition

The resultant reaction ($\overrightarrow{R'}$) is the resultant of the normal reaction $\overrightarrow{N}$ and the static friction force $\overrightarrow{F}$

Angle of Friction

Notes that the measure of the angle included between the normal reaction and the resultant reaction increases as the magnitude of the friction force increases [suppose that the normal reaction is constant] and this value is limiting λ when the friction becomes limiting and this angle in this case called the angle of friction.

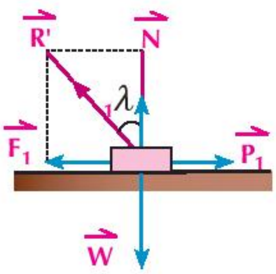


figure (6)

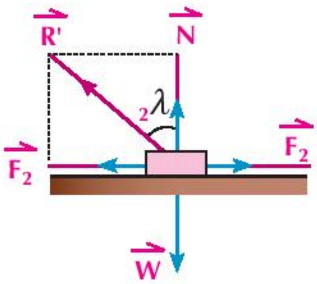


figure (7)

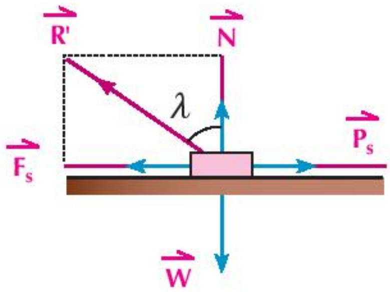


figure (8)

In fig (1) and fig (2) we find: the vector resultant $\overrightarrow{R'}$ is the resultant of the normal reaction $\overrightarrow{N}$ and the friction force $\overrightarrow{F}$, it's magnitude is given by $R' = \sqrt{N^2 + F^2}$

and in fig (3) when the friction force is limiting we have :

$\therefore R' = \sqrt{N^2 + F_s^2}$ $\quad \because F_s = \mu_s N$ $\quad \therefore R' = \sqrt{N^2 + N^2\,{}^2\mu_s}$ $\quad \therefore R' = N\sqrt{1 + {}^2\mu_s}$

The relation between coefficient of friction and angle of friction :

in case the friction is limiting as in shown fig (8) :

we find : $\tan\lambda = \frac{F_s}{N}$ where $\frac{F_s}{N} = \mu_s$

i.e. : $\mu_s = \tan\lambda$

i.e : In the case of limiting friction , the coefficient of friction is equal to the tangent of the angle of friction

Critical thinking: compare between the static and kinetic angle of friction.

Equilibrium of a body on a rough horizantal plane

If a body of weight (w) is in equilibrium on a horizontal rough plane and acted upon by a force p inclined by an angle of measure θ with the horizontal fig (9) the body is equilibrium under the action of :

1) The weight $\vec{w}$ which is directed vertically downward

2) The resultant reaction $\vec{R'}$ and its magnitude is R'

3) The given force $\vec{P}$ with magnitude P. as in fig (9)

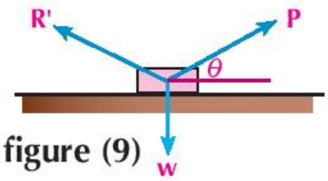


figure (9)

by resolve $\vec{P}$ into two components in the horizontal and vertical, direction then their magnitudes are $P\cos\theta$, $P\sin\theta$.

and by resolve $\vec{R'}$ into two perpendicular components which are the normal reaction $\vec{N}$ and its magnitude N, and the friction force $\vec{F}$ and its magnitude F as shown in fig (10).

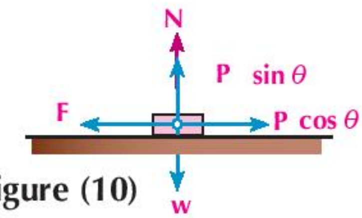


figure (10)

The equations for equilibrium are : $F = P\cos\theta$, $N + P\sin\theta = w$

Example **The acting force on a body**

(1) Karim pushes a box full of books towards his car on the using horizontal, if the weight of the box and books together 124 Newton and coefficient of friction between the road and the box 0.45 then find the magnitude of the force horizontal required by karim to push the box to make it about to move.

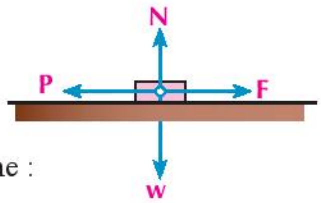


figure (11)

Solution

Suppose that $w = 124$ newton , $\mu_s = 0.45$

From the conditions of equilibrium of the body in the horizontal plane :

$N = w$ **i.e :** $N = 124$ **(1)**

$F = \mu_s N$ and from **(1)** then : $F = 0.45 \times 124 = 55.8$ newton

Try to solve

(1) A mass of weight 32 newton is put on a horizontal rough plane and act on it a horizontal force magnitude P until the mass becomes about to move.

(a) If P= 8 newton, find the coefficient of the static friction between the mass and the plane

(b) If $\mu_s = 0.4$ find P

Example **Friction force**

2 A body of weight 8 kg.wt is placed on a horizontal table, and is connected by a string passing over a smooth pulley at the edge, to a weight of magnitude 1.5 kg.wt which is hanging freely and the body is in equilibrium, find the friction force. If the coefficient of friction between the body and the table is $\frac{1}{4}$. State whether or not the body is about to move.

Solution

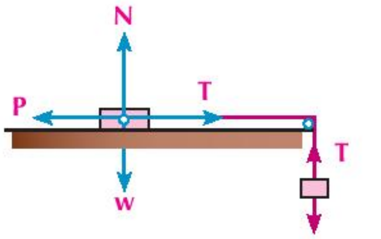


figure (12)

From the equilibrium of the body hanging vertically we find T=1.5 kg.wt and from the equilibrium is placed on a horizontal table then N=W .

$8 = \mu \because$ kg.wt

$\therefore$ Friction force F = T $\therefore$ T = 1.5 kg.wt

To know whether the body is about to move or not determine the limiting static friction F_s

$\because F_s = \mu_s N$ $\therefore F_s = \frac{1}{4} \times 8 = 2$ kg.wt.

$\therefore F \mu >_s N$ the friction is not limiting and the body is not about to move.

Try to solve

2 A particle of weight 20 newton is placed on a horizontal rough plane, if the static friction coefficient between the particle and the plane $\frac{1}{4}$ find:

a The required horizontal force which enough to make the particle is about to move .

b The inclined force which makes an angle measure 30° to the plane and makes the particle is about to move .

Example **Angle of friction**

3 A body of weight 12 kg.wt is placed on a horizontal rough plane, two forces act on the body of magnitudes 4 , 4 kg.wt and include an angle of measure 60° where the two horizontal forces are on the same horizontal plane. If the body is about to move, find the coefficient of friction between the body and the plane also find angle of friction.

Solution

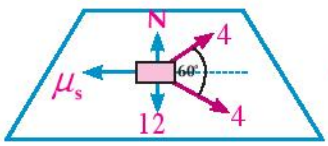


figure (14)

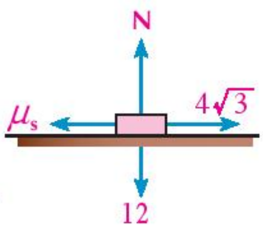


figure (13)

$\because$**The body is about to move:**

the body in equilibrium

$\therefore N = W$ $\therefore N = 12$ kg.wt.

The resultant of the forces 4, 4 kg.wt = limiting friction forces

$\because F = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \alpha}$ the resultant of two forces law

$\therefore F = \sqrt{4^2 + 4^2 + 2 \times 4 \times 4 \times \frac{1}{2}} = 4\sqrt{3}$ kg.wt

$\mu_s N = F \qquad \mu 12 \therefore_s = 4\sqrt{3}$

$\mu \therefore_s = \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3} \quad \mu \therefore_s = \frac{1}{\sqrt{3}}$

$\mu \because_s = \tan\lambda \qquad \therefore \tan\lambda = \frac{\sqrt{3}}{3} \qquad 30 = \lambda \therefore^{\circ}$

Try to solve

3) A particle of weight 6 newton is placed on a horizontal rough plane and two forces in the same plane of magnitudes 2 and 4 newton include an angle of measure 120° act on it, the particle kept at rest prove that the measure of the angle of friction (λ) between the body and the plane must not less than 30°.

and if $\lambda = 45°$ and the directions of the two forces unchange , and the force of magnitude 4 newton without change. Determine the magnitude of the other force for the particle to be about to move.

Example

Theoretical demonstrate

4) A body of weight (w) is placed on a horizontal rough plane and the measure of the angle of friction between the body and the plane is λ located in the vertical plane passing through body weight, the body is attached by a force inclined the horizontal by angle of measure θ the body is about to move. Prove that the magnitude of this force $= \frac{w \sin\lambda}{\cos(\lambda - \theta)}$, then find the magnitude of the smallest value of this force and condition happening.

Solution

$\because$ R' is the resultant of the two forces N , F_s:

$\therefore$ The body is in equilibrium under the action of three forces which are: $\overrightarrow{P}$, $\overrightarrow{W}$, $\overrightarrow{R'}$

and by using Lami's rule:

$\therefore \frac{P}{\sin(180° - \lambda)} = \frac{W}{\sin[90° - (\lambda - \theta)]}$

$\because \frac{P}{\sin\lambda} = \frac{W}{\cos(\lambda - \theta)} \qquad \therefore P = \frac{w \sin\lambda}{\cos(\lambda - \theta)}$

$\because$the required force is smallest value of this force $\overrightarrow{F}$, then the value of $\cos(\lambda - \theta)$ is a maximum possible

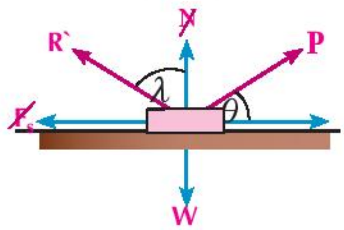


Figure (16)

$\therefore \cos(1 = (\lambda - \theta \qquad \therefore F = w \sin\lambda$ and the condition which must satisfy is :

$\cos(\lambda - \theta) = \cos 0 \qquad 0 = \lambda - \theta \therefore \qquad \lambda = \theta \therefore$

$\therefore$ The condition which must be satisfy is the measure of the angle of inclination of the force to the horizontal equal the measure of the angle of friction

Try to solve

4) A body of weight (w) kg.wt is placed on a rough horizontal plane , the measure of the angle of friction between the body and the plane is (λ), the body is attached by a force inclined the horizontal by an angle of measure (2λ) upwards located in the vertical plane passing through body weight the body is about to move. Prove that the magnitude of this force equal w $\tan\lambda$.

Exercise 1 - 5

First : Complete each of the following :

1. The force resulted from sliding two rough surfaces by the force of ....

2. The friction force vanished and the coefficient of friction equal zero in the .... planes

3. When the static friction force is limiting, then the body is ....

4. The kinetic friction force equal the product of coefficient of kinetic friction at ....

5. The resultant of the normal reaction force and the limiting static friction force is called ....

6. The static friction force less than or equal the product of the coefficient of static friction at ....

7. If the static friction coefficient between a mass of magnitude 40 kg and the ground surface equal 0.45 then the magnitude of the horizontal force which act on the mass and make it about to move equal ....

8. If a body of weight 6 newton is placed on a rough horizontal plane and the static friction force was 4 newton, then the coefficient of static friction is ....

Second : Answer the following questions

9. A boy pushes a stone of weight 56 newton by a horizontal force of magnitude 42 newton on a ridge, the stone was about to move . Find the coefficient of static friction between the stone and the ridge.

10. A body of weight 240 kg.wt is placed on a horizontal rough plane and wanted to attached by a wire inclined to the horizontal by angle of measure 30° , if the coefficient of static friction equal $\frac{\sqrt{3}}{3}$ find the required tension in the wire for the body about to move .

11. A body of weight 39 kg.wt is placed on a horizontal rough plane two forces of magnitudes, 7 and 8 kg.wt and include an angle of measure 60° act on the body so it become about to move. Find the static coefficient friction .

1 - 6 Equilibrium of a body on an Inclined rough plane

We will learn

- The conditions of equilibrium of a body on an inclined rough plane.
- The relation between the measure of the angle of friction and the measure of the angle of inclination of the plane to the horizontal.
- Identifying the coefficient of friction between two contact surfaces (Activity)

Key - term

- Inclined rough plane
- Normal Reaction
- Resultant Reaction
- Angle of Friction
- Coefficient of Friction

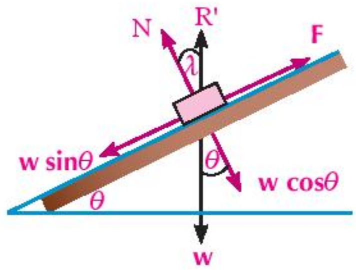
Matrials

- Scientific calculator

We will study is this lesson equilibrium of a body on a rough inclined plane..

We consider the body is in equilibrium on a rough horizontal inclined plane with an angle of measure θ.

The body become in equilibrium under the action of two forces:



(1) The weight of the body $\vec{w}$ that acts vertically down wards and its magnitude is (w)

(2) The resultant reaction and its magnitude (R')

And from the conditions of equilibrium we find that :

The resultant reaction force act vertically up.

and : R' = w .... (1)

we can designate two forces, the friction force and the normal reaction considering the resultant reaction resolved into two directions one of them is parallel to the plane and the other perpendicular to it as shown in the figure (1).

Friction force .

$F = w \sin\theta$ .... (2)

and this force act on opposite direction of the expected motion, this means that is parallel to the line of the greatest slope upwards.

Normal Reaction .

$N = w \cos\theta$ ....(3)

The relation between the static friction angle and the measure of the angle of inclination of the plane to the horizontal.

If we put a body on a rough inclined plane and the body is about to slipping, then the measure of the static friction force equal the measure of the angle of inclination of the plane to the horizontal.

Proof :

∵ The friction is limiting

∴ The resultant reaction force makes an angle with the normal to the plane of measure equal the measure of the static friction force, and its measure is (λ) .

and from the previous figure, we find : $\lambda = \theta$

and we can put the equation in terms of coefficient of friction as follow :

$$\tan \mu = \lambda_s \quad \text{or} \quad \mu_s = \tan \theta$$

For example :

If a body is placed on a rough inclined plane and it was about to move under its weight only, when the angle of inclination of the plane on the horizontal is 30°, then the static coefficient friction $\mu_s = \tan 30° = \frac{1}{\sqrt{3}}$

Example

1 A body of weight 3 newtons is placed on a plane inclined the horizon by angle of measure 30° and the static coefficient friction between the body and the plane equals $\frac{2}{3}$. A force of magnitude 2N act on the body and in the direction of the line of the greatest slope upwards, if the body is in equilibrium determine the friction force state wether or not the motion is about to begin

Solution

By resolve the weight $\overrightarrow{W}$ into two components in the direction of the plane and the normal to it.

1) The tangential component in the direction of the line of the greatest slope downwards and of magnitude $w \sin\theta = 3 \sin 30° = \frac{3}{2}$ newtons

2) The perpendicular component of magnitude $w \cos\theta = 3 \cos 30° = \frac{3\sqrt{3}}{2}$ newtons

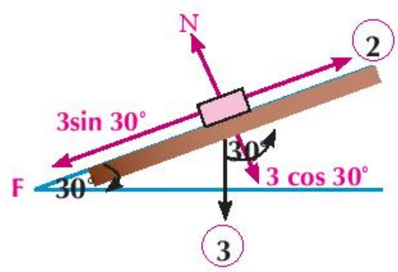
and by comparing the tangential component $w \sin\theta = \frac{3}{2}$ newton , and the magnitude of the force which acts on the body in the direction of the line of the greatest slope upwards = 2 newton we find: $P < w \sin\theta$.

So, the body tends to move upwards, so the friction force must be in opposite direction in the line of greatest slope down wards $\overrightarrow{F}$:

$P = F + w \sin\theta \qquad 2 \therefore = F + \frac{3}{2} \qquad \therefore F = \frac{1}{2}$ newtons

$N = w \cos\theta \qquad \therefore N = 3 \cos 30° \qquad \therefore N = \frac{3}{2}\sqrt{3}$ newtons

the magnitude of the friction = $\frac{1}{2}$ newton acts on the line of the greatest slope down and to know whether the body is about to move or not



We find the $F_s = \mu_s N = \frac{2}{3} \times \frac{3}{2}\sqrt{3} = \sqrt{3}$ newtons

then: $F > F_s$ then the friction is not limiting

∴ the body is not about to move .

Try to solve

1. A particle of weight 2 kg.wt is placed on a plane inclined the horizon by an angle of measure 30° and the static friction coefficient is $\frac{\sqrt{3}}{2}$. a force of magnitude 2.5 kg.wt acts on the body and in the line of the greatest slope up wards if the body is in equilibrium. Determine the friction force and show whether the body is about to move or not?

Critical thinking: If a body is placed on an inclined plane makes with the horizontal an angle if measure (θ), if the measures of static friction angle between the body and the plane is (λ) what is your expectation for the body if:

a) $\lambda > \theta$ b) $\lambda < \theta$

Example

2. A body of weight 10 kg.wt is placed on a rough inclined plane. A force $\overrightarrow{P}$ acts on it in the direction of the line of the greatest slope up. If its known that the body is about to move upwards the plane when P = 6 kg.wt and about to move downwards the plane when P = 4 kg.wt. Find:

   a) The angle of inclination of the plane to the horizontal .

   b) The static friction coefficient .

Solution

When P = 6 kg.wt the body is about to move up the plane and the static friction is limiting and acts, down the plane.

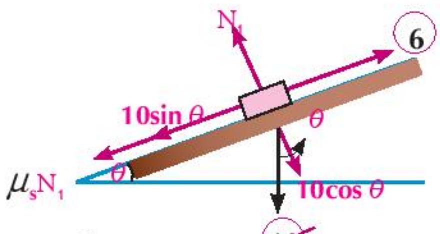


figure (4)

∴ $N_1 = 10 \cos\theta$, $6 = 10 \sin\mu + \theta_s N_1$ eliminating N_1 from the two equations :

$10 \therefore \sin\mu\ 10 + \theta_s \cos$ 1) .... $6 = \theta$)

when P = 4 kg.wt the body is about to move down the plane and the static friction is limiting and acts up the plane .

∴ $N_2 = 10 \cos\mu + 4$, $\theta_s N_2 = 10 \sin\theta$ eliminating N_2

from the two equations :

μ 10 $\therefore_s$ cos 10 = θ sin **2)** ....4 - θ)

from (1) , (2) :

10 = 6 $\therefore$ sin 10 + θ sin 20 $\therefore$ 4 – θ sin 10 = θ

$\therefore$ sin $\theta = \frac{1}{2}$ 30 = θ $\therefore$°

by substituting in (2) μ 10 $\therefore_s$ cos 30° = 10 sin 30° – 4

$\therefore$ $\frac{\sqrt{3}}{2} \times 10$ μ = 5 – 4 $\therefore \mu_s = \frac{1}{5\sqrt{3}} = \frac{\sqrt{3}}{15}$

the static coefficient friction between the body and the plane = $\frac{\sqrt{3}}{15}$

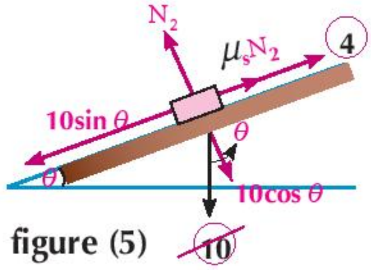


figure (5)

Try to solve

2 A body of weight 30 newtons is placed on a rough inclined plane, it is notice that the body is about to move. If the plane inclined to the horizontal by an angle of measure 30°, If the inclination of the plane to horizontal is increased to 60°, then find:

a The least force which acts on the body parallel to the line of the greatest slope and prevent the body from slipping.

b The force which acts on the body parallel to the line of the greatest slope and make it about to move up the plane.

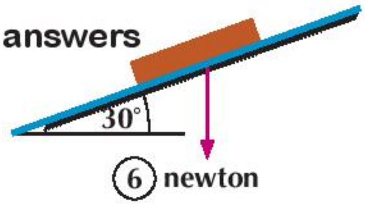
Exercise 1 - 6

First: put the sign (✓) or (✗):

1. The coefficient of friction between two bodies depends on their shapes and their masses.
2. The ratio between the magnitudes of the limiting static friction force and the normal reaction is called the coefficient of friction.
3. The tangent of the angle of the static friction equal the ratio between the limiting friction force and the normal reaction
4. If a body is placed on a rough inclined plane and was about to move, then the static coefficient friction between the body and the plane equals the measure of the angle of inclination to the horizontal.
5. if a body is placed on a rough inclined plane and was about to move, then the measure of the angle of friction equal the measure of the angle of inclination of the plane to the horizontal.
6. The angle of friction is the angle included between the limiting friction force and the resultant reaction.

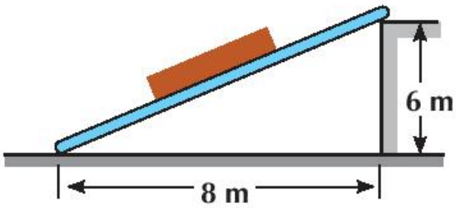
Second: Choose the correct answer from the given answers

7. **In the opposite figure :**if the body is about to move down wards, then the limiting friction equal:



a) 3 b) $2\sqrt{3}$
c) $3\sqrt{3}$ d) 9

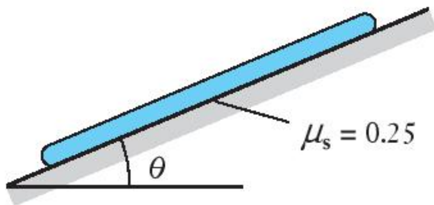
8. **In the opposite figure:** The body is about to move downwards, then the measure of the angle of the static friction equal:



a) 36.87° b) 41.41°
c) 48.59° d) 53.13°

9. **In the opposite figure:**

the body is about to move down wards, then $\lambda =$



a) 14.04° b) 14.48°
c) 75.52° d) 75.87°

Third : Answer the following questions

10. A body of weight 400 gm.wt is placed on a plane inclined by 30° to the horizontal and the coefficient of friction between it and the body equal $\frac{\sqrt{3}}{4}$. A force of magnitude 50 gm.wt acts on it in the direction of the line of the greatest slope upwards. If the body is in equilibrium, determine the friction force and show whether the body is about to move or not.
11. A body of mass 4 kg is placed on a rough inclined plane makes an angle of measure 30° to the horizontal and the coefficient of friction between it and the plane is $\frac{\sqrt{3}}{2}$. Show whether the body sliding on the plane or about to move or the friction is not limiting and find the magnitude and direction of the friction force where upon, then find the magnitude of the

force which acts on the body in the direction of the line of the greatest slope such that the body is about to move upwards the plane.

12. A body of weight (w) is placed on a rough plane inclined to the horizontal by an angle of measure (θ) , it is found that the force which parallel to the line of the greatest slope of the plane and makes the body is about to move upwards the plane equals 2 w sin θ prove that:

a) the measure of the angle of friction $= \theta$ b) the magnitude of the resultant reaction = w

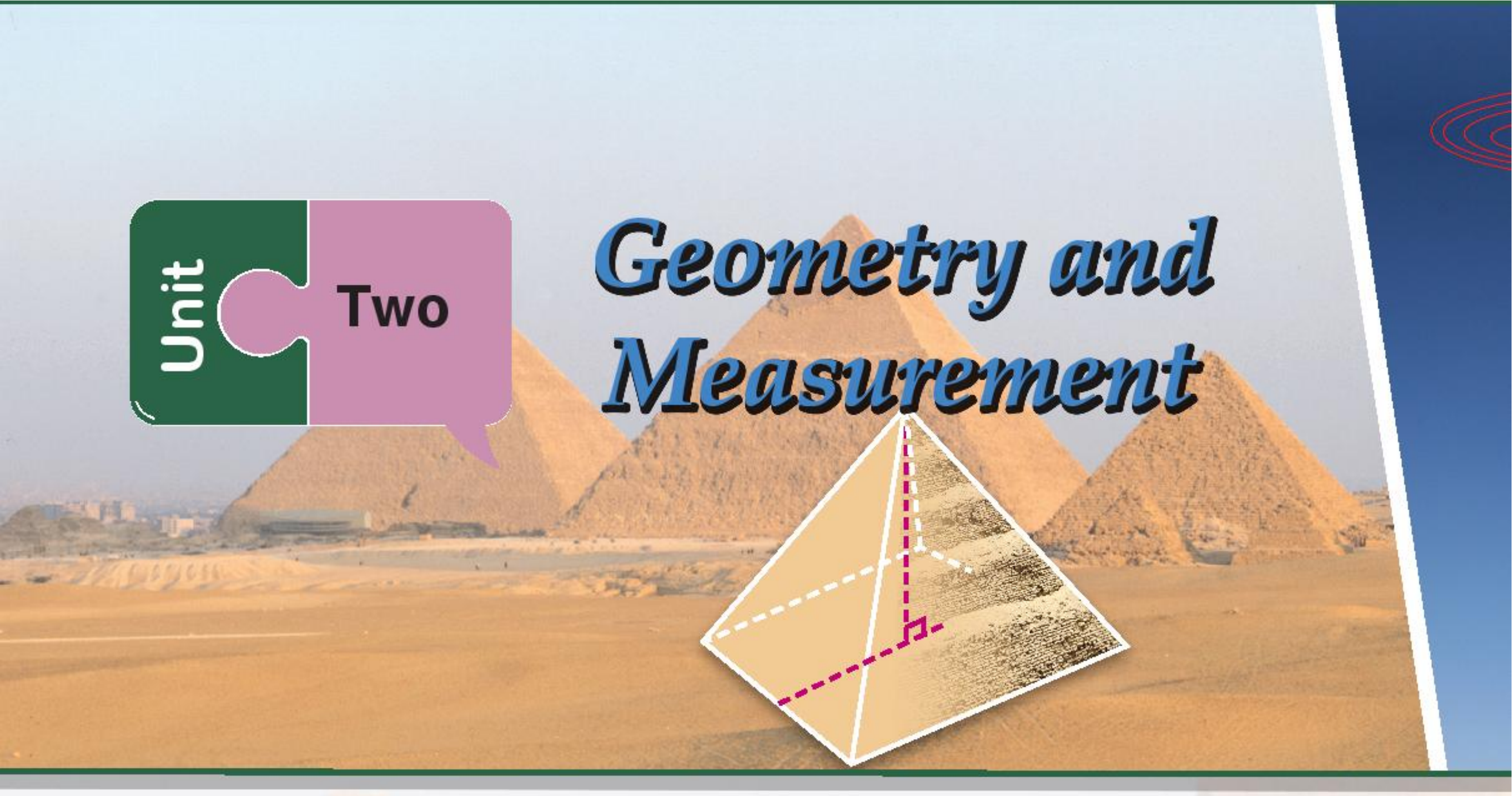
13. A body of weight 25 kg.wt is placed on a rough inclined plane a magnitude force P acts on it in the direction of the line of the greatest slope upwards the plane. If its known that the body is about to move up the plane when P = 15 kg.wt . and it is about to move downwards the plane if P = 10 kg.wt find :

a) the measure of the angle of inclination of the plane to the horizontal

b) the static coefficient friction

14. A body of weight 8 kg.wt is placed on a horizontal rough plane, then the plane incline gradually unitl the body becomes about to move downwards the plane when the measure of the angle of inclination to the horizontal is $30°$. Find the coefficient friction between the body and the plane, and if the body is tied by a string in direction of the line of the greatest slope of with the plane until the body becomes about to move upwards the plane, find:

a) the magnitude of the tension force b) magnitude of the normal reaction



Unit Two
Geometry and Measurement

Introduction

Geometry originated in the beginning linking to the scientific aspect. It was used by the ancient Egyptians in determining the areas of land and construction of the pyramids and temples. They calculated areas of some shapes and volumes of some solids.

When Thales (640 - 546 B.C) went to Alexandria. He liked the ways in which the Egyptians measured the ground and name these ways by the word "Geo-metron" which is taken from the Greek language and consists of two words: the 1st word is Geo which means the land .the 2nd word is metron which means measurement. He was interested in studying Geometry as explicit expressions of abstract subject to proof.

Geometry evolved at the hands of the Greeks (Thales - Pythagoras - Euclid) by the emergence of a series of theories based on some axioms, definitions arranged in a logical accurate system in which included in Euclid's book The principles which consists of 13 parts.

Alexandria continued to be a beacon of knowledge till Arabs came and kept that heritage by translated it into Arabic and added many additions to it and transferred it to Europe in the twelfth century.

In the sixteenth century, Renaissance began in mathematics and the birth of a new science, Decarts (1650 - 1596) introduced the foundations of analytical geometry , the graphical representation of the equations , the express of the geometrical shapes by equations and deduced the equation of the circle $X^2 + y^2 = r^2$.

Euler also reached to the existence of a relationship between the number of faces and the number of edges of any solid has slatted base area, and it is: number of faces + number of heads = number of edges + 2.

Objectives of the unit

By the end of the unit the student should be able to:

- Define the point straight line and the plane in the space
- Recognize some solids such as: pyramid – the regular pyramid – the right pyramid the cone – the right cone and the properties of each.
- Conclude the total surface area and the lateral area of each of the right pyramid and the right cone.
- Deduce the volume of each of the right pyramid – the right cone .
- Find the equation of the circle in terms of coordinates of each of its center and the length of its radius.
- Conclude the general form of the equation of the circle.
- Determine the coordinates of the center of the circle and the length of its radius using the general form of the equation of the circle.
- Apply what he taught in Geometry in modeling mathematical situations.

Basic Terms

- The point
- Straight line
- Plane
- Space
- Vertex
- Base
- Axis
- Circle
- Center
- Radius
- Diameter
- Pyramid
- Cone
- Lateral face
- Lateral edge
- Height
- Slant height
- Regular pyramid
- Right pyramid
- Net of a pyramid
- Right circular cone
- Lateral area
- Surface area

Materials

- Scientific calculator
- Computer - Graphic programs
- Geometrical instruments

Lesson of the unit

Lesson (3-1): straight lines and the plane

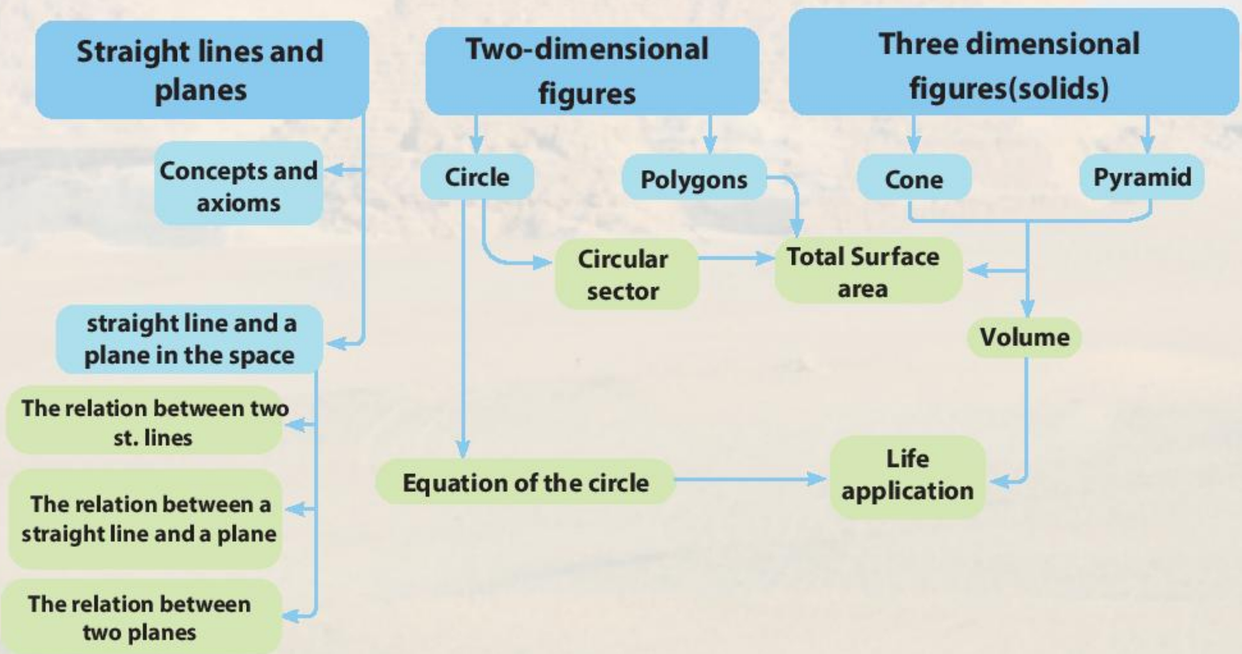
Lesson (3-2): Pyramid and cone

Lesson (3-3): Lateral area and total surface area of a pyramid and a cone

Lesson (3-4): Volume of a pyramid and a cone

Lesson (3-5): equation of a circle

Chart of the unit



2 - 1

The straight lines and the planes in a space

We will learn

- Concepts and geometrical axioms
- The relation between two straight lines in the space
- The relation between a straight line and a plane in the space
- The different positions of two planes in the space

Key - term

- Point
- Straight line
- Plane
- Space.

Matrials

- Geometrical instrument
- Scientific calculator.
- Drawing programs.

Think and discuss

You have studied some mathematical concepts about the point, the straight line and the plane. Can you answer the following questions?

- How can you represent your city on the map of the Arab republic of Egypt?
- How many points are needed to draw a straight line?
- What does the following things related to you: the floor of the classroom , the surface of the table and the surface of the wall
- What does the following things related to you: the surface of the ball, the surface of the dome of the mosque and the surface of the cylinder of gaz.

Activity

Draw two different points A and B on a paperboard.

Use the ruler to join the two points and extend them from each side.

Try to draw another straight line passes through the same points A and B, Can you do that?

What did you deduce from this activity?

Activity

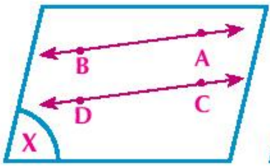
Draw three non-collinear points A, B and C as shown in the figure.

Use a paperboard in a form of a rectangle such that one of its dimensions lies on $\overleftrightarrow{AB}$ Move the plane of the paper so it rotate about $\overleftrightarrow{AB}$ to be on point C.

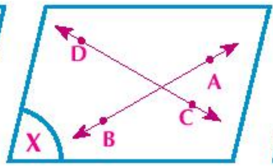
In how many position does the point C lies on the surface of the paper if the paper completes one turn?

Geometrical axioms:

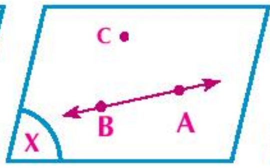
- The straight line is will determine if we determine two points on it.
- The plane in space is determined by one of the following cases :



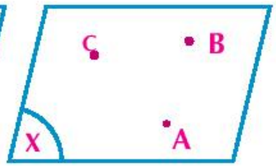
Two parallel straight lines.



Two intersecting straight lines



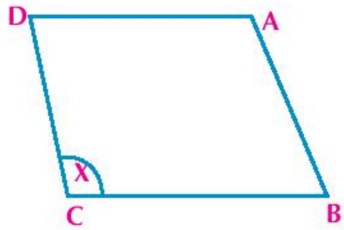
A straight line and a point outside it



Three distinct and non - collinear points

- For any point in the space, there are an infinite number of planes passing through it.

Plane: is a surface with no ends such that any straight line passes through two points on it lies completely on this surface . in the given figure :the plane is denoted by the symbol X .you may use any other symbol as Y , Z , …….or you can denoted it by three letters as ABC , …… ,the plane has no end from its sides and it may represented by a triangle ,a square ,a rectangle ,a parallelogram or a circle ,…… .

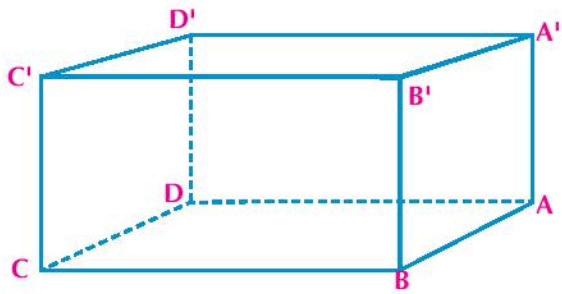


Space: is an infinite number of points contains all figures ,planes and solids we will study.

Example

(1) Meditate the following figure ,and then answer the following questions

- (a) Write three straight lines passes through the point A.
- (b) Write the straight lines passing through both of the points A and B all together
- (c) Write three planes passes through the point A
- (d) Write three planes passing through both of the points A and B all together.



Solution

(a) $\overleftrightarrow{AB}$, $\overleftrightarrow{AA'}$, $\overleftrightarrow{AD}$

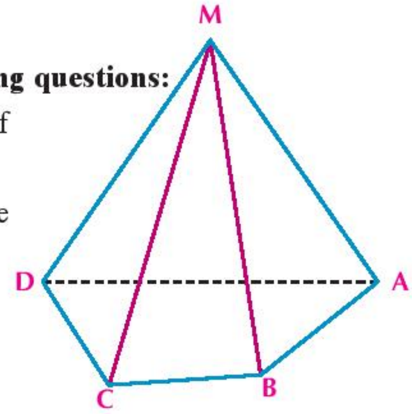
(b) $\overleftrightarrow{AB}$

(c) ABB', ABC, ADD'

(d) ABB', ABC, ABC'D'

Try to solve

1 Meditate the following figure, and then answer the following questions:

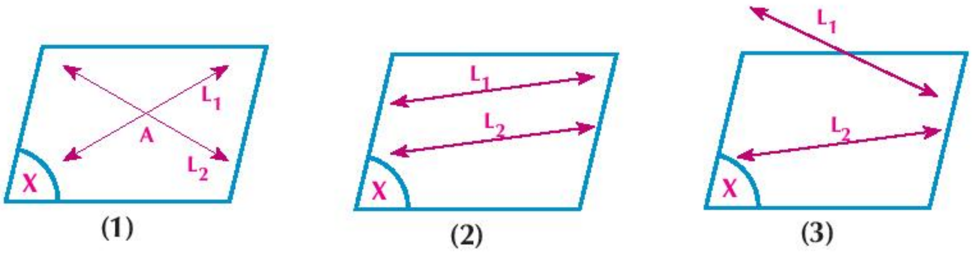


a How many straight lines in this figure? State the names of the straight lines passing through the point A.

b How many planes in this figure? State the names of three planes passing through the point A.

The relation between two straight lines in the space:

Meditate the following figures, and then complete:



1- The two intersected straight lines: are two straight lines lie in the same … ….and having a common …………………… .

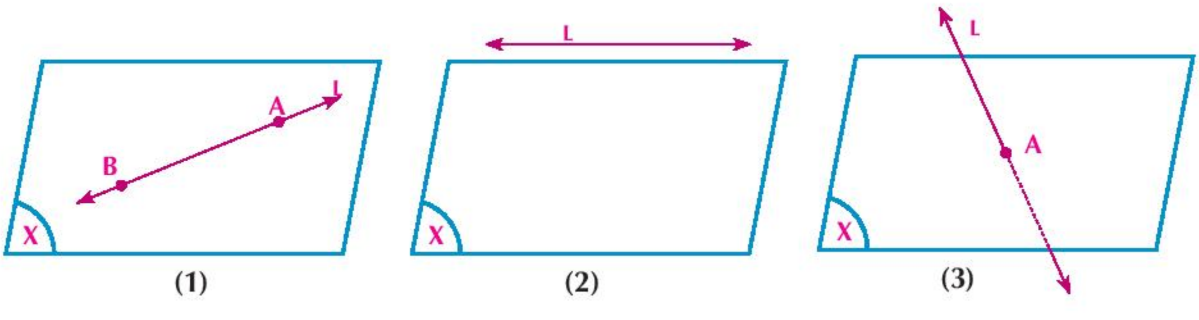
2- The two parallel straight lines: are two straight lines lie in the same … ….and does not have a common …………………… .

3- The two skew straight lines: are two straight lines does not contained in the same … … … … … .. and they are not ……………………

Critical thinking: The two skew straight lines are neither parallel nor intersecting. Explain that.

The relation between a straight line and a plane in the space

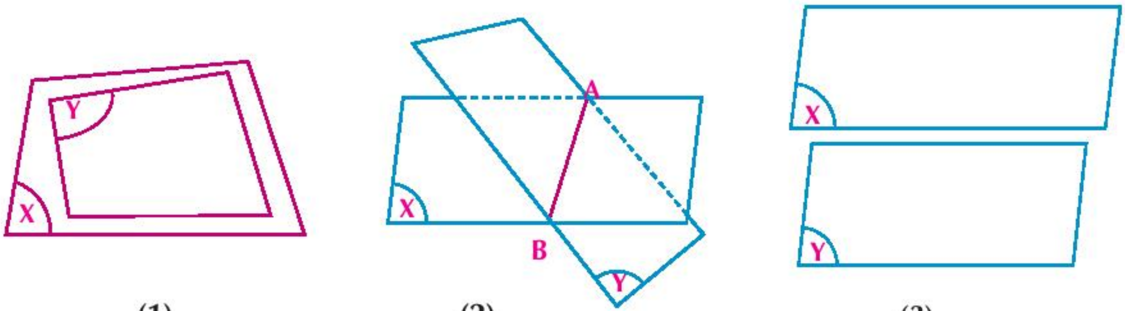
Meditate the following figures, and then complete:



- The straight line is parallel to the plane in figure ……………………
- The straight line is intersected with the plane in figure ……………………
- The straight line contained in the plane in figure ……………………

The relation between two planes in the space

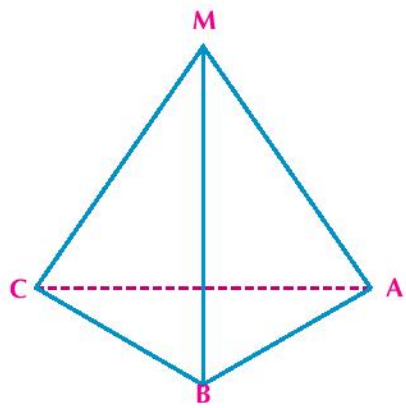
Meditate the following figures, and then complete:



- The two planes are parallel in figure ……..
- The two planes are coincident in figure ……….
- The two planes are intersected in figure ……….

Example

(2) Meditate the following figure, and then complete:



(a) The plane MAB ∩ The plane MBC = ……………

(b) The plane MBC ∩ The plane ABC = ……………

(c) $\overleftrightarrow{MB}$ ∩ The plane ABC = ……………

(d) $\overleftrightarrow{MC} \cap \overleftrightarrow{AB}$ = ……………

(e) The plane MAB ∩ The plane MBC ∩ The plane MAC = ……………

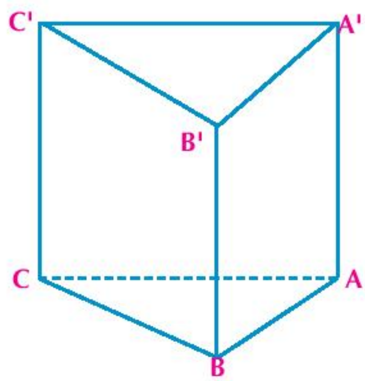
Solution

(a) $\overleftrightarrow{MB}$ (b) $\overleftrightarrow{BC}$ (c) {B}

(d) ϕ (because they are two skew straight lines) (e) {M}

Try to solve

(2) Meditate the following figure, and then complete::



(a) The plane ABB'A' ∩ The plane BCC'B' = ……………

(b) The plane ABC ∩ The plane A'B'C' = ……………

(c) $\overleftrightarrow{AC} \cap \overleftrightarrow{A'C'}$ = ……………

(d) $\overleftrightarrow{BB'}$ ∩ The plane ABC = ……………

Exercise (2-1)

Complete the following:

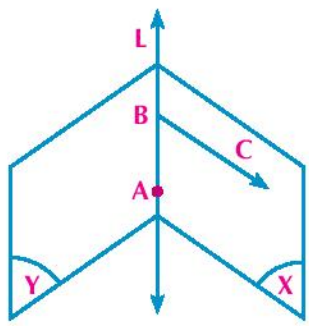
1. If the straight line L // the plane X, then $L \cap X =$ ....
2. If the straight line $L \subset$ the plane X, then $L \cap X =$ ....
3. If the straight line L_1 // the straight line L_2 ,then $L_1 \cap L_2 =$ ....
4. If X and Y are two planes such that: $X \cap Y = \phi$, then X.......... Y
5. The two skew straight lines are neither … ….. nor … … …
6. **State the number of planes that passes through the following:**

   (a) One given point (b) Two different points

   (c) Three collinear points

   (d) Three non-collinear points.

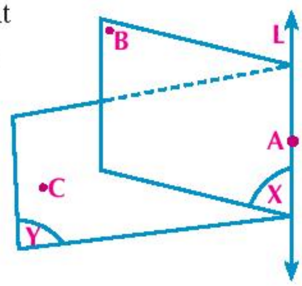
7. Meditate the following figure, and then complete using one of the following symbols ($\in$, $\notin$, $\subset$, $\not\subset$)



   (a) L .... X (b) A X

   (c) C .... Y (d) $\overleftrightarrow{BC}$ .... Y

8. In the opposite figure : X,Y are two planes intersected at the straight line L ,$A \in L$,$B \in X$, $B \notin Y$, $C \in Y$, $C \notin X$ Complete the following:



   (a) The plane X $\cap$ The plane A B C = ...

   (b) The plane Y $\cap$ The plane A B C = ...

   (c) The plane X $\cap$ The plane Y $\cap$ The plane A B C = ...

9. **Meditate the following figure, and then complete the following:**

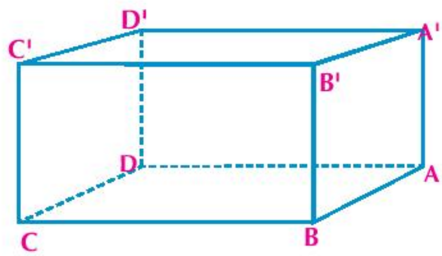
   (a) The plane ABCD // The plane ....

   (b) The plane BCC'B' // The plane ....

   (c) The plane ABB'A' $\cap$ The plane ABCD = ...

   (d) The plane ABB'A' $\cap$ The plane DCC'D'= ....

   (e) The plane DCC'D' $\cap$ The plane A B C D $\cap$ The plane ADD'A'= ....



(10) Put the sign (✓) for the correct answer and the sign (✗) for the incorrect answer where L_1, and L_2 are two straight lines and X,Y are two planes:

(a) If $L_1 \cap L_2 = \phi$ **then** $L_1 // L_2$ **or** L_1, L_2 are skew

(b) If $L_1 \cap X = \phi$ **then** $L_1 // X$

(c) If $L_2 \cap X = L_2$ **then** $L_2 \subset X$

(d) If $L_2 \subset Y$ **then** $L_2 \cap Y = \phi$

(e) If $X \cap Y = \phi$ **then** $X // Y$

(f) If X = Y **then** X , Y are coincident

Choose the correct answer for each of the following:

(11) Any four non-coplanar points form :

(a) Two planes (b) three planes (c) four planes (d) no plane

(12) If two planes have two common points A and B, then they will be:

(a) Coincident

(b) intersected at $\overleftrightarrow{AB}$

(c) Intersected at a straight line parallel to $\overleftrightarrow{AB}$

(d) With a third common point does not belong to $\overleftrightarrow{AB}$

(13) $\overleftrightarrow{AB}$ // the plane X if

(a) $\overline{AB} \cap X = \phi$

(b) A and B lie in two different sides from X

(c) A and B with two different distances from X

(d) $\overleftrightarrow{AB} \cap X = \phi$

(14) The two straight lines L_1 and L_2 are parallel if:

(a) $L_1 \cap L_2 = \phi$

(b) $L_1 \cup L_2$ lie in the same plane

(c) $L_1 \cap L_2 = \phi$, L_1, L_2 located in the same plane.

(d) $L_1 \cap L_2 = \phi$, L_1, L_2 does not located in the same plane.

(15) The two straight lines are skew, if they are:

(a) not parallel.

(b) not coincident.

(c) not located in the same plane.

(d) located in the same plane.

Critical thinking:

(16) Represent by drawing that: if three planes are intersected in pairs, then their lines of intersection will be either parallel or met at a point.

2 - 2 Pyramid and Cone

We will learn

- Properties of some figures:
- Pyramid - Regular Pyramid - Right Pyramid - Cone - Right cone.
- The concept of net solid.
- Conclude properties of the solid from its net.
- Drawing the net of a solid .
- Modeling and solving the problems of mathematical life situations. Using the properties of pyramid and right cone.

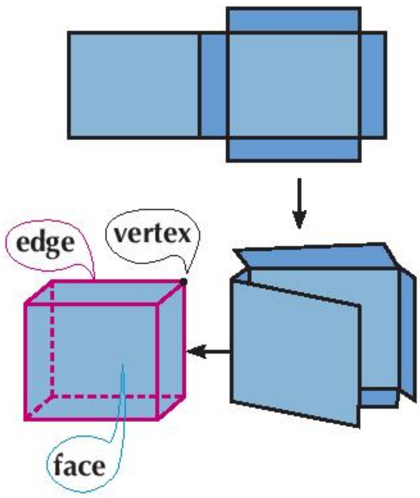
Key - term

- Pyramid - cone - height lateral face - lateral edge - slant height - Regular pyramid - Right pyramid - Net - Right circular cone.

Matrials

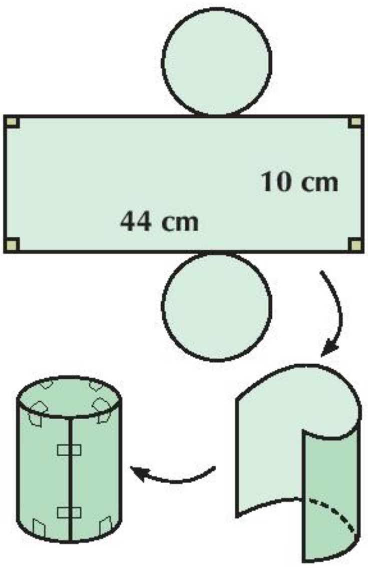
- Geometrical instrument
- Scientific calculator.
- Drawing programs.

Many containers are manufactured by folding the flat cardboard to a three dimensional form to mobilize the factories, products before marketing so they occupy the vaccum of space, such as: cube and cuboid, ...



- ➢ How many faces and vertices does the cube have?
- ➢ How many edges does the cuboid have?
- ➢ Are all faces of the cube identical? Explain your Answer.

We call the figure which can be folded to form a solid by a net solid and from which we deduce the properties of the solid.



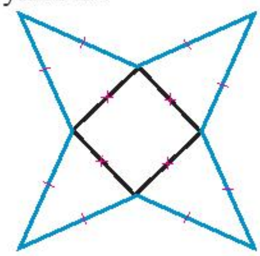
The opposite figure shows a right circular cylinder net, notice:

1 - The bases of the cylinder are identical and each of them has the shape of a circle.

2 - The lateral surface of the cylinder before its folding is a rectangle. whose dimensions are 44 cm, 10 cm then the height of the cylinder will be 10 cm.

What is the length of the radius of the base of the cylinder ?

Think:



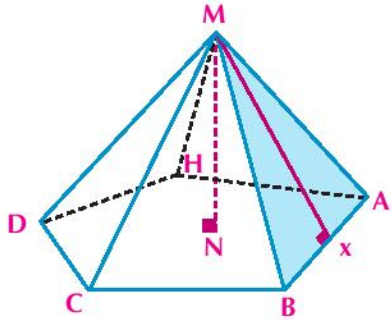
Do you know the name of a solid which will be configured by folding the opposite net? Deduce some of its properties?

Can you draw more than one net for the solid? Explain your answer.

Pyramid:

It's a solid of a single base and all of its other faces are triangles that have the same vertix the pyramid is called triangular, quadrelateral, pentagonal pyramid, according to the number of sides of its base.

Note: In the opposite figure MABCDH is a penta pyramid, with vertix M, and its base is the polygon ABCDH, its lateral faces are the surfaces of triangles MAB, MBC, MCD, MDH, MHA. and its lateral edges are $\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$, $\overline{MH}$.



The height of the pyramid (MN) is the distance between the pyramid's vertix and its base level.

The slant height is the distance between the pyramid's vertix and any of its base edges.

Def.

Regular pyramid

It's a pyramid whose base is a regular polygon its centre is the foot of the perpendicular from the vertix to its base.

Properties of Regular pyramid

1 - Its lateral edges are equal in length.

2 - Its lateral faces are surfaces of isosceles congruent triangles.

3 - Slant heights are equal.

Note

Regular polygon is a polygon of sides that are equal in length and its angles are equal in measures. Its centre is a centre of drawn circle inside or outside it.

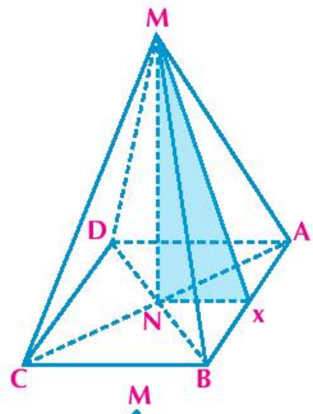
Note:

The perpendicular straight line drawn from the top of the pyramid to the level of its base is perpendicular to any straight line on it.

In the opposite figure: if $\overline{MN}$ is perpendicular to the base level

$\overline{MN} \perp \overline{AC}$, $\overline{MN} \perp \overline{BD}$, $\overline{MN} \perp \overline{NX}$

$\therefore$ The triangle MXN is a right angle triangle at N



Example

(1) MABCD is a regular quadrelateral pyramid. The length of its base side ABCD equals 10 cm and its hieght equals 12 cm. Find its slant height.

Solution:

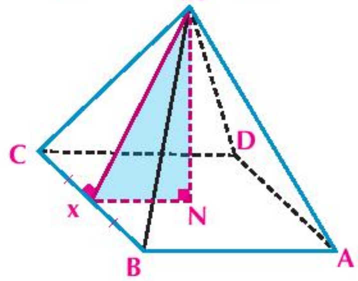
$\because$ The pyramid is regular quadrelateral

$\therefore$ $\overline{MN} \perp$ to the plane ABCD

where N is a point of intersection of the diagonals of the square ABCD, MN = 12 cm.

Let x be the midpoint of $\overline{BC}$ $\therefore$ $\overline{MX} \perp \overline{BC}$ (Why?)

and, $\overline{MX}$ is a slant height of the regular pyramid



In $\triangle$ DBC: N is the mid point of $\overline{DB}$, X is the midpoint of $\overline{BC}$

$\therefore NX = \frac{1}{2} DC = \frac{1}{2} \times 10 = 5$ cm

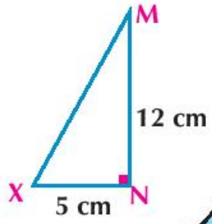
$\because \overline{MN} \perp$ plane ABCD

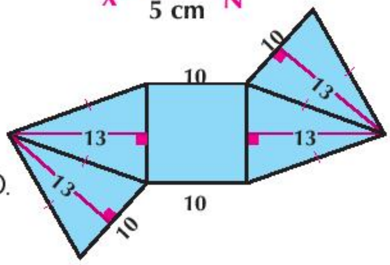
$\therefore \triangle$ MNX is a right angled at N

and, $(MX)^2 = (MN)^2 + (NX)^2 = (12)^2 + (5)^2 = 169$

$\therefore$ The slant height of the pyramid = 13 cm.

and the opposite figure shows one of the net of the pyramid MABCD.





Try to solve

1) MABCD is a regular quad. pyramid its height 20 cm, and its slant height 25 cm. Find the length of the side of the pyramid's base.

Right pyramid

The pyramid is right if and only if the projection of the perpendicular drawn from the vertex of the pyramid to its base passes through its geometrical centre.

Think:

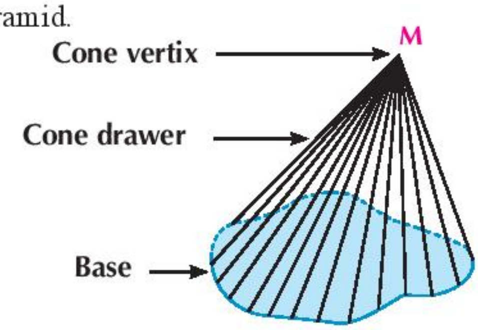
1 - Is the regular pyramid a right pyramid? Explain your answer.

2 - Are the slant height of the right pyramid equal in length?

Note: the pyramid is said to be triangular regular pyramid if all of its faces are equilateral triangles, so each face can represents a base of the pyramid.

Cone

It's a solid with a single base in the form of a closed curve with one vertix, its lateral surface consists of all points of line segments drawn from the vertix to the base of the curve, each of them is known as cone drawer.



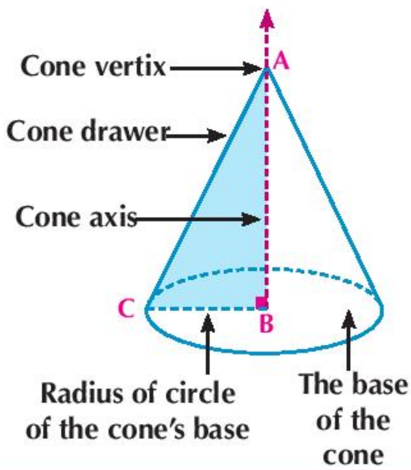
Right circular cone

It's a solid which is generated from the rotation of right angled triangle a compete revolution around one of the sides of the right angle as an axis.

Properties of a right circular cone:

The opposite figure shows a right circular cone constructed from rotation of right- angled triangle at B a complete rotation around $\overleftrightarrow{AB}$ as an axis. We will find that:

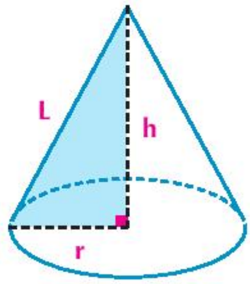
1- $\overline{AC}$ is a cone drawer, A is the vertex of the cone, the point C draw during rotation a circle its centre is the point B. The length of the radius of the circle equals to the length of $\overline{BC}$, the surface of the circle is the base of the cone.



2- $\overleftrightarrow{AB}$ the axis of the cone is perpendicular (⊥) to the base of the cone, the height of the cone equals to the length of $\overline{AB}$.

Example

(2) A right circular cone, the length of its drawer equals 17 cm, and its height equals 15 cm. Find the length of the radius of its circle.



Solution:

Let the length of the cone drawer = L, and its height = h,

the length of the radius of its base circle = r

$\because r^2 = L^2 - h^2$

$\therefore r^2 = (17)^2 - (15)^2 = 64$ $\therefore r = 8$ cm

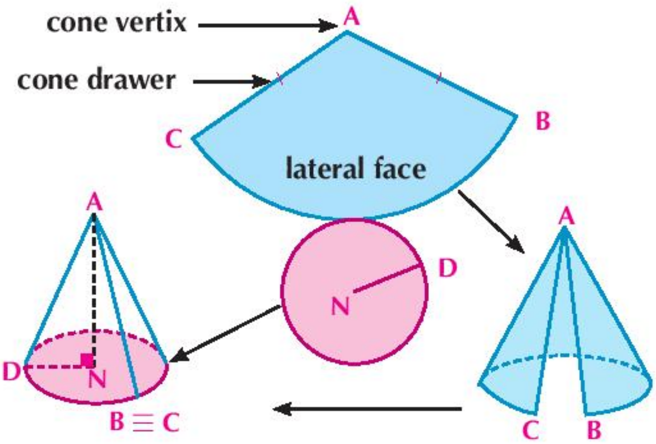
Try to solve

(2) Find in terms of π the circumference and the area of the base of a right circular cone. Its height equals 24 cm, and the length of its drawer equals 26 cm.

Think: A triangle ABC, AB = AC, D is the midpoint of $\overline{BC}$. If the triangle ABC rotates half a complete rotation around the axis $\overleftrightarrow{AD}$. Does a right circular cone arises? Explain your answer.

Right cone net:

The net of the right cone can be folded for forming cone-shaped containers as in the opposite figure, where:



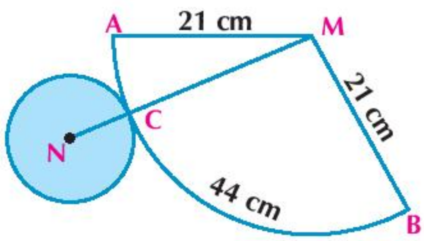
1 - AB = AC = L **(the length of the cone drawer)**

2 - The circular sector ABC shows the lateral surface of the cone.
The length of $\widehat{BC} = 2\pi r$
(r is the length of the radius of the cone's base)

3 - The height of the cone = the length of $\overline{AN}$

Example

(3) The opposite figure shows a net of a right cone, using the given data, find its height ($\pi \simeq \frac{22}{7}$).



Solution:

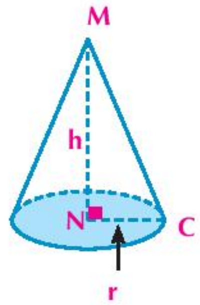
From the net of the cone we note that:

Length of the cone drawer = length of $\overline{MA}$ = 21 cm

Circumference of the cone base = length of $\widehat{AB}$ = 44 cm.

Length of the radius of the cone base = length of $\overline{CN}$ = r

When folding the net of the cone, the opposite figure arises then:



the height of the cone = the length of $\overline{MN}$ = h

$\because 2\pi r = 44$ $\therefore 2 \times \frac{22}{7} \times r = 44$ **then** $r = 7$ cm

$\because h^2 = L^2 - r^2$

$\therefore h^2 = (21)^2 - (7)^2 = 14 \times 28$ **then** $h = 14\sqrt{2}$ cm

$\therefore$ The height of the right circular cone = $14\sqrt{2}$ cm.

Try to solve:

3) In the previous net of the right cone, MA = 41cm, the length of $\overset{\frown}{AB} = 18\pi$ cm. Find the height of the cone.

Critical thinking: Is the following statement true:

the height of the right cone > the length of its drawer? (Explain your answer).

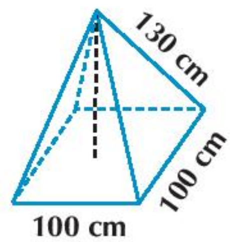
Exercises (3 - 2)

1) In the regular pentagonal pyramid:

A) What the number of its lateral faces?
B) What the number of its faces?
C) What the number of its lateral edges?
D) What the number of its edges?
E) The pyramid has one vertix regardless of the vertices of the base. what is the number of all vertices of pentagonal pyramid? Is your answer prove Euler's rule for any solid, its base is a polygon. **"number of faces + number of vertices = number of edges +2"**

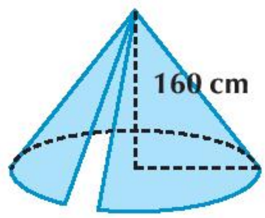
2) In the regular pyramid, arrange the following lengths ascendingly

A) The length of the lateral edge
B) The height of the pyramid
C) The slant height



3) **Civil geometry:** The opposite figure shows a water tank in the form of regular quadrangular pyramid. Using the given data, find each of the slant height and the height of the tank.

4) **Connectivity with scouts:** A tent in the form of right circular cone, its height =160 cm and the circumference of its circular base = 753.6 cm. Find the length of the cone (tent) drawer.



5) **Connecting to tourism:** The great pyramid of Giza (Khofo pyramid), the length of the side of its base is 232 m and its slant height is 186 m. Find the height of the pyramid.

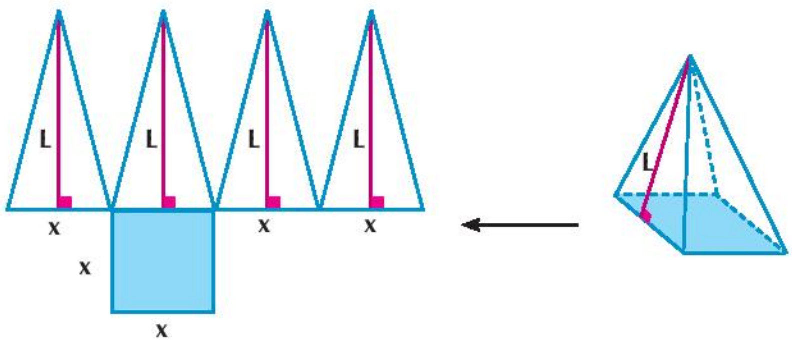
Total area of pyramids and cones

2 - 3

You have already learned the properties of the pyramid and the right circular cone. You have concluded some of them by studying their net. Can you calculate the lateral area and the total surface area of the regular pyramid and the right circular cone from their net? Explain.

Total area of a regular pyramid

The opposite figure shows a regular quadrelateral pyramid and one of its nets.



Note: The lateral faces are congruent isosceles triangles whose slants are with congruent heights (equal in length) and each of them = L.
The base of the pyramid is a regular polygon whose side length = X, then:

The lateral area of the pyramid

= sum of the area of the lateral faces.

$= \frac{1}{2}\ x \times L + \frac{1}{2}\ x \times L + \frac{1}{2}\ x \times L + \frac{1}{2}\ x \times L$

$= \frac{1}{2}\ (x + x + x + x)\ L$

$= \frac{1}{2}$ premiter the base of the pyramid × slant height.

Total area of the pyramid = lateral area of it + the area of its base.

Learn

The lateral area of the regular pyramid = $\frac{1}{2}$ the premiter of its base × its slant height.
The total area of the pyramid = its lateral area + its base area.

We will learn

- Finding the lateral area and the total surface area of the regular pyramid and right cone.
- Modeling and solving mathematical and life problems include the surface area of the pyramid and the right cone.

Key - term

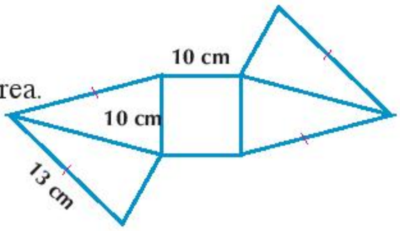
- Lateral surface area (L.S.A)
- Total surface area (T.S.A)

Matrials

- *Scientific calculator*
- *Computer- Graphic program.*

Example

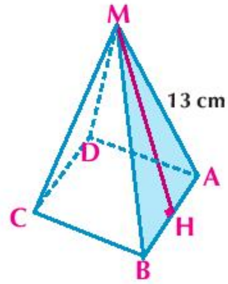
1 Using the opposite net, describe the solid and find its total area.



Solution

The net for a regular quadrangular pyramid.

Its base is square whose side length = 10 cm, the length of its lateral edge 13 cm.



$\because$ The lateral face MAB is an isosceles triangle, $\overline{MH}$ is a slant height.

$\therefore$ H is the mid point of $\overline{AB}$,which means AH = 5 cm

In $\triangle$ MHA which is right angled at H, we find that:

$(MH)^2 = (AM)^2 - (AH)^2$

$(MH)^2 = (13)^2 - (5)^2 = 144$

$\therefore$ M H = 12 cm

$\because$ The lateral area of the regular pyramid = $\frac{1}{2}$ Perimeter of the base × the slant height

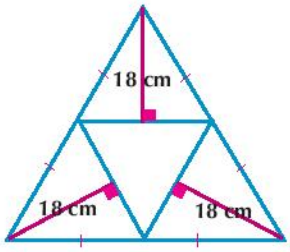
$\therefore$ The lateral area = $\frac{1}{2} \times (10 \times 4) \times 12 = 240 \text{ cm}^2$

$\because$ The area of the pyramid base = $(10)^2 = 100 \text{ cm}^2$

$\therefore$ The total area of the pyramid = $240 + 100 = 340 \text{ cm}^2$

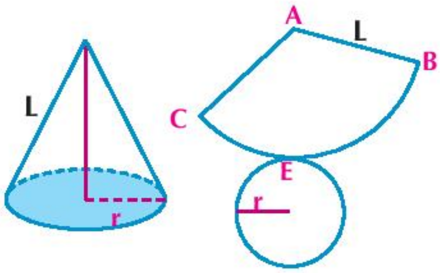
Try to solve

1 Using the opposite net, describe the solid and find its total area.



Total area of the right cone

From the right cone net. In the opposite figure:



The area of the sector ABC = $\frac{1}{2}$ AB × length of $\overset{\frown}{BC}$

$= \frac{1}{2}$ L × perimeter of cone base

$= \frac{1}{2}$ L × $2\pi r = \pi L r$

= Lateral area of the right cone

The total area of the right cone = its lateral surface area + area of its base

Learn

Lateral area of the right cone = $\pi L r$

Total area of the right cone = $\pi L r + \pi r^2 = \pi r (L + r)$

Where L is the length of slant height, r is the radius of the circle.

Remember that

$\theta^{rad} = \frac{L}{r}$

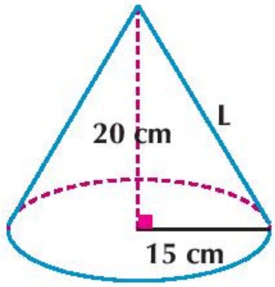
Perimeter of the circular sector = 2r + L

Area of the circular sector

$= \frac{1}{2} L r = \frac{1}{2} \theta^{rad} r^2$

Example

(2) Find the lateral area of a right cone, of base radius 15 cm, and its height 20 cm.



Solution

To find the length of cone drawer L

$\because L^2 = (20)^2 + (15)^2 = 625$

$\therefore L = 25$ cm

$\because$ Lateral area of right cone $= \pi L r$, r = 15 cm

$\therefore$ Lateral area of right cone $= 25 \times 15\pi = 375\pi$ cm^2

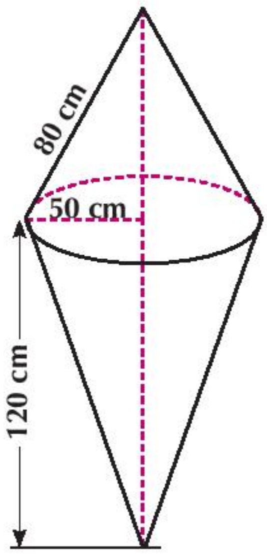
Try to solve

(2) Find the total area of a right cone if the length of its drawer = 17 cm and its height = 15 cm.

Example

(3) **Marine navigation:** The opposite figure shows guide sign (Shamandora) (Buoy) to determine the waterway, and it is in the form of two right cones have a common base.

Find the costs of its painting with a material which resists erosion factor, Note that each square meter of it costs 300 pound



Solution:

The area of the guide sign surface =

lateral area of the 1st cone + lateral area of the 2nd cone

First cone: $L_1 = 80$ cm , $r_1 = 50$ cm

$\therefore$ Lateral area $= 50 \times 80\pi = 4000\pi$ cm^2

Second cone: h = 120 cm , $r_2 = 50$ cm

$\therefore L_2 = \sqrt{(120)^2 + (50)^2} = 130$ cm

$\therefore$ Lateral area $= 50 \times 130\pi = 6500\pi$ cm^2

The area of the guide sign surface $= (4000 + 6500)\pi = 10500\pi$ cm^2

$\simeq 3.299$ squared meter

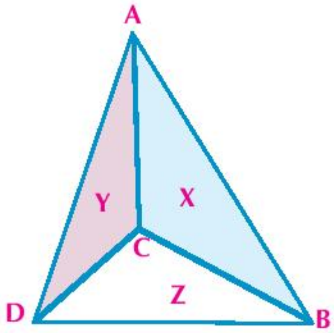
Cost of painting $= 3.299 \times 300 = 989.7$ pound

Try to solve

(3) Lamb cover is in the form of a right cone. The circumference of its base circle = 88 cm, its height = 20 cm. Calculate its area to the nearest square centimeter.

Exercises (3 -3)

1 The opposite figure represents a triangular pyramid, X , Y and Z are three planes. Complete the following



A X ∩ Y = ... **B** X ∩ Z = ...

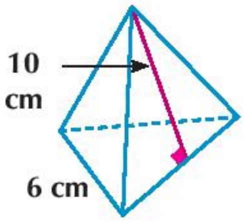
C Y ∩ Z = ... **D** $\overleftrightarrow{AB}$ ∩ X = ...

E $\overleftrightarrow{BC}$ ∩..................... X , $\overleftrightarrow{BC}$ .... Z

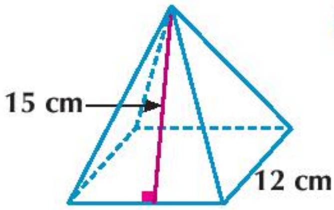
F X ∩ Y ∩ Z = ...

2 Find lateral area, and total area for each regular pyramid, according to the given data

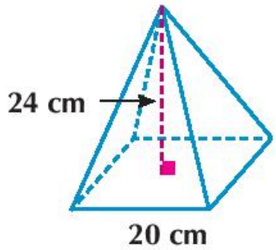
A



B

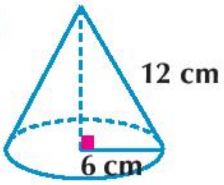


C

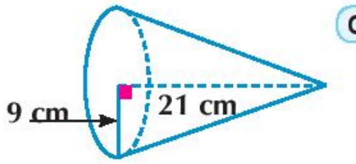


3 Find lateral area, and total area for each right cone, according to the given data.

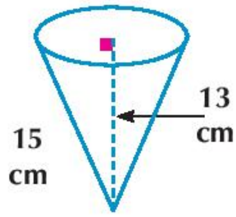
A



B



C



4 Hexagonal regular pyramid, the length of its base side 12 cm and its slant height $10\sqrt{3}$ cm. Find:

A Its lateral area **B** Its total area

5 Find the length of the radius of a right cone, if the length of the cone drawer 15 cm and its total area 154π cm^2.

Volumes of pyramids and cones

2 - 4

We will learn

- Finding the volume of regular pyramid.
- Finding the volume of right cone.
- Modeling and solving mathematical and life problems which include the volume of each of the regular pyramid and right cone.

Key - term

- Vertex
- Base
- Face
- Axis
- Radius
- Volume

Matrials

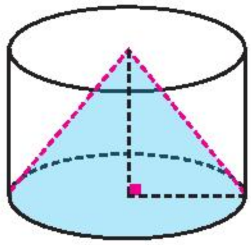
- Scientific calculator
- Computer- Graphic program.

Think and discuss

You have already learned how to calculate the volume of a right prism and the volume of a right circular cylinder.

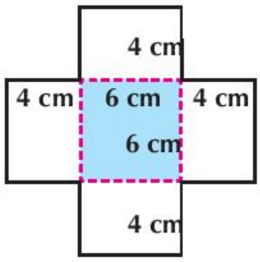
Can you estimate the volume of the pyramid in terms of the volume of the right prism which has the same base area and the same height?

Can you estimate the volume of the a right cone in terms of the volume of a cylinder which has the same base area and the same height?



Activity

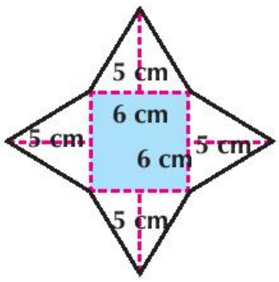
Comparison between the volumes of a pyramid and a prism which have the same area of the base and the same height.



1- Draw on a cardboard the net of the pyramid and prism which are shown in the opposite figure.

2- Cut and fold each net to make two models. One of them is the lateral surface of a quadrangular pyramid, while the other is a right prism, opened from the top.

3- Fill the pyramid with grains of rice or sand and empty it in the prism. Repeat this until the prism is completely filled.



Note that: the contents (grains of rice or sand) that you need to fill the prism, will completely fill three pyramids which means:

The volume of the pyramid = $\frac{1}{3}$ the volume of the prism which have the same base area of the pyramid (b), and the same height of the pyramid (h).

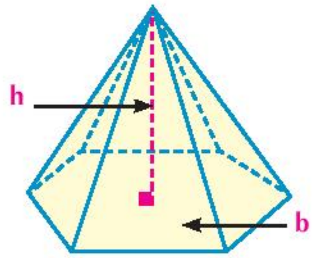
Volume of a Pyramid

Learn

The volume of the pyramid equals one third of the product of the area of its base multiplied by its height.

Which means: the volume of the pyramid = $\frac{1}{3}$ b × h

Where: (b) is the area of the base, (h) is the height of the pyramid.



Example

1 Calculate the volume of regular quadrangular pyramids, the length of its base side is 18 cm and its slant height is 15 cm.

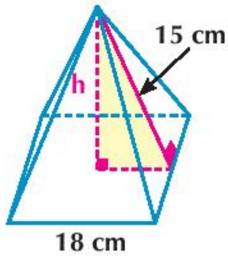
Solution:

First: calculation of the area of the pyramid's base (b)

$\because$ The pyramid is regular quadrangular.

$\therefore$ Its base is a square shape.

The area of the pyramid's base (b) = 18 × 18 = 324 cm^2



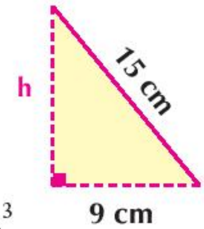
Second: calculation of the pyramid's height (h)

$\because h^2 + (9)^2 = (15)^2$ Pythagouras

$\therefore h^2 = (15)^2 - (9)^2 = 144$, h = 12 cm

$\because$ pyramid's volume = $\frac{1}{3}$b × h

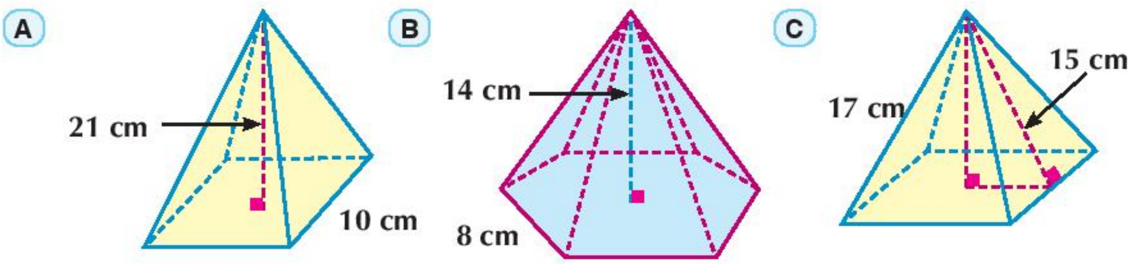
$\therefore$ pyramid's volume = $\frac{1}{3} \times 324 \times 12 = 1296\ cm^3$



> **Remember that**
>
> surface area of a regular polygon of n side, length of each side is x equals $\frac{n}{4} x^2 \cot \frac{\pi}{n}$

Try to solve

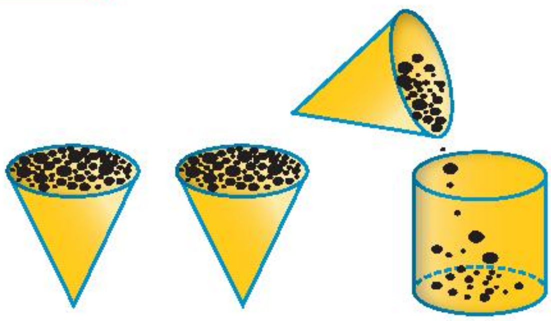
1 Find the volume of a regular pyramid which is shown in each figure using the given date.



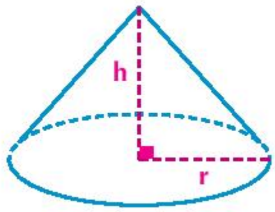
Think: When comparing the volume of right circular cone and right cylinder having the same base area and the same height, we find that:

Volume of a cone = $\frac{1}{3}$ volume of cylinder.

How can you explain that mathematically?



Volume of a cone



Learn

Volume of a cone equals one third product of the area of its base by its height.

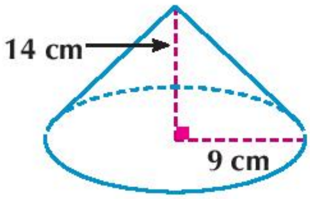
which means: The volume of the cone = $\frac{1}{3}\pi r^2 h$

where (r) is the length of the radius of the cone's circle, (h) is the height of the cone.

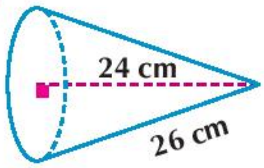
Try to solve

(2) Find the volume of the right cone shown in the figure using the given data.

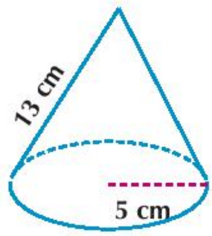
A



B



C



Try to solve

(3) A piece of chocolate in the form of a right cone. Its volume is 27π cm^3, the perimeter of its base 6π cm. Find its height.

Example

(2) **Connecting to Industry:** A pentagonal regular pyramid from copper, the length of its base polygon side is 10 cm, and its height is 42 cm. It's melted and converted to a right circular cone. The length of its base radius is 15 cm. If it had known that 10% of copper had been lost during melting and converting it, find the height of the cone to the nearest one decimal number.

Solution

$\because$ The area of the regular pentagon $= \frac{5}{4} x^2 \cot \frac{\pi}{5}$ (x is the length of its side)

$\therefore$ The area of the pyramid base $= \frac{5}{4} \times 10 \times 10 \cot 36° = \frac{125}{\tan 36°} \simeq 172$ cm^2

$\because$ The volume of the pyramid $= \frac{1}{3}$ area of the base $\times$ the height $= \frac{172}{3} \times 42 = 2408$ cm^3

$\therefore$ The volume of copper in the cone $= \frac{90}{100} \times 2408 = 2167.2$ cm^3

$\frac{\pi}{3}(15)^2 h = 2167.2$ where h is the height of right cone

$\therefore h = \frac{2167.2 \times 3}{225\pi} \simeq 9.2$ cm

Try to solve

4) A cube of wax, the length of its edge is 20 cm. It's melted and converted to a right circular cone, its height is 21 cm. Find the length of the base radius of the cone, if it is known that 12% of wax had been lost during melting and reforming.

Important note: Container capacity is estimated by the volume of the liquid which it contains. To calculate the capacity, the same laws of calculating the volumes are used. The measuring unit of capacity is litre.

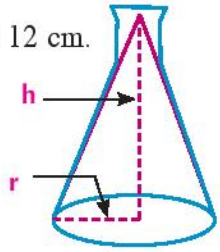
Remember that

Capacity being used for how much a container can hold.

1 litre = 1000 milliliter = 1000 cm^3 = dm^3

Example

3) **Connecting to chemistry:** A conical flask, its capacity 154 ml, its height 12 cm. Find the length of its base radius ($\pi \simeq \frac{22}{7}$)



Solution

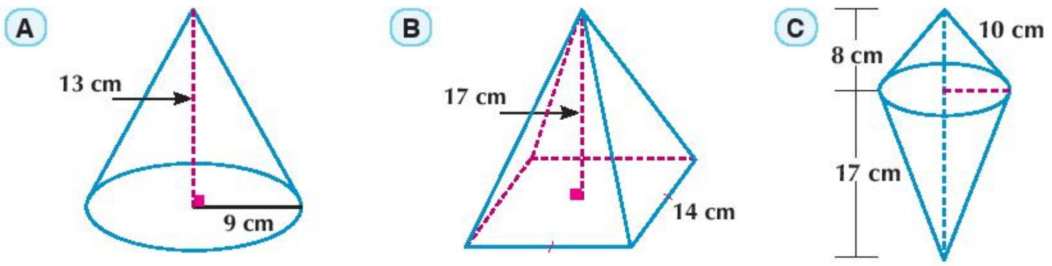
capacity of the flask = volume of right cone = 154 cm^3

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 12 = 154 \quad \therefore r^2 = \frac{49}{4}$

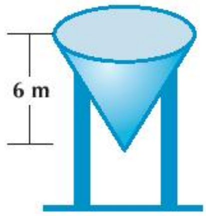
$\therefore r = 3.5$ cm

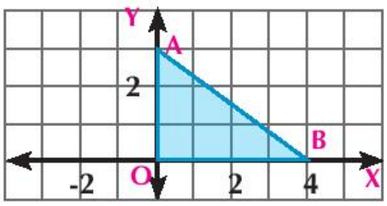
Exercises (3 - 4)

1. Find the volume of a regular quadrangular pyramid. The length of the side of its base =20 cm, and its height =36 cm.
2. Calculate to the nearest one decimal place, the volume of a regular pentagonal pyramid the length of its base side is 40 cm, and its height is 10 cm.
3. A regular quadrangular pyramid, its height 9 cm, and its volume 300 cm^3. Find the length of the side of its base polygon.
4. A regular quadrangular pyramid, the area of its base 700 cm^2, and its slant height 20 cm. Find its volume.
5. Which is greater in volume? A right cone the length of its base radius is 15 cm, and its height is 20 cm, or a regular quadrangular pyramid its height is 40 cm, and the perimeter of its base is 48 cm.
6. Find the volume of a right cone its base perimeter is 44 cm, and its height is 25 cm.
7. Find the volume of a right cone its lateral area is 220 cm^2, and the length of its drawer is 14 cm.
8. Arrange the following figures from the smallest volume to the largest volume.



9. **Civil engineering:** A tank of water in the form of right cone, its volume is 32 πm^3 and its height is 6 m. Find the length of its base radius and its total area.





10. The opposite figure shows a coordinate perpendicular plane. calculate in terms of π the volume of solid generated when revolving triangle ABO one complete revolution around:
    - A) The x -axis
    - B) The y - axis.

2 - 5

Equation of a circle

We will learn

- Writing the equation of the circle in terms of its coordinate centre and the length of its radius.
- The general equation of the circle.
- determining the coordinate of the circle centre, and the length of its radius from the general equation of the circle.

Key - term

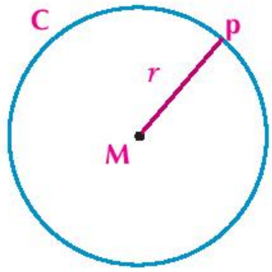
- Circle
- Center
- Radius
- Diameter
- Cartesian plane
- Equation
- General Form

Matrials

- Scientific calculator
- Graph paper

Introduction :

You know that the circle is a set of plane points which are equidistant from a fixed point in the plane.

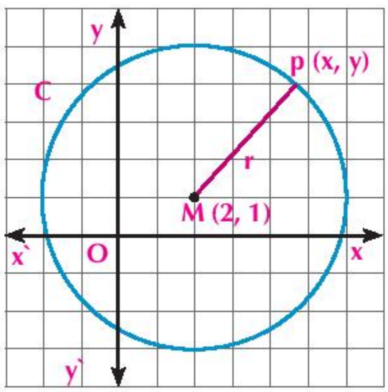


The fixed point is called the "center", and its is usually denoted by the symbol M, and the distance from the centre to any point on the circle is called "radius". It is denoted by the symbol r.

The equation of a circle:

Equation of a circle is a relation between the x-coordinate and the y-coordinate for any point belongs to this circle. Each ordered pair (x, y) satisfies this relation (equation) and represents a point belonging to this circle.

In the perpendicular coordinate plane, if the point P (x, y) belongs to the circle c.



The length of its radius equals 4 units, and its centre is the point M (2.1), then

MP = r = 4, by applying the law of distance between two points, then: $(x - 2)^2 + (y - 1)^2 = (4)^2$

$\therefore (x - 2)^2 + (y - 1)^2 = 16$

which is the equation of the circle c

Remember that

The distance between the two points (x_1, y_1), (x_2, y_2)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

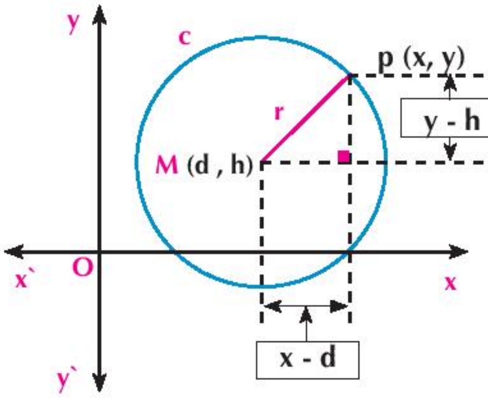
Learn

The equation of a circle

(In terms of the coordinates of its centre and length of its radius)

In a perpendicular coordinates plane:

If the point P (x , y) which belongs to the circle C whose centre is the point M (d, h) and length of its radius equal r , then the equation of the circle is:



$$(x - d)^2 + (y - h)^2 = r^2$$

Example

1) Write the equation of a circle C, its centre is the point M (5 , 2) and the length of its radius equals 6 units.

Solution

Let the point P (x , y) ∈ the circle C

∵ The center of the circle is M (5 , 2), the length of the radius 6 units

∴ d = 5 , h = 2 , r = 6

, then the equation of the circle is: $(x - 5)^2 + (y - 2)^2 = (6)^2$

then: $(x - 5)^2 + (y - 2)^2 = 36$

Try to solve

1) Write the equation of a circle if its centre:

A) M (4 , -3) , the length of its radius equals 5 units.

B) M (7 , -1) , the length of its diameter equals 8 units.

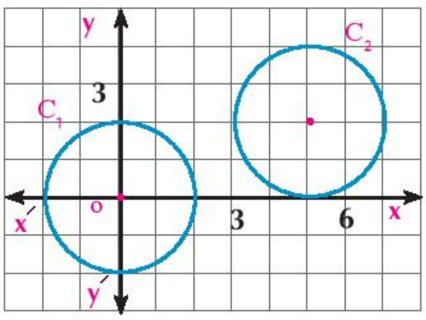
C) M (2 , 0) , the length of its diameter equals $\sqrt{28}$ units.

D) M (0 , -5) , and passes through the point A(-2 , -9)

E) The origin point, the length of its radius equals r units.

Example

2) The opposite figure shows the two circles C_1,C_2. Prove that the two circles are congruent, then find the equation of each of them.



Solution

The two circles are congruent if the two radii have the same length.

The circle C_1: its centre (0 . 0), the length of its radius r_1 = 2 units.

The circle C_2: its centre (5 . 2), the length of its radius r_2 = 2 units.

∵ $r_1 = r_2 = 2$ ∴ The two circles are congruent.

then: the equation of the circle c_1: $x^2 + y^2 = 4$,

the equation of the circle c_2: $(x - 5)^2 + (y - 2)^2 = 4$

Note: The circle C_2 is the image of the circle C_1, by a translation (5 , 2)

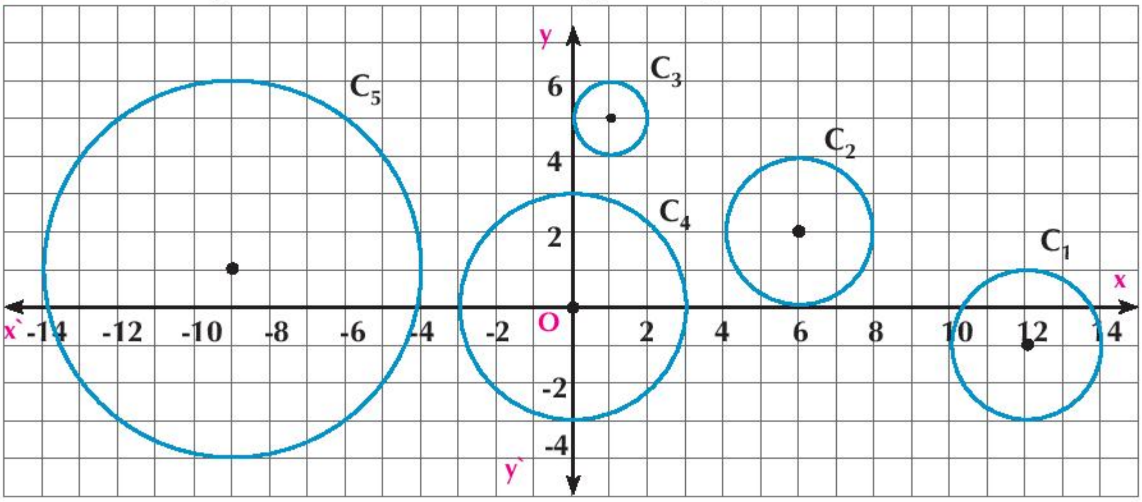
Remember that

The image of the point (h, k) by translation (a, b) is (h+a, k+b)

Critical thinking: If the circle C_3 is the image of the circle C_1 by a translation (-4 , 3), then write the equation of the circle C_3.

Try to solve

2 **A** Write the equation of each circle in the opposite figure:



B Which of the previous circles are congruent? Explain your answer.

Think: what is the position of the point (x_1, y_1) with respect to the circle C: $(x - d)^2 + (y - h)^2 = r^2$ if:

A $(x_1 - d)^2 + (y_1 - h)^2 > r^2$ **B** $(x_1 - d)^2 + (y_1 - h)^2 < r^2$

Example

3 Show that the point (4 , -1) is a point on the circle C its equation: $(x - 3)^2 + (y - 5)^2 = 37$

Solution

By substituting the coordinates of the point (4 , -1) in the right hand side of the equation of the circle.

$\therefore (4 - 3)^2 + (-1 - 5)^2 = 1 + 36 = 37 =$ left hand side

$\therefore$ the point (4 , -1) belongs to the circle C.

Note that: If $(x_1 - 3)^2 + (y_1 - 5)^2 > 37$ then the point (x_1 , y_1) lies outside the circle C.
and if $(x_1 - 3)^2 + (y_1 - 5)^2 < 37$ then the point (x_1 , y_1) lies inside the circle C.

Try to solve

3 Show which of the following points belongs to the circle C, whose equation: $(x - 6)^2 + (y + 1)^2 = 25$, then determine the position of other points with respect to the circle C. where:

A(9 , 3) , B(7 , 5) , C(3 , 3) , E(2 , -3)

Example

(4) Write the equation of the circle whose diameter is $\overline{BA}$ where A(2 , -7) , B(6 , 5)

Remember that

The coordinator of mid point between (x_1, y_1), and (x_2, y_2) is:

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

Solution

Let the point M (d , h) be the center of the circle whose diameter is $\overline{BA}$, so the point M will be the midpoint of $\overline{BA}$.

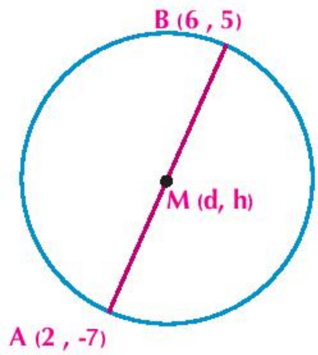
$\therefore$ The coordinates of M: $d = \frac{2 + 6}{2} = 4$, $h = \frac{-7 + 5}{2} = -1$

$r^2 = (AM)^2 = (4 - 2)^2 + [-1 - (-7)]^2$

$= (2)^2 + (6)^2 = 40$

The equation of the circle will be: $(x - 4)^2 + [y - (-1)]^2 = 40$

Which means: $(x - 4)^2 + (y + 1)^2 = 40$



Think: Is the point (6 , 5) satisfy the equation of the circle? Why?

Is the point (6 , -7) belongs to the previous circle, Explain your answer.

Try to solve

(4) Write the equation of the circle if:

(A) Its centre is the point M (-2 , 7), and passes through the point A(2 , 10).

(B) Its centre is the point M (5 , 4), and touches the straight line x = 2

(C) Its centre M lies in the 1st quad. of the coordinates plane where its radius is of length 3 units, and the two straight lines x = 1 , y = 2 are two tangents to it.

Example

(5) Find the coordinates of the center, and length of its radius in each of the following circle:

(A) $(x - 2)^2 + (y + 3)^2 = 17$ (B) $(x + 1)^2 + y^2 = 16$

Solution

We know that the equation of a circle in terms of coordinates of the centre (d , h), and the length of its radius r is :

$(x - d)^2 + (y - h)^2 = r^2$

Compare each of the algebraic expressions in the equation by its corresponding in the given equation. We find:

(A) $x - d = x - 2$ $\therefore d = 2$

$y - h = y + 3$ $\therefore h = -3$

$r^2 = 17$ $\therefore r = \sqrt{17}$

then the centre of the circle will be the point (2 , -3) and the length of its radius equals $\sqrt{17}$ units.

B $x - d = x + 1$ $\therefore d = -1$

$y - h = y$ $\therefore h = 0$

$r^2 = 16$ $\therefore r = 4$

$\therefore$ The centre of the circle is the point (-1 , 0) and the length of its radius equals 4 units.

Try to solve

5 Which of the given circles represents a circle whose centre is (3 , -4) and the length of its radius equals 3 units.

A $(x - 3)^2 + (y - 4)^2 = 9$ **B** $(x + 3)^2 + (y - 4)^2 = 9$

C $(x - 3)^2 + (y + 4)^2 = 9$ **D** $(x + 3)^2 + (y + 4)^2 = 9$

6 Find the coordinates of the center and length of radius of each of the following circles:-

A $(x - 3)^2 + (y + 5)^2 = 15$ **B** $x^2 + (y + 4)^2 = 9$

C $(x + 1)^2 + (y + 7)^2 = \frac{3}{4}$ **D** $(x + 1)^2 = 13 - y^2$

Learn

General form of the equation of a circle

You know that the equation of the circle whose centre (d , h), and the length of its radius equals r units:

is : $(x - d)^2 + (y - h)^2 = r^2$ **by simpliying the expression**

$\therefore x^2 + y^2 - 2dx - 2hy + d^2 + h^2 - r^2 = \text{zero}$ (1)

$\because$ d , h , r constant $\therefore$ The expression $d^2 + h^2 - r^2 = C$ where C is a constant vlue By butting $L = -d$, $k = -h$, $C = d^2 + h^2 - r^2$

Then the equation will be in the form $x^2 + y^2 + 2Lx + 2ky + C = 0$

and it's called the general form of the equation of a circle whose centre (-L , -k), and the length of its radius equals r, where

$r = \sqrt{L^2 + k^2 - C}$, $L^2 + k^2 - C > 0$

Example

6 Find the general form of the equation of the circle whose centre (6 , -3) and the length of its radius equals 5 units.

Solution

$\because$ the centre of the circle in the general form of the equation of a circle is (-L , -K)

, the centre of the circle is (6 , -3) **given**

$\therefore L = -6$, $k = 3$

$\because r = 5$, $C = L^2 + k^2 - r^2$

$\therefore C = (-6)^2 + (3)^2 - (5)^2 = 20$

The general form of the equation of the circle is: $x^2 + y^2 - 12x + 6y + 20 = 0$.

The validity of the solution can be verified using the equation of the circle: $(x - 6)^2 + (y + 3)^2 = 25$ then simplify and compare it to the results.

Try to solve

7) Write the general form of the equation of the circle if:

A) its centre is the point M (-2 , 5), and the length of its radius equals $\sqrt{57}$ units.

B) its centre is the point N (5 , -3), and passes through the point B (2 , 1).

Example

7) Write the general form of the equation of the circle if the two points A(4 , 2) , B(-1 , -3) are the end points of its diameter.

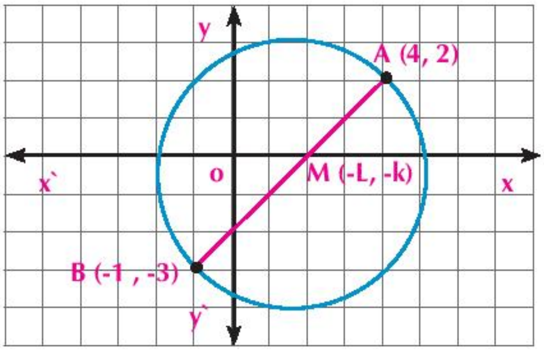
Solution

Let the point M (-L , -k) be the center of the circle whose diameter is $\overline{AB}$

$\therefore$ M is the midpoint of $\overline{AB}$, and the coordinates of the point M is $(\frac{4 - 1}{2} , \frac{2 - 3}{2})$

$\therefore -L = \frac{3}{2}$ $\qquad L = \frac{-3}{2}$

$-k = \frac{-1}{2}$ $\qquad k = \frac{1}{2}$



Substituting by L, k in the general form of the equation of the circle:

$x^2 + y^2 + 2Lx + 2ky + C = 0$

$\therefore x^2 + y^2 - 3x + y + C = 0$ **(1)**

$\because$ The circle passes through the point A (4 , 2), so it verify its equation

$\therefore (4)^2 + (2)^2 - 3(4) + 2 + C = 0 \quad \therefore C = -10$

By substituting in the equation **(1)**

$\therefore$ The general form of the equation of the circle is: $x^2 + y^2 - 3x + y - 10 = 0$

Try to solve

8) If the points A (3 ,-2), B (3 , 8), C(-1 , 0) belong to the same circle. then prove that $\overline{AB}$ is a diameter in it, then write the general form of its equation.

Important note

From the general form of the equation of the circle : $x^2 + y^2 + 2Lx + 2Ky + C = 0$

We conclude that:

First: The equation is of the 2nd degree in x , y

Second: coefficient of x^2 = coefficient of y^2 = unity.

Third: Free form the term xy which means coefficient of xy = 0,

The second degree equation in x, y represents a circle, if the previous three conditions are verified, and $L^2 + K^2 - C > 0$.

Learn

Determine the coordinates of the centre of a circle and its radius

To determine the coordinates of the centre of the circle and the length of its radius from the general form of its equation:

1- Verify firstly to put the equation in the general form where:
the coefficient of x^2 = coefficient y^2 = unity

2- The coordinates of the centre is (-L , -k) $\therefore \left(\frac{-\text{coefficient } x}{2}, \frac{-\text{coefficient } y}{2}\right)$

3- The length of the radius of the circle equal r **where** $r = \sqrt{L^2 + k^2 - C}$, $L^2 + k^2 - C > 0$

Example

8 Which of the following equations represent a circle? And if it is a circle, find its centre and the length of its radius

A $3x^2 + 2y^2 + 6x - 8y - 10 = 0$
B $x^2 + y^2 + 4x + 25 = 0$
C $2x^2 + 2y^2 - 12x + 8y - 30 = 0$
D $4x^2 + 4y^2 = 49$
E $x^2 + y^2 + 2xy + 3 = 0$

Solution

A The coefficient of $x^2 \neq$ the coefficient of y^2 $\therefore$ the equation is not a circle

B The coefficient of x^2 = the coefficient of y^2 = unity, the equation is free of the term containing x y

$L = \frac{4}{2} = 2$, $k = \frac{0}{2} = 0$, $C = 25$

$\therefore L^2 + k^2 - C = (2)^2 + (0)^2 - 25 < 0$

$\therefore$ the equation is not a circle

C Divide both sides by 2 $\therefore x^2 + y^2 - 6x + 4y - 15 = 0$

$\therefore$ The coefficient of x^2 = the coefficient of y^2 = unity, the equation is free of the term containing xy

$L = -3$, $k = 2$, $C = -15$

$\therefore L^2 + k^2 - C = (-3)^2 + (2)^2 - (-15) = 28 > 0$

$\therefore$ The equation is for a circle its centre is (3 , -2) , $r = \sqrt{28} = 2\sqrt{7}$ units

D Divide both sides by 4 $\therefore x^2 + y^2 = \frac{49}{4}$

$\therefore$ The coefficient of x^2 = the coefficient of y^2 = unity , the equation is free of the term containing xy

$L = 0$, $k = 0$, $C = \frac{49}{4}$ $\therefore L^2 + k^2 - C = \frac{49}{4} > 0$

$\therefore$ The equation is a circle, its centre the origin point , $r = \sqrt{\frac{49}{4}} = \frac{7}{2}$ unit

E The equation contains the term xy $\therefore$ The equation is not a circle

Try to solve

9 Which of the following equations represent a circle? And if it is for a circle, find its centre and the length of its radius.

A $x^2 + y^2 - 6x + 4y + 17 = 0$
B $x^2 + y^2 + 4x - 2y = 0$
C $2x^2 + 2y^2 - 4x + 39 = 0$
D $x^2 + y^2 - 2xy - 6 = 0$

Critical thinking: Are the two circles $C_1 : x^2 + y^2 - 10x - 8y + 16 = 0$

$C_2 : x^2 + y^2 + 14x + 10y - 26 = 0$ touch externally? Explain your answer.

Exercises (2 - 5)

Choose the correct answer from those given:

1. The point (2 , 0) lies on the:
   - A) x - axis
   - B) y - axis
   - C) straight line $y = 2x$
   - D) circle $x^2 + y^2 = 9$

2. If A(3 , -7) , B(-3 , 5) , then the coordinates of the midpoint of $\overline{AB}$ is ....
   - A) (0 , 1)
   - B) (1 , 0)
   - C) (0 , -1)
   - D) (-1 , 0)

3. The distance between the two points (2 , 4) , (10 , -2) equal .....
   - A) 9
   - B) 10
   - C) $3\sqrt{10}$
   - D) 6

4. The circle $x^2 + y^2 = 25$ its centre (0 , 0) and passes through the point ....
   - A) (1 , 4)
   - B) (5 , 0)
   - C) (25 , 0)
   - D) (5 , 1)

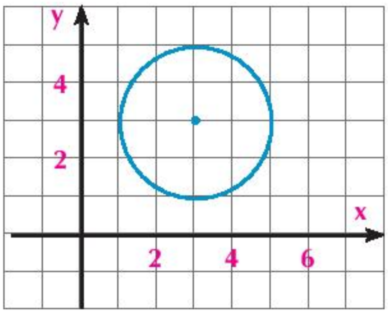
5. The equation of a circle whose centre (3 , -5) and the length of its radius equal 7 units is:
   - A) $(x - 3)^2 + (y - 5)^2 = 49$
   - B) $(x + 3)^2 + (y + 5)^2 = 49$
   - C) $(x + 3)^2 + (y - 5)^2 = 49$
   - D) $(x - 3)^2 + (y + 5)^2 = 49$

6. The circumference of the circle whose equation $x^2 + y^2 = 8$ equals:
   - A) 8π
   - B) 64π
   - C) $2\sqrt{2}\pi$
   - D) $4\sqrt{2}\pi$

7. Write the equation of a circle whose centre M, and the length of its radius r where:
   - A) M (2, 3) , $r = 5$
   - B) M(0 , 0) , $r = 4$
   - C) M (3, 0) , $r = 6$
   - D) M(4 , -5) , $r = \sqrt{7}$
   - E) M (0, -1) , $r = 2\sqrt{3}$
   - F) M(-4 , -3) , $r = \frac{3}{2}$

8 Write the equation of a circle represented by the given figure:

A



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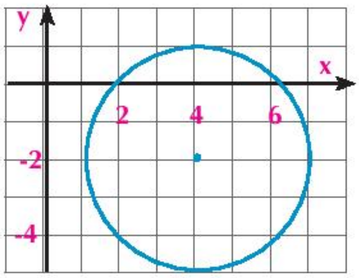

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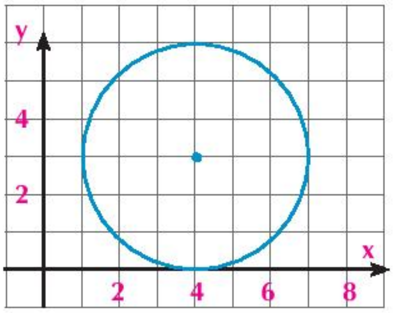

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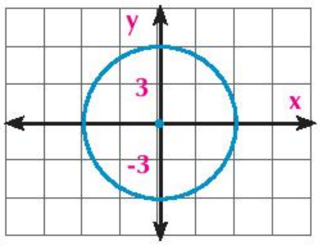

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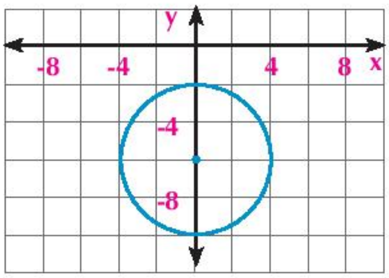

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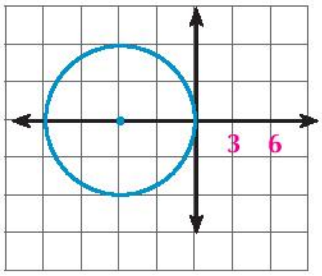

9 Find the equation of a circle if:

A Its centre M (7 , -5), and it passes through the point A(3 , 2).

B $\overline{AB}$ is a diameter in the circle, where A(6 , -4) , B(0 , 2).

C Its centre is the point (5 , -3) and touches the x-axis

10 **Find the centre, and the length of the radius for each of the following circles:**

A $x^2 + y^2 = 27$

B $(x + 3)^2 + (y - 5)^2 = 49$

C $(x - 2)^2 + y^2 = 16$

D $x^2 + (y + 7)^2 = 24$

11 **Write the general form of the equation of a circle in the following cases:**

A Its centre M(3 , 1) ,and the length of its diameter equal 8.

B Its centre M (0 , 0), and it passes through the point A (-1 , 3)

C Its centre M (-5 , 0), and it passes through the point B (3 , 4)

D $\overline{AB}$ is a diameter in it, where A(3 , -7) , B(5 , 1)

12 **Find the centre, and the length of the radius for each of the following circles:**

A $x^2 + y^2 - 4x + 6y - 12 = 0$ B $x^2 + y^2 + 2x = 8$

C $x^2 + y^2 - 6x + 10y = 0$ D $x^2 + y^2 - 8x = 12$

13 **Show which of the following circles are congruent:**

A $x^2 + y^2 - 2x + 4y - 3 = 0$, $x^2 + y^2 + 6x - 11 = 0$

B $x^2 + y^2 - 14x + 37 = 0$, $x^2 + y^2 + 10x + 13 = 0$

14 **Show which of the following equation is for a circle, then find its centre and the length of its radius:**

A $x^2 + y^2 + 8x - 16y - 1 = 0$ B $x^2 + 2y^2 + 6x - 5y = 0$

C $\frac{1}{4}x^2 + \frac{1}{4}y^2 + x - 8 = 0$ D $x^2 + y^2 + 2xy - 12 = 0$

E $x^2 + y^2 - 2x + 4y + 7 = 0$ F $2x^2 + 2y^2 + 3y - 8 = 0$

15 **Find equation of a circle which passes through the two points A(1, 3) , B (2, -4) and its center lies on x-axis.:**