Math.

Third Sec.

General revision 6
Algebra and geometry

General revision 6 Algebra and geometry

(1)The scalar quantities :- The quantities which determined by the magnitude only like (the time , the mass , the speed , ..)

(2) The vector quantities :- The quantities which determined by the magnitude and the direction like (the velocity , the acceleration , the force , …………)

(3) The vector : - represented by a directed line segment

(4) Position vector in the space

The position vector of the point $A=(A_x ,A_y ,A_z)$

with respect to the origin point $O =(0,0,0)$

is the directed segment whose its initial point is O and the terminal point is A

$\vec{A}=(A_x ,A_y ,A_z)$ where

A_x is the component of $\vec{A}$ in the direction of the X – axis

A_y is the component of in the direction of the Y – axis

A_z is the component of $\vec{A}$ in the direction of the Z – axis

(5) The norm of the vector : -

is the length of the directed segment which represents the vector

If $\vec{A} = (A_x, A_y, A_z)$

Then

$$\|\vec{A}\| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

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For example :-

If $\vec{A}=(2,-1,3)$, $\vec{B}=(0,4,-3)$ **then**

(a)The component of $\vec{A}$ **in the direction of the X – axis is 2**

(b) The component of $\vec{B}$ **in the direction of the Z – axis is -3**

(c) The vector $\vec{B}$ **lies in the plane YZ where the component of in the direction of the X – axis vanishes**

(d) $\|\vec{A}\|=\sqrt{(2)^2+(-1)^2+(3)^2}=\sqrt{14}$

$\|\vec{B}\|=\sqrt{(0)^2+(4)^2+(-3)^2}=5$

Vectors multiplication

(1) The scalar product of two vectors (Dot product)

$$\vec{A} \bullet \vec{B} = \|\vec{A}\|\|\vec{B}\| \cos\theta \qquad , \vec{A} \bullet \vec{B} = A_x B_x + A_y B_y$$

Remark

$$\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1 \quad and \qquad \hat{i} \bullet \hat{j} = \hat{j} \bullet \hat{k} = \hat{i} \bullet \hat{k} = 0$$

θ is the smaller angle between the two vectors

The scalar product of two vectors in the orthogonal coordinates system

If $\vec{A} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})$, $\vec{B} = (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

then

$$\vec{A} \bullet \vec{B} = (A_x B_x + A_y B_y + A_z B_z)$$

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Example:-

if $\vec{A} = (3, 2)$, $\vec{B} = 4\hat{i} - 2\hat{j}$

$Then\quad \vec{A} \bullet \vec{B} = ...$

a) 6

b) 8

c) 10

d) 9

Solution

$$\vec{A}\bullet\vec{B}=(A_xB_x+A_yB_y+A_zB_z)$$

$$\vec{A}\bullet\vec{B}=3\times4+2\times-2=8$$

Ans. b

Example:-

if $\vec{A} = (-1, 3, 2)$, $\vec{B} = 4\hat{i} - 2\hat{j} + 5\hat{k}$

$Then\ \vec{A} \bullet \vec{B} = \dots\dots\dots$

a) 16

b) 8

c) 0

d) 4

Solution

$$\vec{A}\bullet\vec{B}=(A_xB_x+A_yB_y+A_zB_z)$$

$$\vec{A}\bullet\vec{B}=-1\times4+3\times-2+2\times5=0$$

Ans . c

The angle between two vectors

$\because \vec{A} \bullet \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos\theta$

$\therefore \cos\theta = \dfrac{\vec{A} \bullet \vec{B}}{\|\vec{A}\| \|\vec{B}\|}$

Remarks

(1) If $\cos\theta = 1$

$\therefore \vec{A}, \vec{B}$ are parallel and in the same direction

(2) If $\cos\theta = -1$

$\therefore \vec{A}, \vec{B}$ are parallel and in the opposite direction

(3) If $\cos\theta = 0$

$\therefore \vec{A}, \vec{B}$ are perpendicular

Example:-

$$If\ \vec{A}=(2\hat{i}+3\hat{j}+4\hat{k})\ ,\vec{B}=(-3\hat{i}+5\hat{j}-\hat{k})$$

Then the measure of the angle between the two vectors = ……..

a) 80^{o}
b) 75^{o}
c) 69^{o}
d) 80.97^{o}

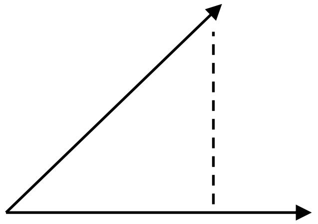
Solution

$$\because \cos\theta = \frac{\vec{A} \bullet \vec{B}}{\|\vec{A}\|\|\vec{B}\|} = \frac{-6+15-4}{\sqrt{4+9+16} \times \sqrt{9+25+1}} = \frac{5}{\sqrt{1015}} \quad ,$$

$$\theta = 80.97^0$$

Ans .d

The components (projection) of a vector in the direction of another vector



If $\vec{A}$, $\vec{B}$ are two vectors then the component of the vector in the direction of the vector $\vec{B}$ which denoted (A_B)

where

$$A_B = \|\vec{A}\| \cos\theta = \frac{\vec{A} \bullet \vec{B}}{\|\vec{B}\|}$$

The vector component of a vector in the direction of another vector

If $\vec{A}, \vec{B}$ are two vectors then the vector component of the vector $\vec{A}$

in the direction of the vector

which denoted $= \|\vec{A}\| \cos\theta \hat{U} = \dfrac{\vec{A} \bullet \vec{B}}{\|\vec{B}\|} \times \dfrac{\vec{B}}{\|\vec{B}\|}$

where $\hat{U} = \dfrac{\vec{B}}{\|\vec{B}\|}$ is the unit vector in direction of $\vec{B}$

Example :-

The algebraic component

of the force in the direction of $\vec{AB}$

where $\vec{F} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

$A = (1,4,0), B = (-1,2,3)$ *is* …………

$a)\sqrt{17}$ $\qquad$ $b)\sqrt{15}$

$c)\sqrt{19}$ $\qquad$ $d)\sqrt{13}$

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Solution

$$\vec{AB} = B - A = (-2, -2, 3)$$

The algebraic component of $\vec{F}$ in direction of $\vec{AB} = \frac{\vec{F} \bullet \vec{AB}}{\|\vec{AB}\|}$

$$\therefore \frac{\vec{F} \bullet \vec{AB}}{\|\vec{AB}\|} = \frac{(2,-3,5)\bullet(-2,-2,3)}{\sqrt{4+4+9}} = \frac{17}{\sqrt{17}} = \sqrt{17}$$

Ans. a

Example :-

The vector component of the force $\vec{F}$ in the direction of $\vec{AB}$ where

$A=(1,4,0), B=(-1,2,3)$ is ....

$a)(-2\hat{i}-2\hat{j}+3\hat{k})$

$b)(-3\hat{i}-2\hat{j}+5\hat{k})$

$c)(2\hat{i}+2\hat{j}-3\hat{k})$

$d)(3\hat{i}+2\hat{j}+5\hat{k})$

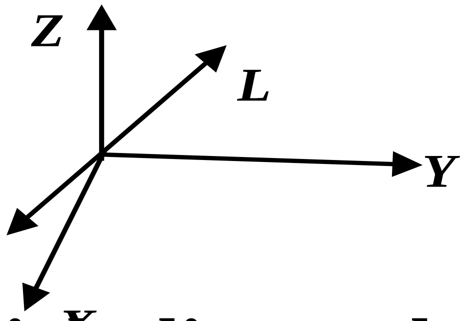
Solution

The vector component of $\vec{F}$ in direction of $\vec{AB}$ $= \dfrac{\vec{F} \bullet \vec{AB}}{\|\vec{AB}\|} \times \hat{U}$

$$\therefore \frac{\vec{F} \bullet \vec{AB}}{\|\vec{AB}\|} \times \frac{\vec{AB}}{\|\vec{AB}\|} = \sqrt{17} \times \frac{(-2,-2,3)}{\sqrt{17}} = (-2\hat{i} - 2\hat{j} + 3\hat{k})$$

Ans.a

The equation of the straight line in the space



The direction vector of the straight line in the space

If θ_x , θ_y , θ_z are the direction angles of a straight line L in the space then

$\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are the direction cosines of this straight line and denoted by (ℓ, m, n) and $\ell = \cos\theta_x$, $m = \cos\theta_y$, $n = \cos\theta_z$ then

$\ell^2 + m^2 + n^2 = 1$ and the vector $\vec{d} = \ell\hat{i} + m\hat{j} + n\hat{k}$ is the unit vector in direction of the straight line and any vector parallel to this vector is called (the direction vector of the straight line)

$\vec{d} = k(\ell\hat{i} + m\hat{j} + n\hat{k}) = (a, b, c)$ where $k \in R$

And the direction ratios of the straight line a, b, c are proportional to (ℓ, m, n)

Example

The direction cosines of the vector $\vec{A} = (-2,1,2)$ **is …………..**

$(a)(-2,1,2)$ $\quad$ $(b)\left(\frac{-2}{3},\frac{1}{3},\frac{2}{3}\right)$ $\quad$ $(c)\left(\frac{-5}{2},5,\frac{5}{2}\right)$ $\quad$ $(d)(-1,1,1)$

Solution

$$\cos\theta_x = \frac{A_x}{\|\vec{A}\|} = \frac{-2}{3}, \cos\theta_y = \frac{A_y}{\|\vec{A}\|} = \frac{1}{3}, \cos\theta_z = \frac{A_z}{\|\vec{A}\|} = \frac{2}{3}$$

The direction cosines $(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$ $\qquad$ $Ans.(b)$

Example:-

Choose the correct answer **(First session 2017)**

The direction cosines of the straight line whose direction vector is $\vec{d} = (-1, 2, 3)$ is…………..

(a) $\left(\frac{-1}{4}, \frac{1}{2}, \frac{3}{4}\right)$

(b) $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

(c) $\left(\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

(d) $\left(\frac{-1}{14}, \frac{1}{7}, \frac{3}{14}\right)$

Solution

$$\left\|\vec{d}\right\| = \sqrt{1+4+9} = \sqrt{14} \qquad , Ans.c$$

The equation of the straight line

If L is a straight line in the space whose direction vector is $\vec{d} = (a, b, c)$ and passes through the point $\vec{A} = (x_1, y_1, z_1)$ and B is a point lies on it whose direction vector is $\vec{r} = (x, y, z)$ then in ΔOAB

$\vec{OB} = \vec{OA} + \vec{AB}$ then

(a) The vector form of the equation of the straight line is

$$\vec{r} = \vec{A} + t\,\vec{d}$$

(b) The parametric equations of the straight line

From the vector equation of the straight line $\vec{r} = \vec{A} + t\,\vec{d}$

$$(x, y, z) = (x_1, y_1, z_1) + t\,(a, b, c)$$

$$\therefore x = x_1 + at, \quad y = y_1 + bt, \quad z = z_1 + ct$$

are the parametric forms

(c) The Cartesian equation of the straight line

From the parametric equations

$$\therefore \frac{x-x_1}{a} = t\ ,\ \frac{y-y_1}{b} = t\ ,\ \frac{z-z_1}{c} = t$$

$$\therefore \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Parallel lines in the space

If $\vec{d}_1 = (a_1, b_1, c_1)$, $\vec{d}_2 = (a_2, b_2, c_2)$ are the two direction vectors of two straight lines L_1, L_2 then $L_1 \parallel L_2$ if and only if $\vec{d}_1 \parallel \vec{d}_2$

Then (1) $\vec{d}_1 = k\,\vec{d}_2$ (2) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (3) $\vec{d}_1 \times \vec{d}_2 = 0$

Example

If the two straight lines $\dfrac{x+1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}, \dfrac{x}{3}=\dfrac{y+1}{4}=\dfrac{z-1}{k}$

are perpendicular then k = ………………

$(a)\ 4$ $(b)\ -4$ $(c)\ 4.5$ $(d)\ -4.5$

Solution

$\vec{d}_1 \cdot \vec{d}_2 = 0$ $\quad (2,3,4)\cdot(3,4,k) = 0$

$\therefore 6 + 12 + 4k = 0$ $\quad k = -4.5$ $\quad Ans.(d)$

Remark

(1) If the two straight lines are parallel and there is a point on one of them satisfying the equation of the other , then the two straight lines are coincident

(2) If $\vec{d}_1$ *and* $\vec{d}_2$ are not parallel then L_1 , L_2

are either intersect or skew

Example

The vector equation of the plane has normal vector $\vec{n} = (1, 1, 1)$ and contains the point (8 , 1 , 8)

$(a)\ (1, 1, 1) \bullet \vec{r} = 17$

$(b)\ \vec{r} = 17$

$(c)\ (1, 1, 1) \bullet \vec{r} = (8, 1, 8)$

$(d)\ \vec{r} = (8, 1, 8)$

Solution

$$\vec{n} \bullet \vec{r} = \vec{n} \bullet \vec{A}$$

$$\therefore (1,1,1) \bullet \vec{r} = (1,1,1) \bullet (8,1,8)$$

$$\therefore (1,1,1) \bullet \vec{r} = 17$$

$$Ans.(a)$$

Example

The equation of the straight line passing through the two points $(4,7,-3)$, $(6,11,6)$ is ……………………..

$(a)\ \frac{x-4}{2}=\frac{y-7}{4}=\frac{z+3}{9}$

$(a)\ \frac{x+3}{9}=\frac{y-7}{4}=\frac{z-4}{2}$

$(c)\ \frac{x+4}{2}=\frac{y+7}{4}=\frac{z-3}{9}$

$(d)\ \frac{x-2}{4}=\frac{y-4}{7}=\frac{z-9}{-3}$

Solution

The direction vector $\vec{d} = (2, 4, 9)$

The equation of the straight line

$$\frac{x-4}{2} = \frac{y-7}{4} = \frac{z+3}{9}$$

$Ans.\ (a)$

Example

The volume of the parallelepiped in which three adjacent sides are represented by the vectors

$\vec{A} = (2,3,-1), \vec{B} = (3,-1,5), \vec{C} = (1,0,4)$ **equals ……. Cubic units**

(a) -30 (b) 32 (c) 30 (d) 12

Solution

The volume of the parallelepiped

$$\vec{A} \bullet \vec{B} \times \vec{C} = \begin{vmatrix} 2 & 3 & -1 \\ 3 & -1 & 5 \\ 1 & 0 & 4 \end{vmatrix} = \left| 2(-4) - 3(7) - 1(1) \right| = 30\ Units \qquad (c)$$

Example

Find the plane equation passes through the point $(-1,2,1)$ and perpendicular to the straight line passing through the points $(-3,1,2),(2,3,4)$

$(a)\ 5x+2y+2z-1=0$

$(b)\ 5x+2y+2z+11=0$

$(c)\ 5x+2y+2z+1=0$

$(d)\ 5x-2y+2z-1=0$

Solution

$\because$ The straight line is perpendicular to the plane

$\therefore$ The direction vector of the straight line is the normal to the plane

$\vec{d} = (2,3,4) - (-3,1,2) = (5\ ,2\ ,2)$ $\therefore \vec{n} = (5\ ,2\ ,2)$

The equation of the plane $\vec{n} \bullet \vec{r} = \vec{n} \bullet \vec{A}$

$(5\ ,2\ ,2) \bullet (x\ ,y\ ,z\ ) = (5\ ,2\ ,2) \bullet (-1,2,1)$

$5x + 2y + 2z - 1 = 0$ $Ans.(a)$

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مع تمنياتنا بالتوفيق

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