Math.

Third Sec.

Final revision

Dynamic 2022

Concept (1) The displacement $\vec{S}$

The displacement of a body $\vec{S}$ is the change of its position vector

$$\vec{S} = \Delta\vec{x} = \vec{x} - \vec{x}_0$$

Concept (2)The instantaneous velocity

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{\Delta t} = \frac{d\vec{x}}{dt} = \frac{d\vec{S}}{dt}$$

Concept (3) The instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{V}(t + \Delta t) - \vec{V}(t)}{\Delta t} = \frac{d\vec{V}}{dt} = \frac{d^2 \vec{x}}{dt^2}$$

Concept (4) If $V = f(x)$, $x = g(t)$ **then**

$$\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} \qquad \therefore a = V \cdot \frac{dv}{dx}$$

Example

A particle moves in a straight line such that its velocity (V) is given by the relation $V^2 + x^2 = 25$, where x is the position of the particle , a is the acceleration then …………

$(a)\ V^2 + a^2 = 25$ $(b)\ a^2 + x^2 = 25$ $(c)\ V^2 + a^2 = 0$ $(d)\ a^2 + x^2 = 0$

$V^2 + x^2 = 25$ **by derivative with respect to** x

$2V\frac{dV}{dx} + 2x = 0$ $\qquad 2a + 2x = 0$ $\qquad \therefore a = -x$

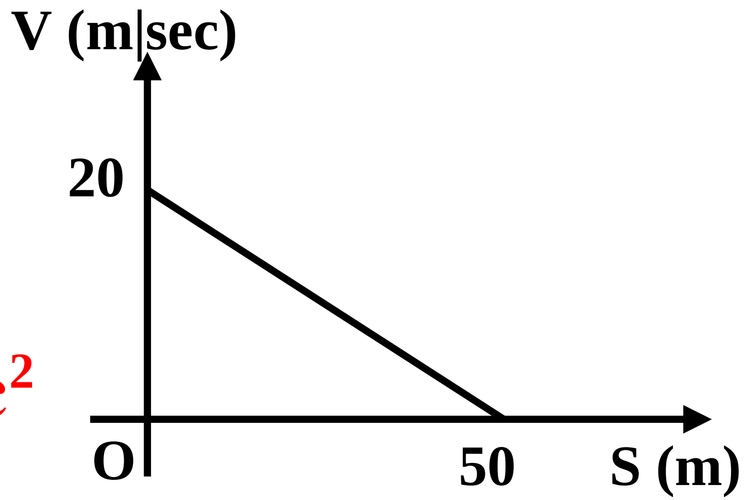
In the given equation $V^2 + a^2 = 25$ $\qquad Ans.(a)$

Example

The opposite figure represents the (velocity – displacement) graph for a body moves in a straight line , then the acceleration

(a) when the displacement vanishes = ….. m| sec^2

$(a) -8$ $(b) 8$ $(c) -20$ $(d) 20$



The slope $\frac{-20}{50}=\frac{-2}{5}$ and y – intercept = 20 units

The equation of the st. line $V = \frac{-2}{5}S + 20$ $\quad \therefore a=\frac{dV}{dt}=\frac{-2}{5}\times\frac{dS}{dt}$

$\therefore a=\frac{dV}{dt}=\frac{-2}{5}\times V$ when the displacement vanished V = 20

$\therefore a=\frac{-2}{5}\times 20=-8$ $\quad Ans.(a)$

Concept (5) $a = \frac{dv}{dt}$ $\therefore \int a\,dt = \int dv$ $\therefore V = \int a\,dt$

And $\int_{v_0}^{v} dv = \int_{0}^{t} a\,dt$

Concept (6) $V = \frac{dx}{dt}$ $\therefore \int V\,dt = \int dx$ $\therefore x = \int V\,dt$

And $\int_{x_0}^{x} dx = \int_{0}^{t} V\,dt$

Concept (7) $a = V\dfrac{dV}{dx}$ $\therefore \int a\,dx = \int V\,dv$ $\therefore \int_{V_0}^{V} V\,dV = \int_{x_0}^{x} a\,dx$

Example

A particle moves in a straight line with acceleration a m/sec^2. Is given as a function in velocity V m / sec as $a = 2V\sqrt{V}$. If the particle started motion from the origin point with velocity 4 m/sec . Then the velocity V at the position $x = 3$ m equals ………… m /sec

$(a)\,9$ $(b)\,4$ $(c)\,25$ $(d)\,16$

$\because a = 2V\sqrt{V}$

$V\dfrac{dV}{dx} = 2V\sqrt{V}$ $\qquad \therefore dV = 2V^{\frac{1}{2}}dx$

$\int V^{-\frac{1}{2}}dV = \int 2dx$ $\qquad \therefore 2\sqrt{V} = 2x + c$

$x_0 = 0 \,,\ V_0 = 4$ $\qquad \therefore 2\sqrt{4} = 0 + c$ $\qquad c = 4$

$\therefore 2\sqrt{V} = 2x + 4$ $\qquad$ at $x = 3$

$\therefore 2\sqrt{V} = 6 + 4$ $\qquad \therefore \sqrt{V} = 5$ $\qquad \therefore V = 25$ $\qquad Ans.(c)$

Concept (8) The momentum $H = mV$

The change in moment= $\Delta H = m(V_2 - V_1)$

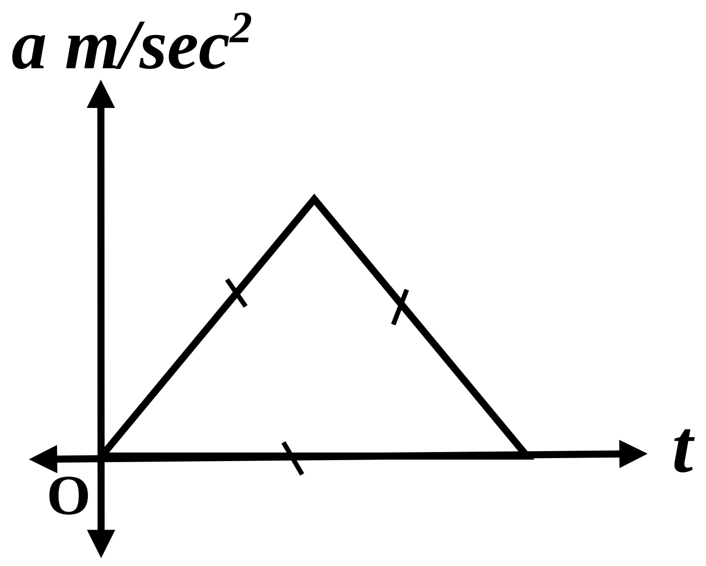
When the acceleration function in time $\Delta H = m\int_{t_1}^{t_2} a\,dt$

Example

The opposite figure represents the (acceleration – time) graph for a body of mass 8 kg , starts its motion from rest .

If the change of its momentum during the time interval $[0, t]$ equals $72\sqrt{3}\ kg.m/sec$.

Then t = ………… sec.



$(a)\ 2$ $(b)\ 4$ $(c)\ 6$ $(d)\ 8$

The change in momentum = $\Delta H = m\int_{t_1}^{t_2} a\,dt$ **= area under the acceleration – time graph**

$$72\sqrt{3} = 8\times\frac{1}{2}\times t^2\sin 60^0$$

$$\therefore 2\sqrt{3}t^2 = 72\sqrt{3}$$

$$\therefore t^2 = 36$$

$$\therefore t = 6\sec$$

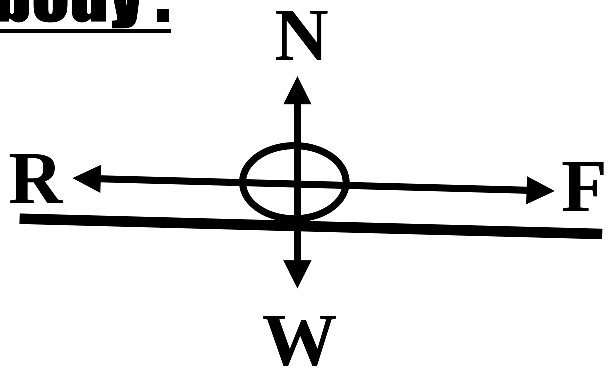
$$Ans.(c)$$

Concept (9) Newton's first law Uniform motion of a body .

(1) uniform motion on a horizontal plane under the action of a horizontal force .

R the resistance of the plane . R = F

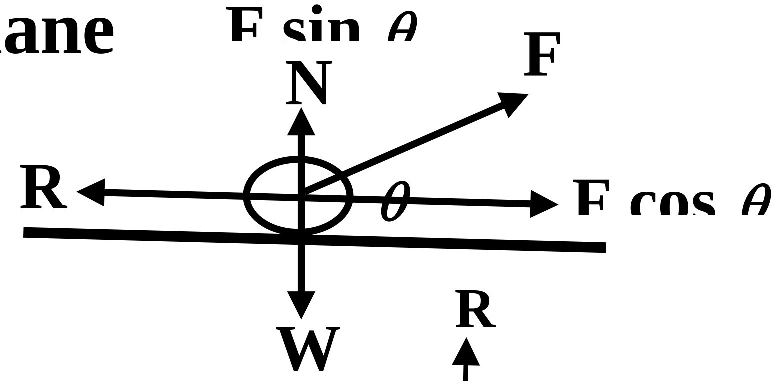
Also N = w



(2) Uniform motion of a body on a horizontal plane under the action of an inclined force .

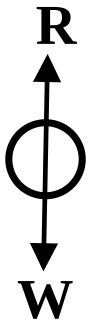
$$F\cos\theta = R$$

$$N + F\sin\theta = W$$



(3) Vertical uniform motion

R = w R the resistance



Example

📖**An engine of mass 30 ton and force 51 ton.wt . pulls a number of train cars each of mass 10 ton , ascends a slope inclined at an angle 30^0 to the horizontal , with a uniform velocity . If the resistance of the motion of the engine and the cars is 10 kg.wt .per each ton of the mass . Then the number of the cars = ………………… car**

(a) 8 **(b) 5** **(c) 9** **(d) 7**

The motion is uniform

$$F = W\sin\theta + R$$

Let the number of cars is x

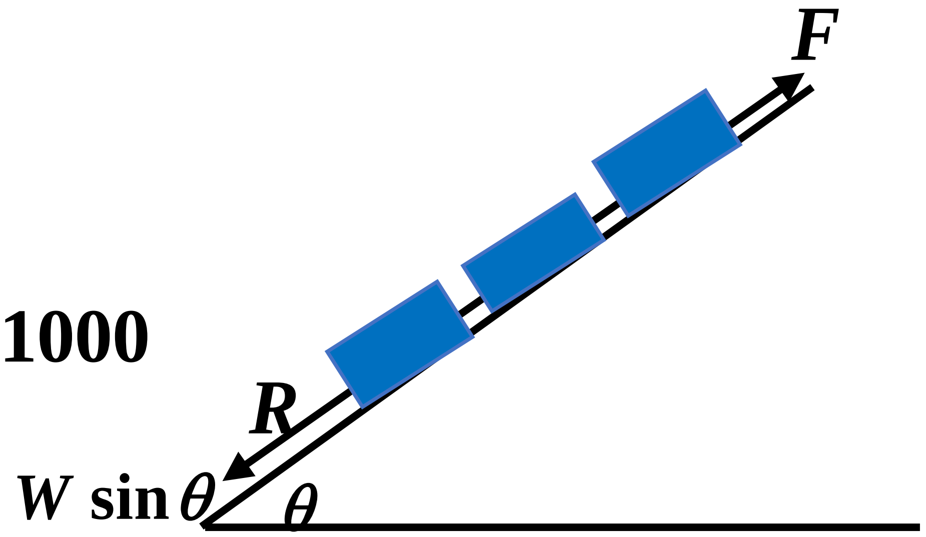
$$W = (30 + 10x) \times 1000 \quad , \quad R = (30 + 10x) \times 10$$

$$(30 + 10x) \times 1000 \times \frac{1}{2} + (30 + 10x) \times 10 = 51 \times 1000$$

$$500 \times (30 + 10x) + 10 \times (30 + 10x) = 51000$$

$$510 \times (30 + 10x) = 51000$$

$$(30 + 10x) = 100 \qquad \therefore \; 10x = 70 \qquad \therefore \; x = 7 \qquad Ans.(d)$$



Concept (10) **Newton's second law**

$$\frac{d}{dt}(\vec{H}) = \vec{F} \qquad \therefore \frac{d}{dt}(m\vec{V}) = \vec{F}$$

There are two cases

(i) When m is constant $\therefore \frac{d}{dt}(m\vec{V}) = m\frac{d}{dt}(\vec{V}) = m\vec{a} \qquad \therefore \vec{F} = m\vec{a}$

(ii) When m is not constant $\therefore \frac{d}{dt}(m\vec{V}) = m\frac{d\vec{V}}{dt} + \vec{V}\frac{dm}{dt}$

Remarks

(1)When the force is a function of time , and the mass is constant then $a = \frac{dV}{dt}$

$\because F = ma \qquad \therefore F = m\frac{dV}{dt} \qquad \therefore \int_{t_1}^{t_2} F dt = m\int_{V_1}^{V_2} dV$

(2)If the force is a function in displacement (s) , and the mass is constant then $a = V\frac{dV}{dS}$

$\because F = ma \qquad \therefore F = mV\frac{dV}{dS} \qquad \therefore \int_{s_1}^{s_2} F dS = m\int_{V_1}^{V_2} V dV$

Example

📖A force $F = 3t + 1$ measured in Newton acts upon a boy at rest of mass 4 kg , starting its motion at the origin point (O) on a straight line then the magnitude of the velocity V when t = 2 sec.is …

(a) 4 (b) 2 (c) 3 (d) 5

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$$\because F = ma \qquad \therefore F = m\frac{dV}{dt} \qquad \therefore m\int_0^v dV = \int_0^2 F\,dt$$

$$4\int_0^v dV = \int_0^2 F\,dt \qquad \therefore 4V = \int_0^2 (3t + 1)\,dt$$

$$\therefore 4V = 8 \qquad \therefore V = 2 \qquad Ans.(b)$$

Example

A body of mass m kg is moving vertically downwards with an acceleration 1 m|sec^2 , under the action of a force acting vertically upward of magnitude 10 kg.wt , and against resistance of magnitude 10 Newton . Then m = …… kg

$(a)\ \frac{135}{11}$ $(b)\ \frac{220}{27}$ $(c)\ \frac{245}{22}$ $(d)\ 10$

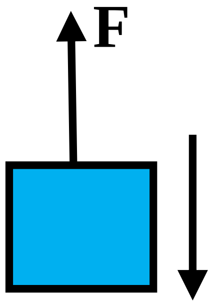
$$mg - F - R = ma$$

$$9.8m - 10\times 9.8 - 10 = m$$

$$8.8m = 108$$

$$\therefore m = \frac{135}{11}\ kg$$

$Ans.(a)$



Example

A body of mass $m = (2t + 1)$ moves a long a straight line parallel to the unit vector $\hat{C}$ and its displacement vector is $\hat{S} = (\frac{1}{2}t^2 + t)\hat{C}$.

Then the magnitude of the force acting at t =2 sec. is …….. Units

(a) 5 (b) 20 (c) 11 (d) 10

$$\vec{V} = \frac{d\vec{S}}{dt} = (t+1)\hat{c}$$

$$\vec{H} = m\vec{V} = (2t+1)(t+1)\hat{c}$$

$$\vec{H} = (2t^2 + 3t + 1)\hat{c}$$

The mass is variable then $\vec{F} = \frac{d\vec{H}}{dt} = (4t + 3)\hat{c}$

at t = 2 sec $\vec{F} = 11\hat{c}$ **F = 11 units** **Ans.(c)**

Concept (11) Impulse : -

$\vec{I} = \vec{F}\, t$ $\qquad I = F\, t$ $\qquad I = m(V' - V) = F\, t$

$$I = \int_{t_1}^{t_2} F\,dt = m(V_2 - V_1)$$

Example

A particle moves in a straight line , its momentum changed by the rate 2t kg.m|sec^2 where t is the time in sec. Then the magnitude of the impulse of the force acting on the particle during the tenth sec. equals …. Newton . sec.

(*a*) 19 **(*b*) 17** **(*c*) 20** **(*d*) 21**

$$\because F = \frac{d}{dt}(H) \qquad \because \frac{dH}{dt} = 2t \qquad \therefore F = 2t$$

$$\because I = \int_{9}^{10} F\,dt \qquad \because I = t^2\Big|_{9}^{10} = 19 \qquad Ans.(a)$$

Concept (12) Collision

If two bodies are collide then

The impulse of the first body on the second = The impulse of the second on the first.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

In the elastic collision the kinetic energy does not change before and after collision

Example

A body of mass 4 kg falls from height 19.6 m above the ground collides with another body of mass 3 kg at rest on the ground and form one body impinges on the ground a distance S cm . Then the value of S given that the resistance of the ground is 2247 kg.wt is ….

$(a)\ 2cm$ $(b)\ 20cm$ $(c)\ 20m$ $(d)\ 2m$

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$m_1 = 4$ kg $\qquad S = 19.6$ m

$V^2 = V_0^2 + 2gS \qquad , V^2 = 0 + 2 \times 9.8 \times 19.6 \qquad \therefore V = 19.6\, m/\sec$

$m_1 = 4$ kg $\quad , V_1 = 19.6 \quad , \quad m_2 = 3$kg $\quad , V_2 = 0$

$$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V$$

$4 \times 19.6 + 0 = 7V \qquad \therefore V = 11.2\, m | \sec$

R = 2247 kg.wt

mg – R = ma **7 × 9.8 – 2247 × 9.8 = 7 a**

a = – 3.136 m / sec^2

V = 0 , V_o = 11.2

$V^2 = V_0^2 + 2aS$ $\backslash\ 0 = (11.2)^2 - 2\,´\,3.136\,´\,S$

$\backslash\ S = 0.02m = 2cm$ $Ans.(a)$

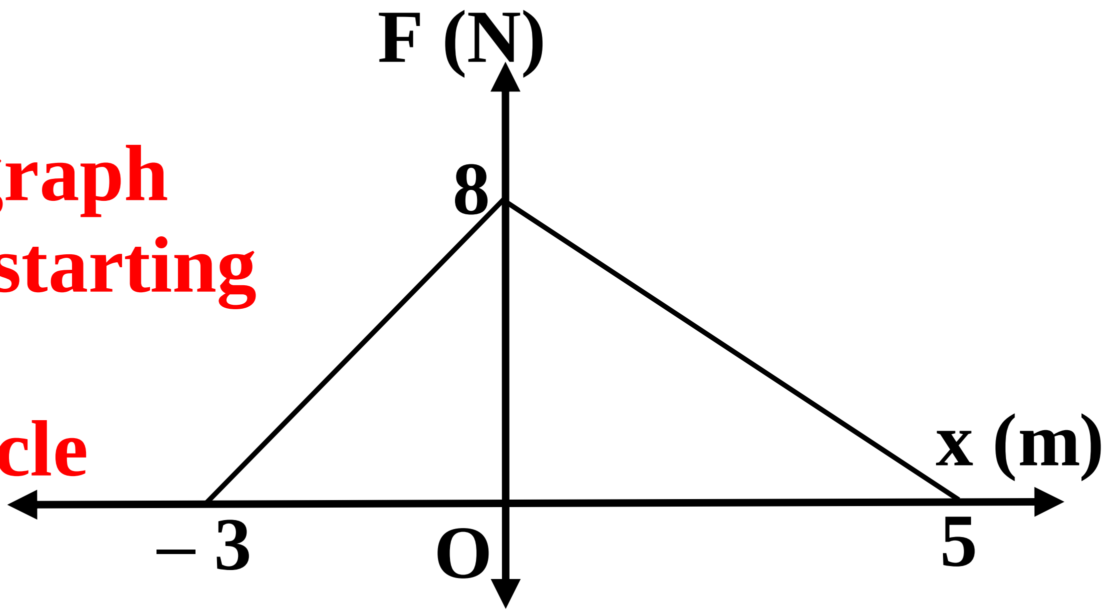
Concept (13) The work $W = \vec{F} \cdot \vec{S}$

$$W = \vec{F} \cdot \vec{S} = \left\|\vec{F}\right\| \times \left\|\vec{S}\right\| \cos\theta$$

The work done by a variable force $W = \int_A^B F \, dS$

Example

The opposite figure :-
represents the (force – position) graph of a body moves in a straight line starting from the position x = – 3 .
Then the position (x) of the particle when the work done by the force = – 148 Joule is x = ……….m



$(a)\,20$ $(b)\,13$ $(c)\,17$ $(d)\,10$

Let the position of the particle at x

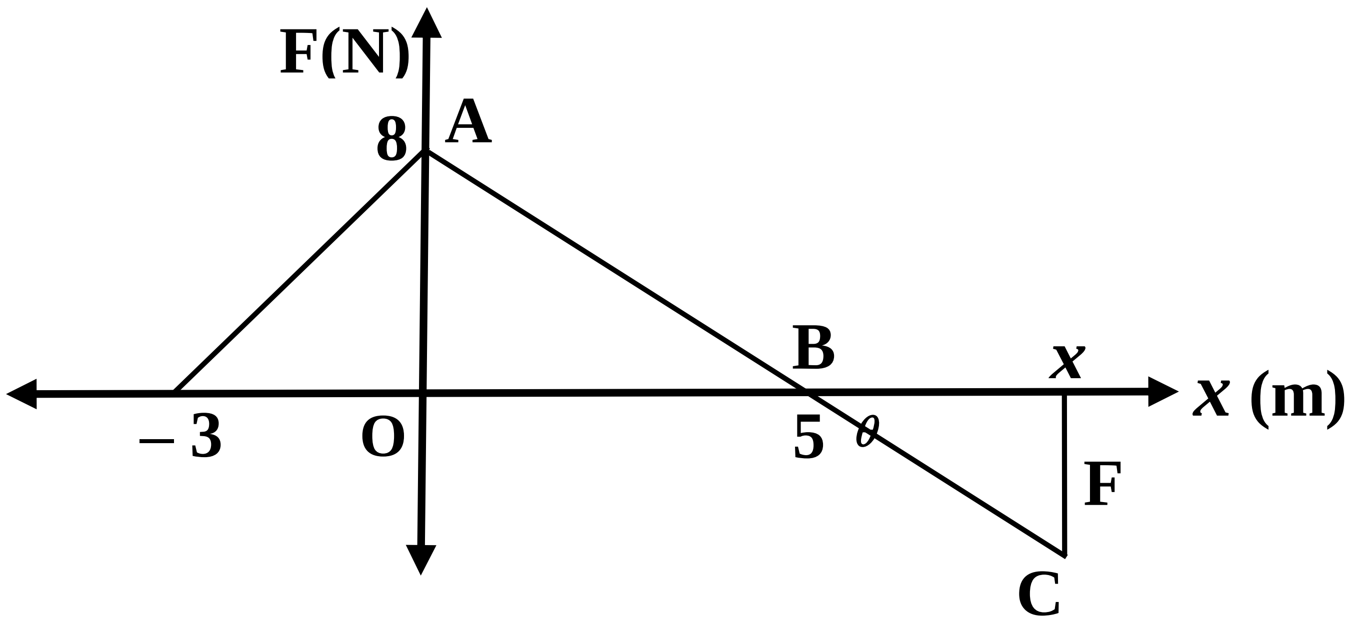
Find the equation of $\overleftrightarrow{AB}$

The slope = $\frac{-8}{5}$

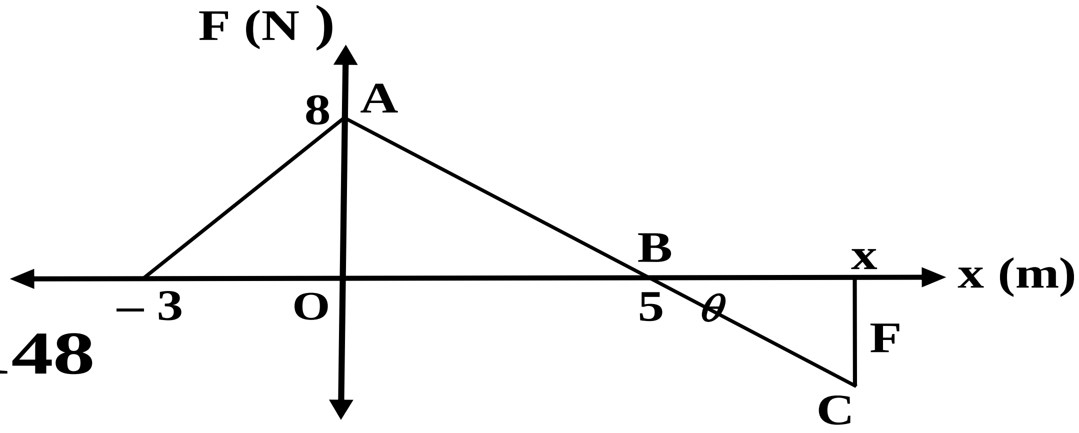
and y – intercept = 8

The equation $F = -\frac{8}{5}x + 8$

$F = \frac{-8}{5}(x - 5)$



The force is a function in (x)



Work $\int_{x_1}^{x_2} F dx$

$$W = \int_{-3}^{5} F dx + \int_{5}^{x} F dx = -148$$

$$W = \frac{1}{2} \times 8 \times 8 + \frac{1}{2}(x-5)(\frac{-8}{5}(x-5)) = -148$$

$$32 - \frac{4}{5}(x-5)^2 = -148 \qquad \therefore (x-5)^2 = 225$$

$$\therefore (x-5) = -15 \qquad \therefore x = -10 \ ref.$$

$$\therefore (x-5) = 15 \qquad \therefore x = 20 \qquad Ans.(a)$$

Concept (14) **The kinetic Energy**

The kinetic energy of a body denoted by T is defined as the product of half the mass of the particle times the square of the magnitude of its velocity.

$$T = \frac{1}{2} m V^2$$

Concept (15) <u>Principle of work and energy</u> .

The change in kinetic energy of a particle when it is moving from an initial position to a final position is equal to the work done by the force acting on it during the displacement between these two positions

$$T - T_0 = W .$$

Concept (16) **Potential Energy**

The potential energy of a partial at a certain instant denoted by P is defined as the work done by acting forces on the body if it moved it from its position at this instant to a fixed position on the straight line on which motion occurs.

Rule (1)

The change in the potential energy of a particle is equal to negative the work done by the force during the motion

$$P - P_0 = -W$$

Rule (2)

The sum of kinetic and potential energies is constant during motion

$$T + P = T_0 + P_0$$

Concept (17) <u>The power</u>

The power : - is the time rate of doing work or

The power : - is the work done in unit time power = $\frac{dw}{dt}$

The power = $\frac{dw}{dt} = \frac{d}{dt}(FS) = F\frac{dS}{dt} = F \times V$

The average power = $\frac{Work}{time}$

The work $\int_{t_1}^{t_2} Power \; dt$

Example

A force $\vec{F}$ of magnitude 7.5 kg.wt acts upon a body and moves it in a straight line , if the velocity of the body at a certain moment is 36 km|hr , then the power resulted from the force at the moment cannot equal to …………

$(a)\ 80\, kg.wt.m|\sec$ $(b)\ 735\, watt$

$(c)\ 50\, kg.wt.m|\sec$ $(d)\ 700\, watt$

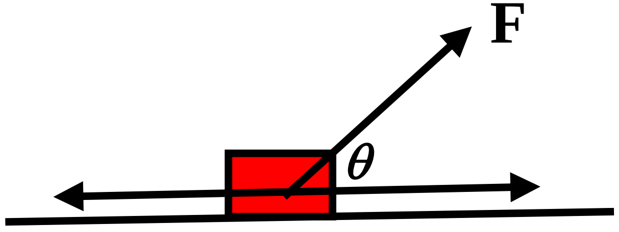
The power $= F \times V = 7.5 \times 9.8 \times 36 \times \frac{5}{18} = 735\, watt$

The power $\in [0\ ,\ 735]$ $\therefore 80 \times 9.8 = 784 \notin [0\ ,\ 735]$

The power resulted cannot equal 80 kg.wt m|sec (a)

Another solution

The power = $\vec{F} \cdot \vec{V} = F \times V \cos\theta$



The power = $7.5 \times 9.8 \times 36 \times \frac{5}{18} \cos\theta$

The power = $735 \cos\theta$ *watt* $\because -1 \le \cos\theta \le 1$

The power $\in [0 , 735]$ $\therefore 80 \times 9.8 = 784 \notin [0 , 735]$

The power resulted cannot equal 80 kg.wt m|sec (a)

Example

A particle moves in a straight line where its velocity (V) is given as a function in position (x) by the relation $V^2 = \ln x \quad : x > 1$, if (a) is the acceleration of the motion , then ………

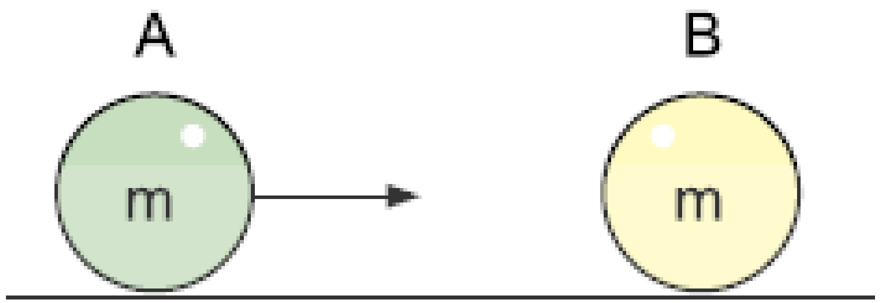
$(a)\ 2ax = 1$ $(b)\ ax^2 = 1$ $(c)\ ax = 2$ $(d)\ ax^2 = 2$

$$V^2 = \ln x \qquad 2V \times \frac{dV}{dt} = \frac{1}{x} \times \frac{dx}{dt}$$

$$2V \times a = \frac{1}{x} \times V \qquad \therefore 2ax = 1 \qquad Ans.(a)$$

Example

A , B are two smooth balls and the mass of each is m kg , the ball A is moving in a straight line on a smooth horizontal plane with velocity 8 m/ sec. If the ball A collided with the rested ball B with an elastic collision, then the velocity of the ball A after collision directly = …..



(a) Zero

(b) 8 m/sec in the opposite direction

(c) 4 m/ sec in the opposite direction

(d) 4 m /sec in the same direction

The collision is elastic then there is no shortage in the kinetic energy

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$$m \times 8 + zero = mV_1' + mV_2' \qquad \therefore V_1' + V_2' = 8 \qquad (1)$$

$$\frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2 = \frac{1}{2}m(V_1')^2 + \frac{1}{2}m(V_2')^2$$

$$64 + zero = (V_1')^2 + (V_2')^2 \qquad \therefore (V_1')^2 + (V_2')^2 = 64 \qquad (2)$$

Solve (1) , (2)

$$\therefore (V_1')^2 + (8 - V_1')^2 = 64 \qquad \therefore (V_1')^2 + 64 - 16V_1' + (V_1')^2 = 64$$

$$\therefore 2(V_1')^2 - 16V_1' = 0 \qquad \therefore V_1'(V_1' - 8) = 0$$

$$\therefore V_1' = 8 \ \ ref \qquad \therefore V_1' = zero \qquad Ans.(a)$$

الأستاذ / سامي فاضل   كبير معلمي الرياضيات

مع تمنياتنا بالتوفيق

الإدارة العامة للتعليم الإلكتروني