Arab Republic of Egypt
Ministry of Education &
Technical Education
Central Administration of
Book Affairs

3

MATHEMATICS

For Preparatory Year three

Student's Book

Second Term

Authors

Mr. **Omar Fouad Gaballa**

Pref. Dr. **Afaf Abo-ElFotoh Saleh**

Dr. **Essam Wasfy Rouphaiel**

Mr. **Serafiem Elias Shander**

Mr. **Kamal Younes Kabsha**

Revised by

Mr. **Samir Mohamed Sedawy**

Consultant of Mathematics

Mr. **Gamal Elshahed**

2021 - 2022

Introduction

Dear students :

It is extremely great pleasure to introduce the mathematics book for third preparatory. We have been specially cautious to make learning mathematics enjoyable and useful since it has many practical applications in real life as well as in other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate mathematicians, roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining patterns of positive thinking which pave your way to creativity.

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration.

Our great interest here is to help you get the information independently in order to improve your self-study skills.

Calculators and computer sets are used when needed. Exercises, practices, general exams, activities, unit tests, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

Contents

Algebra

Geometry

Unit 4 : The Circle

Geometry

Unit 5 : Angles and Arcs in the circle

MATHEMATICAL NOTATION

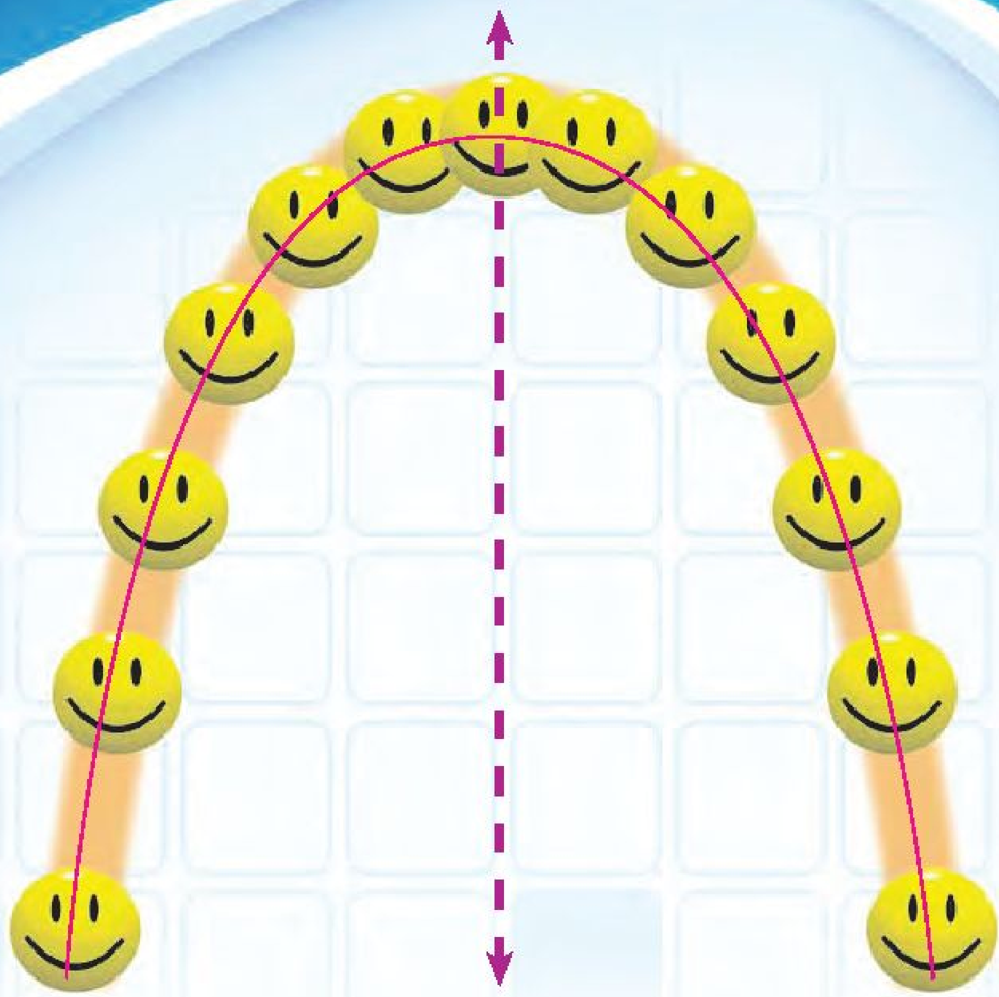
$\mathbb{N}$	The set of natural numbers	$\perp$	Perpendicular to
$\mathbb{Z}$	The set of integers	$//$	Parallel to
$\mathbb{Q}$	The set of rational numbers	$\overline{AB}$	line segment AB
$\mathbb{Q}`$	The set of irrational numbers	$\overrightarrow{AB}$	Ray AB
$\mathbb{R}$	The set of real number	$\overleftrightarrow{AB}$	Straight line AB
$\sqrt{A}$	The Square root of A	$m(\angle A)$	Measure of angle A
$\sqrt[3]{A}$	The Cube root of A	$m(\widehat{AB})$	Measure of arc AB
$[a, b]$	Closed interval	$\sim$	Similarity
$]a, b[$	Open interval	$>$	Greater than
$[a, b[$	Half-open interval	$\geqslant$	Greater than or equal to
$]a, b]$	Half-open interval	$<$	Less than
$[a, \infty[$	Infinite interval	$\leqslant$	Less than or equal to
$\equiv$	Is congruent to	$p(e)$	Probability of occurring event
$n(A)$	Number of elements A	$\overline{x}$	Mean
s	Sample space	σ	Standard deviation
		Σ	Sum

Algebra

Unit (1)

Equations

Algebraic Fractional Functions and the operations on them



One of the players threw the ball so, It took the direction shown in the figure.

This figure represents one of the functions which you will study and is called "a quadratic function".

Solving two equations of first degree in two variables graphically and algebraically

1 - 1

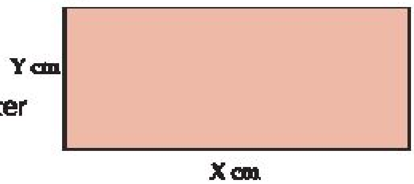
Think and Discuss

A rectangle of a perimeter 30cm. What are the possible values of its length and width. If the length of the rectangle = x cm and the width of the rectangle = y cm

then :

the length + width = $\frac{1}{2}$ the perimeter

$\therefore$ x + y = 15



- This equation is called the equation of first degree in two variables.
- Solving this equation means finding an ordered pair of the real number is satisfying equation.
- Can (-5, 20) be a solution of the previous equation. Explain your answer. Dear student: Solve this problem after the following.
- You can solve this equation by putting it in one of the two forms:

(1) y = 15 - x   or   (2) x = 15 - y

By giving one of the two variables any value, you can calculate the value of the other variable.

If x $\in$ R then the substitution set is R × R thus there are infinite number of solutions of the equation of the first degree, in which each of them is in an ordered pair. (x , y) where its first projection x and its second projection y.

when x = 8 $\therefore$ y = 15 - 8 = 7 $\therefore$ (8 , 7) is a solution of the equation
when x = 9.5 $\therefore$ y = 15 - 9.5 = 5.5 $\therefore$ (9.5 , 5.5) is a solution of the equation
when x = $4\sqrt{7}$ $\therefore$ y = $15 - 4\sqrt{7}$
$\therefore$ ($4\sqrt{7}$, $15 - 4\sqrt{7}$) is a solution of the equation

First: Solving equations of the first degree in two variables graphically :

Examples

(1) Find the solution set of the equation 2 x - y = 1

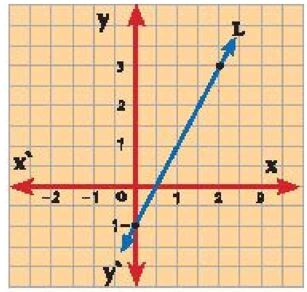
What you'll learn

- Solving two equations of first degree in two variables.

Key terms

- Equation of first degree.
- Graphical solution.
- Substitution set.
- Algebraic solution.
- Solution set.

Solution



Write the equation in the form $y = 2x - 1$

By putting $x = 0$ $\therefore$ $y = -1$ $\therefore$ (0, -1) is a solution of the equation

By putting $x = 2$ $\therefore$ $y = 3$ $\therefore$ (2, 3) is a solution of the equation

And by drawing the straight line L passing throught the two represented points of the two ordered pairs (0 , -1) ,(2 , 3).

We find that every point $\in$ L is a solution to the equation.

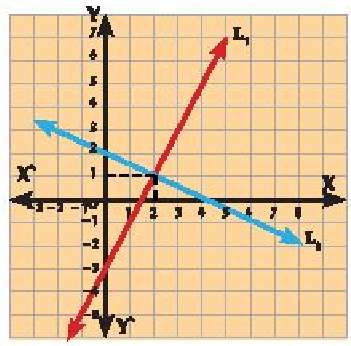
i.e for the equation $2x - y = 1$ their is an infinite number of solutions

Tell another four solutions for this equation?

2 Find the solution set of the following two equations graphically:

$L_1 : y = 2x - 3$, $L_2 : x + 2y = 4$

Solution



In the equation $y = 2x - 3$

By putting $X= 0$ $\therefore$ $y = -3$ $\therefore$ (0, -3) is a solution of this equation

By putting $X = 4$ $\therefore$ $y = 5$ $\therefore$ (4, 5) is a solution of this equation

Thus: L_1 in the opposite figure represents the solution set of this of equation (1)

By putting the equation $x + 2y = 4$ int the form $x = 4 - 2y$

By putting $y = 0$ $\therefore$ $x = 4$ $\therefore$ (4 , 0) is a solution of this equation

By putting $y = 1$ $\therefore$ $x = 2$ $\therefore$ (2 , 1) is a solution of this equation

This : L_2 in the opposite figure represents the solution set of the equation (2)

In the figure $L_1 \cap L_2$ is the point (2 , 1)

$\therefore$ The solution set of the two equations is {(2 , 1)}

Drill

Find the solution set for each pair in the following equations graphically :

1. $2x + y = 0$ $x + 2y = 3$
2. $y = 3x - 1$ $x - y + 1 = 0$

Example 3

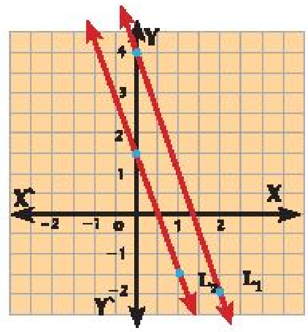
Find graphically the solution set for each pair of the following equations:

First: $3x + y = 4$ **(1)** , $2y + 6x = 3$ **(2)**

Second: $3x + 2y = 6$ **(1)** , $y = 3 - \frac{3}{2}x$ **(2)**

Solution

First:



Put the equation **(1)** in the form $y = 4 - 3x$

By Putting $x = 0$ $\therefore$ $y = 4$ thus, (0, 4) is a solution of the equation

By Putting $x = 2$ $\therefore$ $y = -2$ thus, (2, -2) is a solution of the equation

L_1 represents a solution set of the equation **(1)**

By putting the equation **(2)** in the form $y = \frac{3 - 6x}{2}$

By Putting $x = 0$ $\therefore$ $y = \frac{3}{2}$ thus, $(0, \frac{3}{2})$ is a solution of the equation

By Putting $x = 1$ $\therefore$ $y = \frac{-3}{2}$ thus, $(1, \frac{-3}{2})$ is a solution of the equation

and L_2 is a solution of the equation **(2)**

$\because$ $L_1 \cap L_2 = \phi$ $\therefore$ No solution for the two equations together.

i.e there is no solution of the two equations **(1)** , **(2)** when $L_1 // L_2$

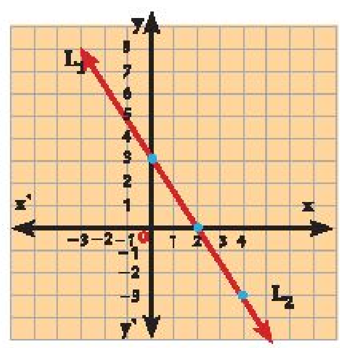
From the Analytical Geometry:

The slope of $L_1 = \frac{-3}{1} = -3$ The slope of $L_2 = \frac{-6}{2} = -3$ $\therefore$ $L_1 // L_2$

Second:

By Putting the equation **(2)** in the form of $2y = 6 - 3X$

i.e. $3X + 2Y = 6$ is the same as equation (1) the graph shown illustrates the graphical representation of the two equations by two coincident straight lines.



We say that: The two equations **(1)** and **(2)** have an infinite number of solutions. The solution set is $\{(x, y): y = 3 - \frac{3}{2}x\}$

Drill

Graphically find the solution set for each pair in the following equations:

1. $3x + y = 5$, $y + 3x = 8$
2. $2x + y = 4$, $8 - 2y = 4x$

Second: Solving two equations of first degree in two variables algabrically.

Solving two simultaneous equations of first degree in two variables is being done by removing one of the two variables where we get an equation of first degree in one variable. Solving this equation gives the value of this variable and by substituting in one of the given equations we get the value of the other which we removed.

Example 4

Find the solution set of the two equations

$2x - y = 3$ **(1)** , $x + 2y = 4$ **(2)**

Solution **(Substitution method)**

From the equation **(1)** , $y = 2x - 3$

by substitution in the equation **(2)** $\therefore x + 2(2x - 3) = 4$

thus : $x + 4x - 6 = 4$ $\therefore 5x = 10$ $\therefore x = 2$

Substituting in equation **(1)** $\therefore y = 2 \times 2 - 3$ $\therefore y = 1$

$\therefore$ The common solution set of the two equations = {(2, 1)}

Another solution **(Omitting method)**

Omitting one of the two variables in the two equations (by adding or subtracting) to get a third equation in one variable, and by solving the resulted equation we find the value of this variable.

$2x - y = 3$ **(1)** , $x + 2y = 4$ **(2)**

By multiplying the two sides of the equation (1) × 2 $\therefore 4x - 2y = 6$ **(3)**

Adding **(2)** and **(3)** $\therefore 5x = 10$ $\therefore x = 2$

Substituting in **(1)** $\therefore 2 \times 2 - y = 3$ $\therefore y = 1$

$\therefore$ The common solution set of the two equations is = {(2, 1)}.

Drill

1. Find algebraically, the solution set of each pair of the following equations:
   - **A** $3x + 4y = 24$, $x - 2y + 2 = 0$
   - **B** $3x + 2y = 4$, $x - 3y = 5$
2. What is the number of solutions of each pair in the following equations:
   - **A** $7x + 4y = 6$, $5x - 2y = 14$
   - **B** $3x + 4y = -4$, $5x - 2y = 15$
   - **C** $9x + 6y = 24$, $3x + 2y = 8$

Example 5

Find the values of a , b knowing that (3 , -1) is the solution of the two equations.

$a x + b y - 5 = 0$, $3 a x + b y = 17$

Solution

$\because$ (3, -1) is the solution of the two equations

$\therefore$ (3, -1) is the solution of the equations $a x + b y - 5 = 0$

$\therefore$ $3 a - b - 5 = 0$ **i.e. :** $3a - b = 5$ (1)

, (3, -1) is the solution of the equations $3 a x + b y = 17$

$\therefore$ $9 a - b = 17$ (2)

Substracting both sides of equation (1) from both sides of equation (2) we get :

$6 a = 12$ $\therefore$ $a = 2$

Substituting in equation (1)

$3 \times 2 - b = 5$ $\therefore$ $b = 1$

Example 6

A two-digit number of sum of its digits is 11. If the two digits are reversed, then the resulted number is 27 more than the original number. What is the original number ?

Solution

Consider that the units digit is x and the tens digit is y .

$\therefore$ $x + y = 11$ .... (1)

	units digit	tens digit	the value of the number
The original number	x	y	$x + 10 y$
The sum after reversing digits	y	x	$y + 10 x$

The resulted number after reversed its two digits - the original number = 27

$\therefore$ $(y + 10 x) - (x + 10 y) = 27$ $\therefore$ $y + 10 x - x - 10 y = 27$

$\therefore$ $9 x - 9 y = 27$ **by dividing by 9** $\therefore$ $x - y = 3$ .... (2)

By adding both equations (1) and (2)

$\therefore$ $2x = 14$ $\therefore$ $x = 7$ substituting in the equation .... (1)

$\therefore$ $7 + y = 11$ $\therefore$ $y = 4$ $\therefore$ the number is 47

Exercises 1-1

First : Complete the following :

A. The solution set of the two equations $x + y = 0$, $y - 5 = 0$ is ....

B. The solution set of the two equations $x + 3y = 4$, $3y + x = 1$ is ....

C. The solution set of the two equations $4x + y = 6$, $8x + 2y = 12$ is ....

D. If the two straight lines which represent the two equations $x + 3y = 4$, $x + ay = 0$ are parallel, then a = ...

E. If there is only one solution for the two equations $x + 2y = 1$ and $2x + ky = 2$, then k cannot equal ....

Second : Choose the correct answer from the given answers:

1. The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersected in :
   A. The origin B. First quadrant C. Second quadrant D. Fourth quadrant

2. The solution set of the two equations $x - 2y = 1$, $3x + y = 10$ is :
   A. {(5, 2)} B. {(2, 4)} C. {(1, 3)} D. {(3, 1)}

3. If there are infinite numbers of solutions of the two equations $x + 4y = 7$, $3x + ky = 21$ then k :
   A. 4 B. 7 C. 12 D. 21

Third:

1. **Find the solution set for each pair of the following two equations algebraically and graphically:**
   A. $y = x + 4$, $x + y = 4$
   B. $x - y = 4$, $3x + 2y = 7$
   C. $3x + 4y = 11$, $2x + y - 4 = 0$
   D. $3x - y + 4 = 0$, $y = 2x + 3$
   E. $2x + y = 1$, $x + 2y = 5$
   F. $x + 2y = 8$, $3x + y = 9$

2. If the number of the teams participating in the African cup of Nations is 16 teams, and the number of non-Arab teams is 4 more than three times the Arab teams, Find the number of the participating Arabic teams in the championship.

3. Two acute angles in a right angled triangle. The difference between their measures is 50. Find the measure of each angle.

4. Two supplementary angles, the twice of the measure of their bigger equals seven times the measure of the smallest. Find the measure of each angle.

5. If the sum of the ages of Ahmed and Osama is now 43 years and after 5 years, the difference between both ages will be 3 years. Find the age of each them after 7 years.

6. A rectangle with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm, Find the area of the rectangle.

1 - 2

(2) Solving an equation of second degree in one unknown graphically and Algebraically

What you'll learn

- (2) Solving an equation of second degree in one unknown graphically and Algebraically.

Key terms

- Graphical solution
- Algebraic solution
- Solution set

Think and Discuss

We have reperesented graphically the quadratic function f where :

$f(x) = a x^2 + b x + c$, a, b , c $\in$ R, $a \neq 0$

The corresponding equation is $f(x) = 0 \Rightarrow ax^2 + b x + c = 0$

You have previously solved this equation by factorizing.

To solve the equation : $x^2 - 4x + 3 = 0$

We factorize the left side of the equation to be :

(x - ....) (x .... 1) = 0

$\therefore$ x - .... = 0 or (x - 1) = 0

$\therefore$ x = ... or x = ...

$\therefore$ The solution set is { .... , ... }

First: the graphical solution:

To solve $a x^2 + b x + c = 0$ graphically we follow the steps:

- We draw the function curve of $f(x) = a x^2 + b x + c$ where $a \neq 0$
- Identify the set of x coordinates of the points of intersection of the function curve with the x-axis, thus we get the solution of the equation.

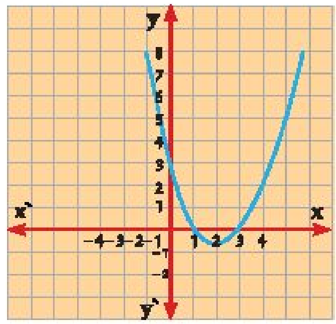
Example 1

Draw the graphical representation of the function f where $f(x) = x^2 - 4 x + 3$ in the interval [- 1, 5]

From the drawing, find the solution set of the equation $x^2 - 4 x + 3 = 0$

Solution

Identify some ordered pairs (x, y) which belong to the function f, whose first projection $x \in [-1, 5]$



f (-1) = 8, f (0) = 3, f (1) = 0,

f (2) = -1, f (3) = 0, f (4) = 3, f (5) = 8

Put the ordered pairs in a table as follows:

x	5	4	3	2	1	0	-1
y = f(x)	8	3	0	-1	0	3	8

Plot on the coordinate plane the points which represent these ordered pairs, then draw a curve passing through these points.

From the drawing we find that the function curve f intersects the x-axis in two points (3, 0), (1, 0) the two numbers 1, 3 are called the two roots of the equation $x^2 - 4x + 3 = 0$.

Thus, the solution set of the equation is {1 , 3}

Drill

1. Draw the graphical form of the function f where $f(x) = x^2 + 2x + 1$ in the interval [-4, 2] and from the drawing find the solution set of the equation: $x^2 + 2x + 1 = 0$
2. Draw the graphical form of the function f where $f(x) = -x^2 + 6x - 11$ in the interval [0, 6] and from the drawing find the solution set of the equation: $x^2 - 6x + 11 = 0$

Second : The algebraic solution by using the general rule:

Think and Discuss

Solving the equation : $x^2 - 6x + 7 = 0$ using the idea of completing the square.

Complete : $\because$ $x^2 - 6x + 9 + 7 - 9 = 0$

$\therefore$ (x - ....)^2 - 2 = 0   (x - ....)^2 = 2

x - .... = $\sqrt{2}$ or   x .... = - $\sqrt{2}$

x = ... + $\sqrt{2}$ or   x = ... - $\sqrt{2}$

$\therefore$ x = ... $\pm\sqrt{2}$

You can solve an equation of second degree : $a x^2 + b x + c = 0$ where $a, b, c \in R$, $a \neq 0$ using the rule

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } a \neq 0,\ a, b, c \in R$$

Examples

2 Find the solution set of the equation $3x^2 = 5x - 1$ rounding the results to two decimal places.

Solution

$\because 3x^2 = 5x - 1 \qquad \therefore 3x^2 - 5x + 1 = 0$

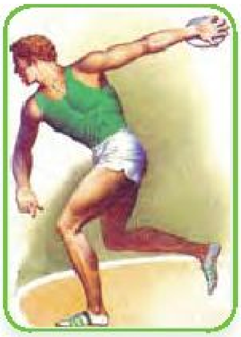
$\therefore a = 3, b = -5, c = 1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{13}}{6} = \frac{5 \pm 3.61}{6}$$

As for $x = \frac{5 + 3.61}{6} = 1.44$ or $x = \frac{5 - 3.61}{6} = 0.23$

$\therefore$ The solution set is : {1.44, 0.23}

3 In a disk throwing race the path way of the disk to one of the players follows the relation : $y = -0.043 x^2 + 4.9 x + 13$ where x represents the horizontal distance in meters, y represents the disk height from the floor surface. Find the horizontal distance at which the disk falls to the nearset hundred.



Solution

$\because a = -0.043, b = 4.9, c = 13$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(4.9) \pm \sqrt{(4.9)^2 - 4 \times (-0.043) \times 13}}{2 \times (-0.043)}$$

$$= \frac{(-4.9) \pm \sqrt{26.246}}{-0.086} = \frac{-4.9 \pm 5.123}{-0.086}$$

$\therefore x = \frac{-4.9 + 5.123}{-0.086} = -2.59$ (refused) why ?

or $x = \frac{-4.9 - 5.123}{-0.086} = 116.5465116$ meters

$\therefore$ The horizontal distance where the disk lands is 116.55 meters

Exercises 1-2

1 Find the solution set for each of the following equations, using the general rule, rounding the results to three decimals.

A $x^2 - 2x - 6 = 0$ **B** $x^2 + 3x - 3 = 0$ **C** $2x^2 - 4x + 1 = 0$

D $3x^2 - 6x + 1 = 0$ **E** $x(x - 1) = 4$ **F** $(x - 3)^2 - 5x = 0$

G $x + \frac{4}{x} = 6$ **H** $\frac{8}{x^2} + \frac{1}{x} = 1$ **I** $\frac{x}{3} = \frac{1}{5 - x}$

2 Draw the graphical representation of the function f in the given interval, then find the solution set of the equation f(x) = 0 Rounding the results to one decimal digit in each of the folloing :

A $f(x) = x^2 - 2x - 4$ in the interval $[-2, 4[$

B $f(x) = 2x^2 + 5x$ in the interval $[-4, 2[$

C $f(x) = 3x - x^2 + 2$ in the interval $[-1, 4]$

D $f(x) = x(x - 5) + 3$ in the interval $[0, 5]$

E $f(x) = 2x^2 - 3(2 - x)$ in the interval $[-3, 2[$

F $f(x) = 2x(x - 1) - 3(x + 2) + 5$ in the interval $[-1, 3]$

G $f(x) = (x - 3)^2 - (x - 3) - 4$ in the interval $[7, 1]$

3 Draw a graphical representation of the function f where $f(x) = 6x - x^2 - 9$ in the interval $[0, 5]$ and from the drawing find:

A The maximum value and the minimum value of the function

B The solution set of the equation $6x - x^2 - 9 = 0$

4 A man waters his garden with a hose where the water is pumped through in a pathway. Identified by the relation: $y = -0.06x^2 + 1.2x + 0.8$ where x is the horizontal distance that the water, can reach in meters, y is the hight of water from the floor surface in meter. Find to the nearest centimeter the maximum horizontal distance the water can reach.

5 A snake saw a hawk at a height of 160 meters and hawk flying at a speed of 24 meter/minute to pounce on it. If hawk is launching vertically downwards according to the relation $d = V_o t + 4.9t^2$. where d is the distance by meter, V_o is the launching speed in meter / minute and t is the time in minutes. Find the time the snake takes to escape before the hawk reaches it.

1 - 3

Solving two equations in two variables, one of them is of the first degree and the other is of the second degree

What you'll learn

- Solving two equations in two variables one of them is of the first degree and the other of the second degree.

Key terms

- Equation of the first degree
- Equation of the second degree
- Solution set

Introductions:

You know that the equation $2x - y = 3$ is an equation of the first degree in two variables while the equations: $x^2 + y = 5$ and $xy = 2$ are equations of the second degree in two veriables. why?

We will solve the two equations in two varibles one of them is of the first degree and the other of the second degree, by the substitution method as shown in the following examples.

Mental Math: If $x + y = 10$ and $x^2 - y^2 = 40$ then find $x - y$.

Examples

1 Find algebraically the solution set of the two equations:

$y + 2x + 1 = 0$, $4x^2 + y^2 - 3xy = 1$

Solution

From the first equation: $y = -(2x + 1)$

Substituting in second equation.

$\therefore 4x^2 + [-(2x + 1)]^2 - 3x[-(2x + 1)] = 1$

$\therefore 4x^2 + 4x^2 + 4x + 1 + 6x^2 + 3x - 1 = 0$

$\therefore 14x^2 + 7x = 0$ $\therefore 7x(2x + 1) = 0$

$\therefore x = 0$ or $2x + 1 = 0$ i.e $x = \frac{-1}{2}$

Substituting for the values of x in first equation :

When $x = 0$ $\therefore y = -(0 + 1) = -1,$

When $x = \frac{-1}{2}$ $\therefore y = -(2 \times \frac{-1}{2} + 1) = 0$

$\therefore$ The solution set is : $\{(0, -1), (\frac{-1}{2}, 0)\}$

2 A rectangle of a perimeter 14 cm and area 12 cm^2. Find its two dimensions.

Solution

Supose the two dimensions of the rectangle are x and y.

$\because$ The rectangle perimeter = **2 (Length + Width)**

$\therefore$ $14 = 2(x + y)$ ..... **divide both sides by 2**

$\therefore$ $x + y = 7$ i.e. $y = 7 - x$ .... (1)

$\because$ The rectangle area = **length** × **width** $\therefore$ $xy = 12$ .... (2)

Substituting from equation (1) in equation (2)

$\therefore$ $x(7 - x) = 12$ $7x - x^2 = 12$

$\therefore$ $x^2 - 7x + 12 = 0$ $(x - 3)(x - 4) = 0$

$\therefore$ $x = 3$ or $x = 4$ substitute in equation (1)

when : $x = 3$ $\therefore$ $y = 7 - 3 = 4$,

when : $x = 4$ $\therefore$ $y = 7 - 4 = 3$ the lenght and width of the rectangle are 3 cm and 4 cm.

Exercises 1-3

First: Choose the correct answer from the given answers:

1. The solution set of the two equations $x - y = 0$ and $xy = 9$ is :
   A $\{(0, 0)\}$ B $\{(-3, -3)\}$ C $\{(3, 3)\}$ D $\{(-3, -3), (3, 3)\}$
2. One of the solutions for the two equation: $x - y = 2$, $x^2 + y^2 = 20$ is :
   A (-4, 2) B (2, -4) C (3, 1) D (4, 2)
3. If the sum of two positive numbers is 7 and their product is 12 then the two numbers are :
   A 2, 5 B 2, 6 C 3, 4 D 1, 6

Second:

1. Find the solution set for each of the following equations:
   A $y - x = 2$ and $x^2 + xy - 4 = 0$
   B $x + 2y = 4$ and $x^2 + xy + y^2 = 7$
   C $x - 2y - 1 = 0$ and $x^2 - xy = 0$
   D $y + 2x = 7$, $2x^2 + x + 3y = 19$
   E $x - y = 10$ and $x^2 - 4xy + y^2 = 52$
   F $x + y = 2$, $\frac{1}{x} + \frac{1}{y} = 2$ where $(x, y \neq 0)$
2. Consider a digit in units digit is twice the digit in the tens place of a two-digits number. If the product of the two digits equals the half of the original number, what is this number?
3. A length of the rectangle is 3 cm more than its width and its area is 28 cm^2. Find its perimeter.
4. A right angled triangle of hypotenuse length 13cm and its perimeter is 30 cm. Find the lengths of the other two sides.
5. For a rhombus, the difference between the lengths of its diagonals equals 4 cm and its perimeter is 40 cm, find the lengths of the diagonals.
6. A point moves on the straight line $5x - 2y = 1$ where its y-coordinate is twice of the square of its x-coordinate. Find the coordinates of this point.

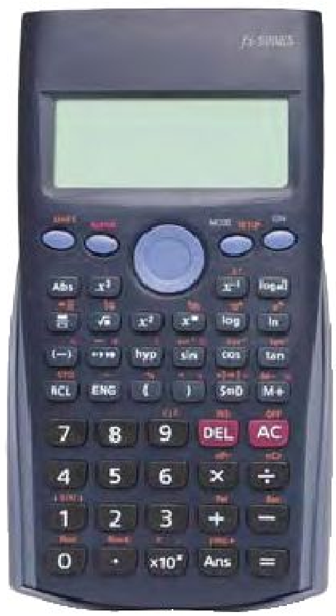
Activity

Connecting with technology

1 Solving two simultaneous equations of first degree in two unknowns:

To check the solution of two equations : $X + 2y = 8$ and $3X + y = 9$ (for example) using the calculator and do the following steps:

press the operations button (MODE) and choose from the menu (EQN) by writing the written number before it, or press the (EXE) button in some calculators, then choose the linear equation: ($an\,X + bn\,Y = cn$) enter the coefficients ((Y),(X)), and the absolute term ((cn)) to the first equation then to second equation. Notice that pressing the button of ((=)) gives the value of ((Y),(X)), and this is the solution of the equation.

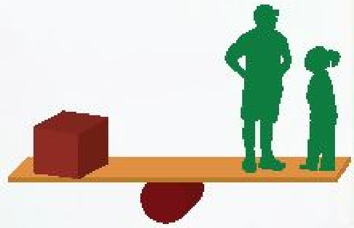


2 Solving an equation of second degree in one unknown :

Repeat the same previous steps in the first three lines, then choose ($ax^2 + b\,x + c = 0$) type the cofficients (a) , (b) and (c) and press the button of ((=)) or (EXE) after each digit, keeping press on the enter button gives the two values of (X) directly.

Activity

1 *The opposite figure :* A man and his daughter are standing on a side of a seesaw. If man's weight is 80 kg on the other side of seesaw there is a stone of weight three times the weight of the girl then the seesaw is completely balanced then find the girls, weight?



2 *The following figures :* show weight of apples and bananas of each one of the apples is identical, and also for the bananas, find the balance reading in figure C. Determine the gage position on the drawing.

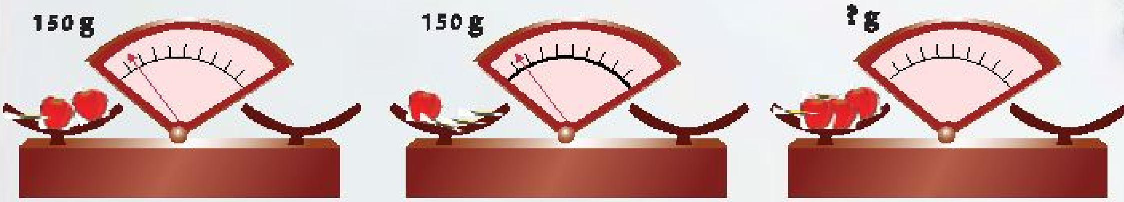


Figure (A) Figure (B) Figure (C)

Unit test

1 Complete the following :

A If $(5, x - 7) = (y + 1, - 5)$ then $x + y =$ ....

B The function f where $f(X) = X^6 + 2X^4 - 3$ is a polynomial function of degree ....

C If the curve of the function f where $f(x) = x^2 - a$ passes throught the point (1 , 0) then a = ...

2 Find the solution set of the following equations:

A $x + 3y = 7$ and $5x - y = 3$ graphically and algebraically.

B $x^2 - 4x + 1 = 0$ using the rule, rounding the results to nearest two decimal places.

C $y - x = 3$ and $x^2 + y^2 - xy = 13$

3 Represent the function f where $f(X) = x^2 - 2x - 1$ in the interval $[-2, 4]$ graphically and find :

A The equation of axis of symmetry

B Solution set of the equation $x^2 - 2x - 1 = 0$

4 If the sum of two numbers is 90 and their product is 2000, then find the two numbers.

5 A bike rider moved from city A in the direction of east to city B. From city B, he moves north to city C to travel a distance of 14 km. If the sum of the squares of the traveled distance is 100 km^2. Find the shortest distance between city A and C.

6 When a dolphin jumps on water surface, its pathway follows the relation :
$y = -0.2X^2 + 2x$ where y is the height of the dolphin above water and x is the horizontal distance in feet.
Find the horizontal distance that the dolphin covers when it jumps from water.

2 - 1 Set of zeroes of a polynomial function

You Will learn

- Find zeroes of the polynomial function.

Key terms

- Polynomial function.
- Set of zeroes of the polynomial function.

Think and Discuss

if f : R $\longrightarrow$ R where $f(x) = x^3 - 3x^2 + 2x$ is a polynomial function of third degree in X. calculate : f (0) , f (1), and f (2) **what do you notice?**

We notice that : f (0) = 0 , f (1) = 0 , f (2) = 0

So 0 , 1 and 2 are called the set of zeroes of the function.

Generally if f : R $\longrightarrow$ R is a polynomial in x, then the set of values of x which makes f (x) = 0 is called the set of zeroes of the function f and its denoted by the sumbol Z (f).

i.e : Z (f) is the solution set of the equation f (x) = 0

In general, to get the zeros of the function f, put f(x) = 0 and solve the resulted equation to find the set of values of x.

Example

Find Z(f) for each of the following polynomial :

1. $f_1(x) = 2x - 4$
2. $f_2(x) = x^2 - 9$
3. $f_3(x) = 5$
4. $f_4(x) = 0$
5. $f_5(x) = x^2 + 4$
6. $f_6(x) = x^6 - 32x$
7. $f_7(x) = x^2 + x + 1$

Solution

1. $f_1(x) = 2x - 4$ put $f_1(x) = 0$ $\therefore 2x - 4 = 0$
   i.e $2x = 4$ $\therefore x = 2$ $\therefore z(f_1) = \{2\}$.

2 $f_2(x) = x^2 - 9$

put $f_2(x) = 0$ $\therefore$ $x^2 - 9 = 0$

i.e $x^2 = 9$ $\therefore$ $x = \pm 3$ $\therefore$ $z(f_2) = \{-3, 3\}$.

3 $f_3(x) = 5$

$\therefore$ there is no real number that makes $f_3(x) = 0$ $\therefore$ $z(f_3)$ is ϕ

4 $f_4(x) = 0$

$\therefore$ all the real numbers R are zeroes to this function $\therefore$ $z(f_4)$ is R

5 put $x^2 + 4 = 0$

$\therefore$ $x^2 = -4$ $\therefore$ $x = \pm\sqrt{-4} \notin R$ $\therefore$ $z(f_5)$ is ϕ

6 put $x^6 - 32x = 0$

$\therefore$ $x(x^5 - 32) = 0$ $\therefore$ $x = 0$, $x^5 = 32$

when $x^5 = 2^5$ $\therefore$ $x = 2$ $\therefore$ $z(f_6) = \{0, 2\}$

7 put $x^2 + x + 1 = 0$

the expression $x^2 + x + 1$ could not be factorized so we use the rule to solve the quadratic

equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = 1$, $b = 1$, $c = 1$

$\therefore$ $x = \frac{-1 \pm \sqrt{-3}}{2} \notin R$

$\therefore$ there is no solutions then $z(f_7) = \phi$

Drill

1 **Find the set of zeroes of the following functions :**

a) $f(x) = x^3 - 4x^2$ b) $f(x) = x^2 - 2x + 1$ c) $f(x) = x^2 - 2x - 1$

d) $f(x) = x^4 - x^2$ e) $f(x) = x^2 - x + 1$ f) $f(x) = x^2 - 2$

Exercises 2-1

First : **Choose the correct answer :**

1. The set of zeroes of f : where $f(x) = -3x$ is:
   - a. {0}
   - b. {-3}
   - c. {-3 , 0}
   - d. R

2. The set of zeroes of f : where $f(x) = x(x^2 - 2x + 1)$ is :
   - a. {0 , 1}
   - b. {0 , -1}
   - c. {-1 , 0}
   - d. {1}

3. If z(f) = { 2 } , $f(x) = x^3 - m$, then m equals :
   - a. $\sqrt[3]{2}$
   - b. 2
   - c. 4
   - d. 8

4. If z(f) = { 5 } , $f(x) = x^3 - 3x^2 + a$ then a equals :
   - a. -50
   - b. -5
   - c. 5
   - d. 50

5. If z(f) = { 1 , -2 } , $f(x) = x^2 + x + a$ then a equals :
   - a. 28
   - b. 1
   - c. - 1
   - d. - 2

Second: **1. Find the set of zeroes of the polynomial which are known by the following rules in R.**

- a. $f(x) = (x - 1)(x - 2)$
- b. $f(x) = x^2 - 2x$
- c. $f(x) = x^2 - 16$
- d. $f(x) = 25 - 9x^2$
- e. $f(x) = 2x^3 - 18x$
- f. $f(x) = 5x^3 - 20x$
- g. $f(x) = x^3 - 125$
- h. $f(x) = 2x^3 + 16$
- i. $f(x) = 2x^4 + 54x$
- j. $f(x) = 6x^2 + x - 12$
- k. $f(x) = x^3 + 2x^2 - 15x$
- l. $f(x) = 2x^4 + x^3 - 6x^2$
- m. $f(x) = x(x - 5) - 14$
- n. $f(x) = (x - 2)(x + 3) + 4$
- o. $f(x) = x^3 + x^2 - 2x - 8$
- p. $f(x) = x^3 - 3x^2 - 4x + 12$

2. If $f(x) = x^3 - 2x^2 - 75$ **Prove that** the number 5 is the one of the zeroes of this function.

3. If {-3 , 3} is the set of zeroes of the function f where : $f(x) = x^2 + a$ ***Find*** **the value of** a.

4. If the set of zeroes of the function f where $f(x) = ax^2 + bx + 15$ is { 3 , 5 }
   Find **the values of a and b .**

2 - 2

Algebraic fractional function

What you'll learn

- Algebraic fractional function.

Key terms

- Polynomial function.
- The domain of algebraic fraction.
- The common domain for two algebraic fractions.

Think and Discuss

you have previously learned the rational number which is in the form $\frac{a}{b}$ where $a, b \in Z$, $b \neq 0$

If $p: R \longrightarrow R$, $p(x) = x + 3$,

$f: R \longrightarrow R$, $f(x) = x^2 - 4$.

1. ***Find*** the domain of f and p .
2. If $n(x) = \frac{p(x)}{f(x)}$ can you find the domain of n when you know the domain of each of p and f ?

From the previous, we deduce the following:

n is called an algebraic fractional function or an algebraic fraction where $n(x) = \frac{x + 3}{x^2 - 4}$

The domain in this case is R except for the values of x which makes the fraction unknown (set of zeroes of the denominator).

i.e : the domain of n(x) is R - {-2 , 2}

If p and f are two polynomail functions and z (f) is the set of zeroes of f, then the function n where

$$n : R - z(f) \longrightarrow R \ , \ n(x) = \frac{p(x)}{f(x)}$$

is called real algebraic fractional function or briefly called an algebraic fraction.

Note that : the domain of algebraic fractional function = R - the set of zeroes of the denominator.

Drill

1 **Identify the domain of each of the following algebraic fractional function then *find* n(0), n (2), n (-2):**

a $n(x) = \frac{x+3}{4}$ b $n(x) = \frac{x-2}{2x}$ c $n(x) = \frac{1}{x+2}$

d $n(x) = \frac{x^2+9}{x^2-16}$ e $n(x) = \frac{x^2+1}{x^2-x}$ f $n(x) = \frac{x^2-1}{x^2+1}$

2 **If the domain of the function n :** $n(x) = \frac{x-1}{x^2 - a x + 9}$ is R - {3} *then find* the value of a.

The common domain of two or more algebraic fraction:

The set of real numbers where the fractions are identified together completely (at the same time).

Example

If n_1 , n_2 are two algebraic fractions where :

$n_1(x) = \frac{1}{x-1}$, $n_2(x) = \frac{3}{x^2-4}$ *then calculate* the common domain of n_1 , n_2

Solution

Let m_1 the domain of n_1 , m_2 the domain of n_2.

$\therefore$ m_1 = R - {1} , m_2 = R - {-2 , 2} then the common domain of the two fractions n_1 , $n_2 = m_1 \cap m_2$

where : $m_1 \cap m_2$ = {(R - {1}} $\cap$ {R - {-2 , 2}} = R - {-2 , 1 , 2}

Remark : For any value of the variable x which belongs to the common domain then , each of $n_1(x)$ and $n_2(x)$ are defined (existed).

Generally :

If n_1 and n_2 are two algebraic fractions, and if the domain of $n_1 = R - X_1$ (where X_1 , the set of zeroes of the denominator of n_1) of the domain $n_2 = R - X_2$ (where X_2 , the set of zeroes of the denominator of n_2)

then the common domain of the two fractions n_1 and $n_2 = R - (X_1 \cup X_2)$

= R - the set of zeroes of the two denominators of the two fractions.

$\therefore$ the common domain of a number of algebraic fractions

= R - the set of zeroes of the denoinators of these fractions

Drill

Find the common domain for each of the following :

1. $n_1(x) = \frac{1}{x}$, $n_2(x) = \frac{2}{x+1}$
2. $n_1(x) = \frac{3}{x^2 - x}$, $n_2(x) = \frac{2x-3}{x^2-1}$
3. $n_1(x) = \frac{3}{x-2}$, $n_2(x) \frac{5}{x+2}$, $n_3(x) = \frac{x}{x^3 - 4x}$
4. $n_1(x) = \frac{x^2-4}{x^2-5x+6}$, $n_2(x) = \frac{3x}{x^2-x}$, $n_3(x) = \frac{x^2-3x-4}{x^2+x-2}$

Exercises 2-2

Find **the common domain to the sets of the following algebraic fractions :**

1. $\frac{x}{3}$, $\frac{3}{x}$
2. $\frac{1}{2x}$, $\frac{x-1}{5}$
3. $\frac{x+2}{x+5}$, $\frac{x-4}{x-7}$
4. $\frac{4}{x-4}$, $\frac{x-5}{5x}$
5. $\frac{x}{x^2-4}$, $\frac{3}{2-x}$
6. $\frac{5}{x-2}$, $\frac{x+1}{x^2-2x}$
7. $\frac{1}{x^3-1}$, $\frac{x}{1-x^2}$
8. $\frac{x^2+4}{x^2-4}$, $\frac{7}{x^2+4x+4}$
9. $\frac{x-2}{x+4}$, $\frac{7}{x-3}$, $\frac{x}{x^2+4}$
10. $\frac{x+3}{2}$, $\frac{3}{x^2-9}$, $\frac{3x}{x^2-3x}$
11. $\frac{x^2}{x-3}$, $\frac{7}{x+3}$, $\frac{-2x}{x^3+27}$
12. $\frac{4x-3}{x^2-x}$, $\frac{x-1}{x^2+16}$, $\frac{5x}{x^2-2x-3}$
13. $\frac{x^2-4}{x^2-5x+6}$, $\frac{7}{x^2-9}$, $\frac{x^2-3x-4}{x^2+x-2}$

2 - 3

Equality of two algebraic fractions

What you'll learn

- The concept the equality of two algebraic fractions.
- How to determine when two algebraic fractions are equal.

Key terms

- Reducing an algebraic fraction.
- Equality of two algebraic fractions.

Reducing the algebraic fraction

Think and Discuss

If n is an algebraic fraction where : $n(x) = \dfrac{x^2 + x}{x^2 - 1}$

Complete :

1. The domain of n = ...
2. The common factor between the numerator and denominator after factorizing both of them perfect facterization is .... ≠ zero where x doesn't take the value of ....
3. The algebraic fraction in the simplest form after removing the common factor = ...
4. Does the domain of the algebraic fraction change after putting it in the simplest form ?

From the previous, we deduce that :

Putting the algebraic fraction in the simplest form is called reducing the algebraic fraction.
Follow the following steps to reduce an algebraic fraction :

1. ***Factorize both the numerator and denominator perfectly.***
2. ***Identify the domain of the algebraic fraction before removing the common factors in the numerator and denominator.***
3. ***Remove the common factor in both the numerator and denominator to get the simplest form.***

Definition: It is said that the algebraic fraction is in its simplest form if there are no common fractons between its numerator and denominator.

Example 1

If $n(x) = \dfrac{x^3 + x^2 - 6x}{x^4 - 13x^2 + 36}$ then reduce n(x) in the simplest form showing the domain of n.

Solution

$\because$ $n(x) = \dfrac{x^3 + x^2 - 6x}{x^4 - 13x^2 + 36} = \dfrac{x(x^2 + x - 6)}{(x^2 - 4)(x^2 - 9)} = \dfrac{x(x+3)(x-2)}{(x+2)(x-2)(x+3)(x-3)}$

$\therefore$ the domain of n(x) = R - {-3 , -2 , 2 , 3}.

$\therefore$ $n(x) = \dfrac{x}{(x+2)(x-3)}$ then cancel (x + 3) , (x - 2) from the numerator and denominator.

Equality of two algebraic fraction to be equal

Think and Discuss

Find **$n_1(x)$ and $n_2(x)$ in the simplest form showing the domain of the following :**

1. $n_1(x) = \dfrac{x+3}{x^2 - 9}$, $n_2(x) = \dfrac{2}{2x - 6}$
2. $n_1(x) = \dfrac{2x}{2x + 4}$, $n_2(x) = \dfrac{x^2 + 2x}{x^2 + 4x + 4}$

Does $n_1 = n_2$ in each case ? Explain your answer.

From the previous we deduce that :

1. $n_1(x) = \dfrac{x+3}{(x+3)(x-3)} = \dfrac{1}{x-3}$ **and the domain of** n_1 = R - {-3 , 3}

   $n_2(x) = \dfrac{2}{2(x-3)} = \dfrac{1}{x-3}$ **and the domain of** n_2 = R - {-3}

   i.e.: n_1 and n_2 are reduced to the same fraction but the domain of $n_1 \neq$ the domain n_2

2. $n_1(x) = \dfrac{2x}{2(x+2)} = \dfrac{x}{x+2}$ **and the domain of** n_1 = R - {-2}

   $n_2(x) = \dfrac{x(x+2)}{(x+2)^2} = \dfrac{x}{x+2}$ **and the domain of** n_2 = R - {-2}

   i.e.: n_1 and n_2 are reduced to the same form, **and the domain of** n_1 = **and the domain of** n_2

From the previous, we deduce that :

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e: $n_1 = n_2$) if the two following conditions are satisfied.
the domain of n_1 = the domain of n_2 , $n_1(x) = n_2(x)$ for each $x \in$ the common domain.

Examples

2 If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$ **prove that :** $n_1 = n_2$

Solution

$\because n_1(x) = \frac{x^2}{x^3 - x^2} = \frac{x^2}{x^2(x-1)}$ $\therefore n_1(x) = \frac{1}{x-1}$

the domain of $n_1 = R - \{0 , 1\}$ (1)

$\because n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x} = \frac{x(x^2 + x + 1)}{x(x^3 - 1)} = \frac{x(x^2 + x + 1)}{x(x-1)(x^2 + x + 1)}$

$\therefore n_2(x) = \frac{1}{x-1}$

the domain of $n_2 = R - \{0 , 1\}$ (2)

from (1) **and** (2)

$\because$ the domain of n_1 = the domain of n_2 , $n_1(x) = n_2(x)$ for each $x \in R - \{0 , 1\}$

$\therefore n_1 = n_2$

3 If $n_1(x) = \frac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \frac{x^3 - x^2 - 6x}{x^3 - 9x}$

prove that $n_1(x) = n_2(x)$ for the values of x which belong to the common domain and find the domain.

Solution

$\because n_1(x) = \frac{x^2 - 4}{x^2 + x - 6} = \frac{(x+2)(x-2)}{(x+3)(x-2)} = \frac{x+2}{x+3}$

the domain of $n_1 = R - \{-3 , 2\}$ (1)

$\because n_2(x) = \frac{x^3 - x^2 - 6x}{x^3 - 9x} = \frac{x(x-3)(x+2)}{x(x+3)(x-3)} = \frac{x+2}{x+3}$

and the domain of $n_2 = R - \{-3, 0, 3\}$ (2)

from (1) and (2)

we notice that : $n_1(x)$, $n_2(x)$ are reduced to the same fraction $\frac{x+2}{x+3}$.

but the domain of $n_1 \neq$ domain of n_2 so $n_1 \neq n_2$.

we can say that : $n_1(x) = n_2(x)$ take the same values if x belongs to the common domain for the two functions n_1, n_2 $R - \{-3, 0, 2, 3\}$.

Drill

Complete the following :

1. The simplest from of the function $f(x) = \frac{4x^2 - 2x}{2x}$, $x \neq 0$ is ....

2. The common domain of the function n_1, n_2 where $n_1(x) = \frac{x-2}{x^2-4}$,
   $n_2(x) = \frac{1}{x+1}$ is ....

3. if $n_1(x) = \frac{1+a}{x-2}$, $n_2(x) = \frac{4}{x-2}$ and $n_1(x) = n_2(x)$ then $a =$ ....

4. If the simplest form of the algebraical fraction $n(x) = \frac{x^2 - 4x + 4}{x^2 - a}$ is $n(x) = \frac{x-2}{x+2}$ then $a =$ ....

5. If $n_1(x) = \frac{-7}{x+2}$, $n_2(x) = \frac{x}{x-k}$ and the common domain of two function n_1, n_2
   is $R - \{-2, 7\}$ then $k =$ ....

Exercises 2-3

1. **Simplifiy each of the following fractions to the simplest form, showing its domain :**

   A. $\frac{x^2-4}{x^3-8}$
   B. $\frac{x+1}{x^2+3x+2}$
   C. $\frac{x^2-4}{x^2-5x+6}$
   D. $\frac{x^3-1}{(x-1)(x+5)}$
   E. $\frac{x^2-6x+9}{2x^3-18x}$
   F. $\frac{x^3+1}{x^3-x^2+x}$

G $\frac{2x^2+7x+6}{4x^2+4x-3}$ H $\frac{(x-2)^2-1}{x(x-3)}$ I $\frac{x^3+x^2-2}{x-1}$

2 In each of the following show whether $n_1 = n_2$ or not and why ?

A $n_1(x) = \frac{x-1}{x}$, $n_2(x) = \frac{(x-1)(x^2+1)}{x(x^2+1)}$

B $n_1(x) = \frac{x^2-4}{x^2+x-6}$, $n_2(x) = \frac{x^2-x-6}{x^2-9}$

C $n_1(x) = \frac{2x^3+6x}{(x-1)(x^2+3)}$, $n_2(x) = \frac{2x}{x-1}$

D $n_1(x) = \frac{x^3+1}{x^3-x^2+x}$, $n_2(x) = \frac{x^3+x^2+x+1}{x^3+x}$

3 In each of the following prove that : $n_1 = n_2$

A $n_1(x) = \frac{1}{x}$, $n_2(x) = \frac{x^2+4}{x^3+4x}$

B $n_1(x) = \frac{2x}{2x+8}$, $n_2(x) = \frac{x^2+4x}{x^2+8x+16}$

C $n_1(x) = \frac{x^3-1}{x^3+x^2+x}$, $n_2(x) = \frac{(x-1)(x^2+1)}{x^3+x}$

d $n_1(x) = \frac{x^3+x}{x^3+x^2+x+1}$, $n_2(x) = \frac{x}{x+1}$

4 ***Find*** the common domain of functions n_1 , n_2 for each of the following :

A $n_1(x) = \frac{x+2}{3}$, $n_2(x) = \frac{x-3}{x}$

B $n_1(x) = \frac{-5}{x^2-1}$, $n_2(x) = \frac{2}{x}$

C $n_1(x) = \frac{x-5}{3-x}$, $n_2(x) = \frac{3x}{x^2+1}$

D $n_1(x) = \frac{x}{x^3-8}$, $n_2(x) = \frac{11}{x^2-4}$

E $n_1(x) = \frac{3x+1}{7x}$, $n_2(x) = \frac{x^2+1}{x^4-81}$

F $n_1(x) = \frac{x^2+9x+20}{x^2-16}$, $n_2(x) = \frac{x^2+5}{x^2-4x}$

2 - 4

Operations on Algebraic fractions

First : Adding and subtracting the algebraic fractions

Think and Discuss

1. If $\frac{a}{b}$, $\frac{c}{b}$ are two rational numbers then find each of the following : $\frac{a}{b}+\frac{c}{b}$, $\frac{a}{b}-\frac{c}{b}$

2. If $\frac{a}{b}$, $\frac{c}{d}$ two rational numbers then find each of the following : $\frac{a}{b}+\frac{c}{d}$, $\frac{a}{b}-\frac{c}{d}$

From the previous, we can do the operation of adding or subtracting of two algebraic fractions :

If $x \in$ the common domain of the two algebraic fractions n_1, n_2 where :

1. $n_1(x)=\frac{f_1(x)}{f_2(x)}$, $n_2(x)=\frac{f_3(x)}{f_2(x)}$

   (two algebraic fractions having a common denominator)

   then : $n_1(x)+n_2(x)=\frac{f_1(x)}{f_2(x)}+\frac{f_3(x)}{f_2(x)}=\frac{f_1(x)+f_3(x)}{f_2(x)}$,

   $n_1(x)-n_2(x)=\frac{f_1(x)}{f_2(x)}-\frac{f_3(x)}{f_2(x)}=\frac{f_1(x)}{f_2(x)}+\frac{-f_3(x)}{f_2(x)}$

2. $n_1(x)=\frac{f_1(x)}{f_2(x)}$, $n_2(x)=\frac{f_3(x)}{f_4(x)}$

   (two algebraica fractions having two different denominators)

   then : $n_1(x)+n_2(x)=\frac{f_1(x)}{f_2(x)}+\frac{f_3(x)}{f_4(x)}$

   $=\frac{f_1(x)\times f_4(x)+f_3(x)\times f_2(x)}{f_2(x)\times f_4(x)}$,

   $n_1(x)-n_2(x)=\frac{f_1(x)}{f_2(x)}-\frac{f_3(x)}{f_4(x)}=\frac{f_1(x)\times f_4(x)-f_3(x)\times f_2(x)}{f_2(x)\times f_4(x)}$

What you'll learn

- Doing the operations of (+ , - , × , ÷) on the algebraic fractions

Key terms

- Additive inverse of the algebraic fractions.
- multiplicative inverse on the algebraic fractions.

Examples

1 If $n_1(x) = \frac{x}{x^2 + 2x}$, $n_2(x) = \frac{x+2}{x^2 - 4}$

Find $n(x) = n_1(x) + n_2(x)$ show the domain of n.

Solution

$\because$ $n(x) = n_1(x) + n_2(x)$

$\therefore$ $n(x) = \frac{x}{x^2 + 2x} + \frac{x+2}{x^2 - 4} = \frac{x}{x(x+2)} + \frac{x+2}{(x-2)(x+2)}$

domain n = R - {-2 , 0 , 2}

$\therefore$ $n(x) = \frac{1}{x+2} + \frac{1}{x-2} = \frac{x-2+x+2}{(x+2)(x-2)} = \frac{2x}{(x+2)(x-2)}$

2 ***Find :*** n(x) in the simplest form showing the domain of n where :

$n(x) = \frac{3x-4}{x^2 - 5x + 6} + \frac{2x+6}{x^2 + x - 6}$

Solution

$\because$ $n(x) = \frac{3x-4}{(x-2)(x-3)} + \frac{2(x+3)}{(x-2)(x+3)}$

domain n = R - {-3 , 2 , 3}

$\therefore$ $n(x) = \frac{3x-4}{(x-2)(x-3)} + \frac{2}{x-2}$

$\because$ L.C.M. of denominators = (x - 3)(x - 2) by multiplying the two terms of the second fraction in (x - 3)

$\therefore$ $n(x) = \frac{3x-4}{(x-2)(x-3)} + \frac{2(x-3)}{(x-2)(x-3)} = \frac{3x-4+2x-6}{(x-2)(x-3)}$

$= \frac{5x-10}{(x-2)(x-3)} = \frac{5(x-2)}{(x-2)(x-3)} = \frac{5}{x-3}$

3 *Find* **n(x) in the simplest form showing the domain of n where :**

$n(x) = \frac{12}{12x^2 - 3} + \frac{2}{2x - 4x^2}$, then find n(0) , n(-1) if possible.

Solution

∵ $n(x) = \frac{12}{12x^2 - 3} + \frac{2}{-4x^2 + 2x}$

$= \frac{12}{12x^2 - 3} + \frac{2}{-(4x^2 - 2x)}$ *(descending order) according to the powers of x*

$= \frac{12}{3(4x^2 - 1)} - \frac{2}{2x(2x - 1)} = \frac{4}{(2x + 1)(2x - 1)} - \frac{1}{x(2x - 1)}$ *(Factorize)*

domain n = $\mathbb{R} - \{\frac{-1}{2}, 0, \frac{1}{2}\}$

L.C.M of denominators = $x(2x + 1)(2x - 1)$

∴ $n(x) = \frac{4x}{x(2x + 1)(2x - 1)} - \frac{2x + 1}{x(2x + 1)(2x - 1)}$

∴ $n(x) = \frac{4x - (2x + 1)}{x(2x + 1)(2x - 1)} = \frac{4x - 2x - 1}{x(2x + 1)(2x - 1)}$

$= \frac{2x - 1}{x(2x + 1)(2x - 1)} = \frac{1}{x(2x + 1)}$

n(0) does not exist because zero ∉ the function domain of n,

$n(-1) = \frac{1}{-1 \times (-2 + 1)} = \frac{1}{-1 \times -1} = 1$

Drill

Find **n(x) in the simplest form showing its domain where :**

1. $n(x) = \frac{x - 2}{x} + \frac{3 + x}{2x}$
2. $n(x) = \frac{2x}{x + 2} + \frac{4}{x + 2}$
3. $n(x) = \frac{2}{x + 3} + \frac{x + 3}{x^2 + 3x}$
4. $n(x) = \frac{x}{x - 4} - \frac{x + 4}{x^2 - 16}$
5. $n(x) = \frac{3}{x - 1} - \frac{2}{x - 1}$
6. $n(x) = \frac{5}{x - 3} + \frac{4}{3 - x}$
7. $n(x) = \frac{x^2}{x - 1} + \frac{x}{1 - x}$
8. $n(x) = \frac{x}{x - 2} - \frac{x}{x + 2}$
9. $n(x) = \frac{x + 3}{2x} - \frac{x}{2x - 1}$
10. $n(x) = \frac{3}{x + 1} + \frac{2x + 1}{1 - x^2}$

Second: Multiplying and dividing the algebraic fractions

Think and Discuss

For each algebric fraction n (x) ≠ 0, there is a multiplicative inverse. It is the reciprocal of the fraction and denoted by n^{-1} (X).

If $n(x) = \frac{x+2}{x+5}$, then $n^{-1}(x) = \frac{x+5}{x+2}$ where the domain of n = R - {-5}, the domain of n^{-1} = R -{-2, -5} and then $n(x) \times n^{-1}(x) = 1$

From the previous, we can do a multiplication or division of two algebraic fractions as follows :

If n_1 , n_2 are two algebraic fractions where:

$n_1(x) = \frac{f_1(x)}{f_2(x)}$, $n_2(x) = \frac{f_3(x)}{f_4(x)}$ *then :*

1. $n_1(x) \times n_2(x) = \frac{f_1(x)}{f_2(x)} \times \frac{f_3(x)}{f_4(x)} = \frac{f_1(x) \times f_3(x)}{f_2(x) \times f_4(x)}$

   where $x \in$ *the common domain of the two algebraic fractions* n_1 , n_2

   i.e. $R - (Z(f_2) \cup Z(f_4))$

2. $n_1(x) \div n_2(x) = \frac{f_1(x)}{f_2(x)} \div \frac{f_3(x)}{f_4(x)} = \frac{f_1(x)}{f_2(x)} \times \frac{f_4(x)}{f_3(x)}$

   then, the domain of $n_1 \div n_2$ *is the common domain of* n_1 , n_2 , n_2^{-1}

   i.e. $R - (Z(f_2) \cup Z(f_3) \cup Z(f_4))$

Examples

4 If $f(x) = \frac{x+1}{x^2 - x - 2} \times \frac{x^2 + 3x - 10}{3x^2 + 16x + 5}$

then find **f(x) in the simplest form and identify its domain, then find f(0) , f(-1) if possible.**

Solution

$f(x) = \frac{x+1}{(x-2)(x+1)} \times \frac{(x+5)(x-2)}{(3x+1)(x+5)}$

$= \frac{(x+1)(x+5)(x-2)}{(x-2)(x+1)(3x+1)(x+5)} = \frac{1}{3x+1}$ *(The simplest form)*

the domain f = R - $\{-5 , -1 , -\frac{1}{3} , 2\}$, f(0) = 1,

f(-1) it is not exist because $-1 \notin$ the domain of f .

5 If $f(x) = \frac{x^2-9}{2x^2+3x} \div \frac{3x^2+6x-45}{4x^2-9}$

then find **n(x) in the simplest form showing the domain of n .**

Solution

$\because n(x) = \frac{x^2-9}{2x^2+3x} \div \frac{3(x^2+2x-15)}{4x^2-9}$ $\therefore n(x) = \frac{(x+3)(x-3)}{x(2x+3)} \div \frac{3(x+5)(x-3)}{(2x+3)(2x-3)}$

domain of n = $R - \{0 , -\frac{3}{2}, \frac{3}{2}, -5 , 3\}$

$\therefore n(x) = \frac{(x+3)(x-3)}{x(2x+3)} \times \frac{(2x+3)(2x-3)}{3(x+5)(x-3)}$

$= \frac{(x+3)(x-3)(2x+3)(2x-3)}{3x(2x+3)(x+5)(x-3)} = \frac{(x+3)(2x-3)}{3x(x+5)}$

Drill

Third: Find n(x) in the simplest form identifying a domain in each of the following:

1 $n(x) = \frac{x^2+x+1}{x} \times \frac{x^2-x}{x^3-1}$

2 $n(x) = \frac{x^3-1}{x^2-x} \times \frac{x+3}{x^2+x+1}$

3 $n(x) = \frac{3x-15}{x+3} \div \frac{5x-25}{4x+12}$

4 $n(x) = \frac{x^2+2x-3}{x+3} \div \frac{x^2-1}{x+1}$

Exercises 2-4

Find n (x) in the simplest form showing the domain of n :

1 $n(x) = \frac{x-6}{2x^2-15x+18} + \frac{x-5}{15-13x+2x^2}$

2 $n(x) = \frac{x^2-8x+12}{x^2-4x+4} + \frac{x^2-4x-5}{x^2-7x+10}$

3 $n(x) = \frac{x^2+2x+4}{x^3-8} - \frac{9-x^2}{x^2+x-6}$

4 $n(x) = \frac{x-3}{x^2-7x+12} - \frac{x-3}{3-x}$

5 $n(x) = \frac{x-5}{2x^2-13x+15} + \frac{x+3}{15x-18-2x^2}$

6 $n(x) = \frac{x^2-3x}{2x^2-x-6} \div \frac{2x^2-3x}{4x^2-9}$

7 $n(x) = \frac{x^2-12x+36}{x^2-6x} \times \frac{4x+24}{36-x^2}$

8 $n(x) = \frac{x^2-2x+1}{x^3-1} \div \frac{x-1}{x^2+x+1}$

9 $n(x) = \frac{x^2-2x-15}{x^2-9} \div \frac{2x-10}{x^2-6x+9}$

10 $n(x) = \frac{x^2-3x+2}{1-x^2} \div \frac{3x-15}{x^2-6x+5}$

Activity

If $n_1(x) = x + \frac{1}{x-2}$, $n_2(x) = 4x + \frac{4}{x-2}$

and $n(x) = n_1(x) \div n_2(x)$ then find :

1. domain of n (x)
2. n (x) in its simplest form
3. n (1), n (5) if possible

Unit test

First **Complete the following :**

1. The simplest form of the function f, where $f(x) = \frac{3x}{x+1} \div \frac{x}{x+1}$ is .... and its domain is ....
2. If the algebraic fraction $\frac{x-a}{x-3}$ has a multiplicative inverse of $\frac{x-3}{x+2}$, then a = ...
3. If $n_1(x) = \frac{x+1}{x-2}$, $n_2(x) = \frac{x^2+x}{x^2-2x}$ then the common domain in which $n_1 = n_2$ is ....

Second

1. Find the common domain for which $f_1(x)$ and $f_2(x)$ are equal, where :

   $f_1(x) = \frac{x^2+x-12}{x^2+5x+4}$, $f_2(x) = \frac{x^2-2x-3}{x^2+2x+1}$

2. If $f(x) = \frac{x^2-49}{x^3-8} \div \frac{x+7}{x-2}$ then find n (x) in the simplest form and identify its domain dand find f (1).
3. If $n_1(x) = \frac{x^2}{x^3-x^2}$, $n_2(x) = \frac{x^3+x^2+x}{x^4-x}$ prove that $n_1 = n_2$
4. If the domain of the function n where $n(x) = \frac{b}{x} + \frac{9}{x+a}$ is R - {0, 4}, n (5) = 2 find the values of a , b .
5. Find the function in its simplest form and identify its domain :

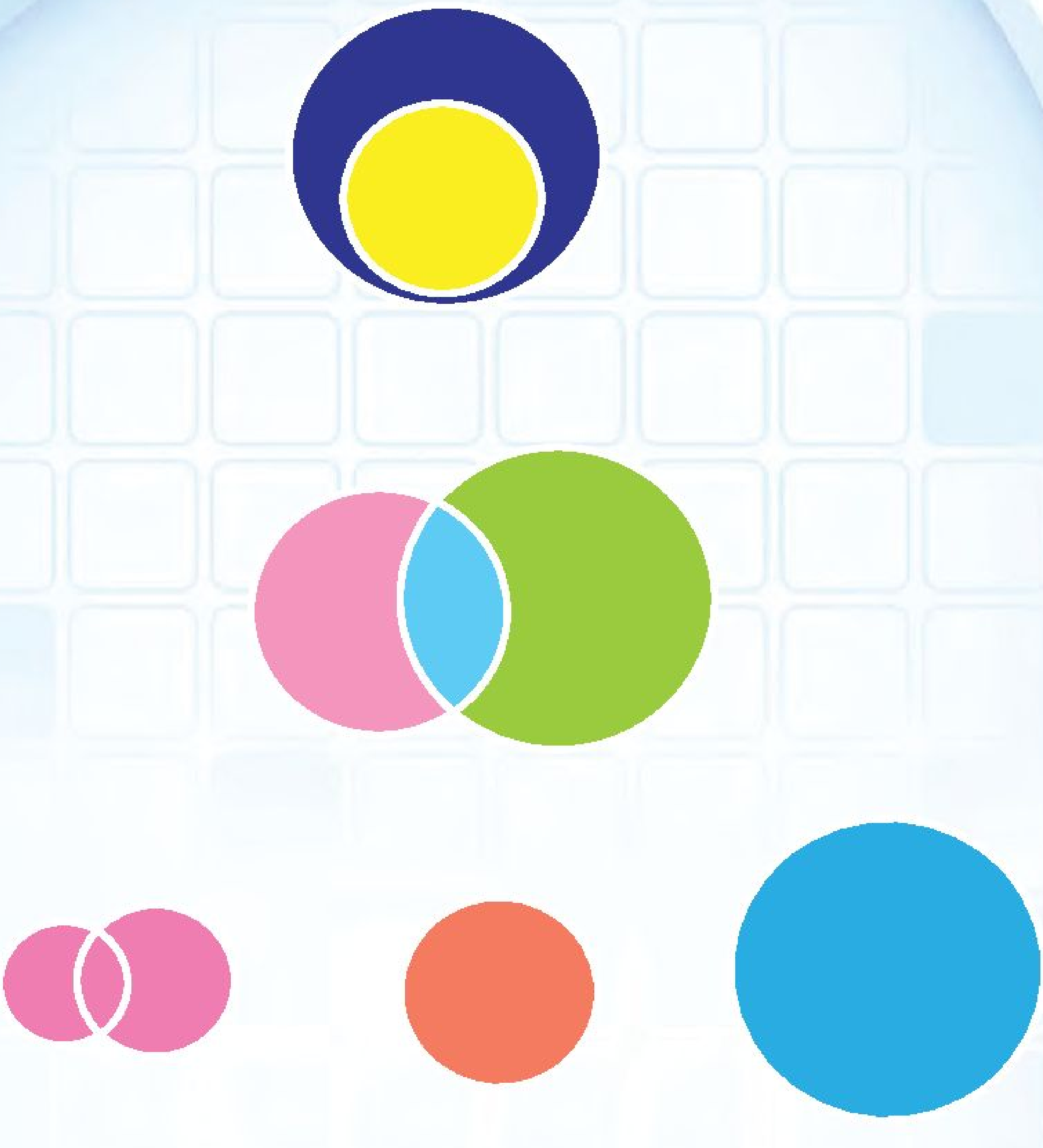
   first : $n(x) = \frac{x^2-x}{x^2-1} + \frac{x-5}{x^2-6x+5}$ **second :** $n(x) = \frac{x^3-1}{x^2-2x+1} \times \frac{2x-2}{x^2+x+1}$

6. **If** $n(x) = \frac{x^2-2x}{(x-2)(x^2+2)}$

   first : find $n^{-1}(x)$ and identify its domain. **second :** if $n^{-1}(x) = 3$ what is the value of x.

Probability

Unit 3 : Probability



3 - 1

Operation on events

What you'll learn

- ☆ Do operations on events (intersection, union).

Key terms

- ☆ Union
- ☆ Intersection
- ☆ Two mutually exclusive events.
- ☆ Venn diagram

Think and Discuss

A regular dice is rolled once randomly and the upper face is observed as :

1. Sample space (S) = { ...., ..., ..., ..., ..., ...}.
2. The event of having 7 is .... and the event is called .... and the probability of appearnce = ...
3. The event of getting a number less than 9 is .... and the event is called .... and the probability of appearance = ...
4. The event of getting a prime even number is .... and it is a subset of .... and the probability of occurrance = $\frac{\dots\dots}{\dots\dots}$

If A is an event of S i e $A \subset S$ then $P(A) = \frac{n\ (A)}{n\ (S)}$

where n (A): number of elements of the event A, n (S) is the number of elements of sample space S, and P (A) is the probability of occurring event (A).

we notice that : probability can be written as a fraction or percentage as follows :

impossible event	less likely	Equally likely as unlikely	More likely	Certain event
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
0%	25%	50%	75%	100%

Drill

1. A box contains 3 white balls and 4 red balls. If a ball is randomly drawn, then calculate the probability that the ball drawn is ...... :

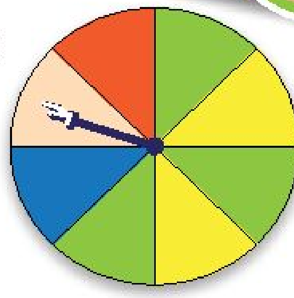
   A white. B white or red. C blue.

2 **The opposite figure is a spinner divided into eight equal colored sectors *Find*** the probability that the indicater stops on :

A the green color.

B the yellow color.

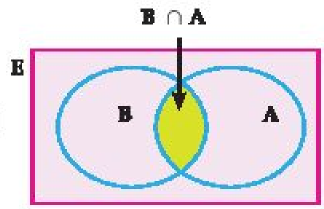
C the blue color.



Operations on events :

Events are subset of the sample space (S), so oparetions on events are similar to the operations on sets such as union and intersection. When the sample sapce (S) is considered the universal set, we can represent events and operations on the sample space by using Venn diagrams:

First: intersection

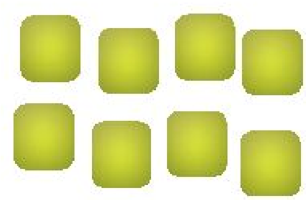


If A and B are two events from a sample space (S), then the intersection of the two events A and B which are denoted by the symbol $A \cap B$ means the events A and B occur together.

Note that : It is said that an event occured if the outcome of the experiment is an element of the elements of the set expressing this event.

Example

A set of identical cards numbered from 1 to 8 with no repetition mixed up and well, if a card is drawn randomly.



1 write down the sample space.

2 write down the following events.

A Event A : The drawn card has an even number.

B Event B : The drawn card has a prime number.

C Event C : The drawn card has a number divisible by 4.

3 Use Venn diagram to calculate the probability of :

A occurring A and B together.

B occurring A and C together.

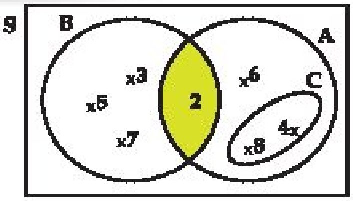
C occurring B and C together.

Solution

1 $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $n(S) = 8$

2 A $A = \{2, 4, 6, 8\}$ B $B = \{2, 3, 5, 7\}$ C $C = \{4, 8\}$

3. Use the venn diagram opposite and find :



A. The probability of the occurrence of events A and B together means A ∩ B where :

B. A ∩ B = {2} it is a one element set ∴ n (A ∩ B) = 1

∴ the probability of the occurrence of events A and B together = P (A ∩ B)

$$= \frac{n\ (A \cap B)}{n\ (S)} = \frac{1}{8}$$

C. The probability of the occurrence of the events A and C together means A ∩ C where :

A ∩ C = {4 , 8 } ∴ n (A ∩ C) = 2

∴ The probability of the occurrence of the events A and C together = P (A ∩ C)

$$= \frac{n\ (A \cap C)}{n\ (S)} = \frac{2}{8} = \frac{1}{4}$$

the probability of the occurrence of the events B and C together means B ∩ C where :

B ∩ C = ϕ (because B and C are two separate or distant sets), n (B ∩ C) = zero

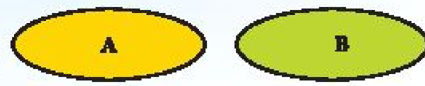
∴ The probability of the occurrence of two events B and C together = P (B ∩ C)

$$= \frac{n\ (B \cap C)}{n\ (S)} = \frac{0}{8} = \text{zero}$$

Notice that : the two events B and C cannot occur at the same time so we say A and B are mutually exclusive events.

Mutually exclusive events.

It is said that A and B are mutually exclusive events if A ∩ B = ϕ



and it is said that a set of events are mutually exclusive if every pair is mutually exclusive.

Drill

A regular dice is rolled once :

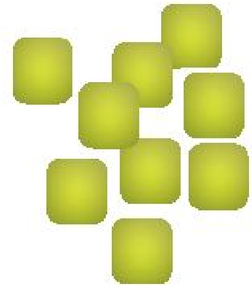
1. Write down the sample space.
2. Write the following events:
   - A. A = the event of getting an even number.
   - B. B = the event of getting an odd number.
   - C. C = the event of getting an a prime even number.
3. Find the following probabilities of :
   - A. The occurrence of two events A and B together.
   - C. The occurrence of two events A and C together.

Second : Union

If A and B are two events from the sample sapce (S) then the union of the two events which is denoted by the symbol A ∪ B means the occurrance of the two events A or B or both i.e occurance of at least one event.

Example

1 9 identical cards numbered from 1 to 9 a card was drawn randomally.



First Write down the sample space.

Second Write down the following events :

A Getting a card with an even number.

B Getting a card with a number divisible by 3.

C Getting a card with a prime number greater than by 5.

Third use the venn diagram to calculate the probability of :

a Occurrence of A or B

b Occurrence of A or C

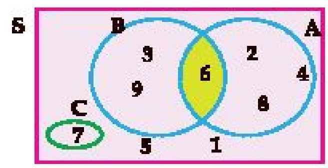
c Find $P(A) + P(B) - P(A \cap B)$, $P(A \cup B)$ what do you notice ?

Solution

First $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $n(S) = 9$

Second $A = \{2, 4, 6, 8\}$, $n(A) = 4$, $B = \{3, 6, 9\}$, $n(B) = 3$, $C = \{7\}$, $n(C) = 1$

Third **In the venn opposite diagram :**



A Occurrence of A or B means $A \cup B$

where : $A \cup B = \{2, 3, 4, 6, 8, 9\}$, $n(A \cup B) = 6$

$\therefore$ probability of the occurrence of A or B $= P(A \cup B) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{6}{9} = \dfrac{2}{3}$

B Occurrence of A or C means $A \cup C$ they are two distant sets.

then $A \cup C = \{2, 4, 6, 7, 8\}$, $n(A \cup C) = 5$

$\therefore$ probability of the occurrence of A or C $= P(A \cup C) = \dfrac{n(A \cup C)}{n(S)} = \dfrac{5}{9}$

C $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{9}$, $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{9}$

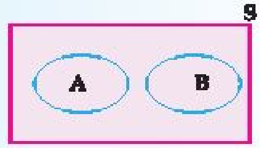
$A \cap B = \{6\}$ $\therefore P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{9}$

$P(A) + P(B) - P(A \cap B) = \dfrac{4}{9} + \dfrac{3}{9} - \dfrac{1}{9} = \dfrac{2}{3}$ **(1)**

, $P(A \cup B) = \dfrac{2}{3}$ **(2)**

from **(1), and (2)** we get $P(A) + P(B) - P(A \cap B) = P(A \cup B)$

Remark: From the opposite figure, A and B are mutually exclusive events from the sample space S, then :



1. $A \cap B = \phi$
2. $P(A \cap B) = \frac{\text{number of elements of } \phi}{\text{number of elements of } S} = \frac{\text{Zero}}{\text{number of elements of } S} = \text{Zero}$

Notic that A and B are mutually exclusive events.

Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ but $(A \cap B) = \text{zero}$

$\therefore P(A \cup B) = \frac{4}{9} + \frac{1}{9} - \text{zero}$

$= \frac{5}{9}$ As previously found

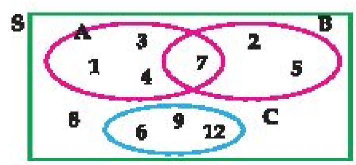
i.e if A and B are two mutually exclusive events then $P(A \cup B) = P(A) + P(B)$

Drill

1. If A and B are two events in the sample space of a random experiment complete :

A	B	C
$P(A) = 0.2$	$P(A) = 0.55$	$P(A) = \ldots\ldots\ldots$
$P(B) = 0.6$	$P(B) = \frac{3}{10}$	$P(B) = \frac{1}{4}$
$P(A \cap B) = 0.3$	$P(A \cap B) = \ldots$	$P(A \cap B) = \text{zero}$
$P(A \cup B) = \ldots\ldots$	$P(A \cup B) = \frac{13}{20}$	$P(A \cup B) = 0.9$

2. use the venn opposite diagram to find:



A. $P(A \cap B)$, $P(A \cup B)$
B. $P(A \cap C)$, $P(A \cup C)$
C. $P(B \cap C)$, $P(B \cup C)$

Exercises 3-1

First If A and B are two events in the sample space of a random experiment : complete:

1. $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$ then find $P(A \cup B)$
2. $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cup B) = \frac{5}{8}$ then find $P(A \cap B)$
3. $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ then find $P(A \cup B)$ in the following cases :

A. $P(A \cap B) = \frac{1}{8}$

B. A , B are mutually exclusive events.

Second **Choose the correct answer :**

1. If A and B are two mutually exclusive events then P (A ∩ B) equals

   A ϕ B zero C 0.56 D 1

2. If A ⊂ B , then P (A ∪ B) equals:

   A zero B P (A) C P (B) D P (A ∩ B)

3. If a regular coin is tossed once, then the probability of getting head or tail is:

   A 0 % B 25 % C 50 % D 100%

4. If a die is rolled once, then the probability of getting an odd number and even number together equals:

   A zero B $\frac{1}{2}$ C $\frac{3}{4}$ D 1

Third

1. A box contains 12 balls 5 of them are blue, 4 are red and the left are white. A ball is randomly drawn from the box. Find the probability that the drawn ball is :

   A blue B not red C blue or red

2. A bag contains 20 identical cards numbered from 1 to 20, a card is randomly drawn. Find the probability that the number on the card is:

   A divisible by 5. B an odd number and divisible by 5.

3. If A, and B are two events from a sample space of a random experiment, and $P(B) = \frac{1}{12}$, $P(A \cup B) = \frac{1}{3}$ then find P (A) If :

   A A and B are mutually exclusive events.

   B B ⊂ A

4. A card is randomly drawn from 20 identical card numbered from 1 to 20 calculate the probability that the number on the card is :

   A divisible by 3.

   B divisible by 5.

   C divisible by 3 and divisible by 5.

   D divisible by 3 or divisible by 5.

3 - 2

Complementary event and the difference between two events

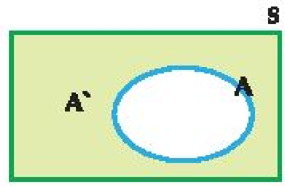
What you'll learn

- The concept of the complementary even
- The concept of the difference between two events.

Key terms

- complementary event
- difference between two events.

Think and Discuss



In the venn diagram opposite :

If S is the universal set, $A \subset S$

then the complementery set of A is A`

Complete :

1. $A \cup A^{`}$ = ......... , $A \cap A^{`}$ = ...
2. If S = {1, 2, 3, 4, 5, 6, 7} A = {2, 4, 6} then: A` = {.............}.

From the previous, we notice that : If S is the sample space of a random experiment and one ball is drawn from a box having identical balls numbered from 1 to 7 and observing the number on it.

A is the event of getting even number : A = {2, 4, 6}

A` is the event of getting an odd number : A` = {1, 3, 5, 7} and it is a complementry event to A .

The complementry event :

i.e : If $A \subset S$ then A` is the complementry event to event A

where $A \cup A^{`} = S$ , $A \cap A^{`} = \phi$

i.e the event and the complementary event are two mutually exclusive events.

Drill

If S the sample space of a random experiment, $A \subset S$, A` is the complementry event to the event A and S = { 1 , 2 , 3 , 4 , 5 , 6 }.

Complete the following table and record your observation.

event A	event A`	P (A)	P (A`)	P (A) + P (A`)
{2, 4, 6}				
	{3, 6}			
{5}				
{1, 2, 3, 4, 5, 6}				

From the previous table, notice that : $P(A) + P(A') = 1$ **then :** $P(A') = 1 - P(A)$, $P(A) = 1 - P(A')$

Note : $P(A) + P(A') = P(S) = 1$

Example

1 A classroom contains 40 students. 18 of them read Al-Akhbar newspaper, 15 read Al Ahram news paper and 8 read both newspapers. If a student is selected randomly calculate the probability that the student :

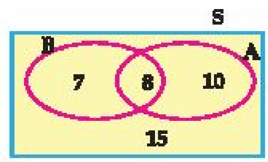
A reads Al-Akhbar newspaper

B doesn't read Al-Akhbar newspaper

C reads Al-Ahram newspaper

D reads both newspaper.

Solution



Let the event A be reading Al Akhbar newspaper and the event B reading Al Ahram newspaper. then $A \cap B$ is the event of reading both newspapers.

then $n(S) = 40$, $n(A) = 18$, $n(B) = 15$, $n(A \cap B) = 8$

A event A : Read Al Akhbar newspaper then $P(A) = \frac{n(A)}{n(S)} = \frac{18}{40} = \frac{9}{20}$

B Does not read Al Akhbar is the complementary event of the event A and it is A`.

$$\therefore P(A') = \frac{\text{number of elements of s set } A'}{n(S)} = \frac{15+7}{40} = \frac{22}{40} = \frac{11}{20}$$

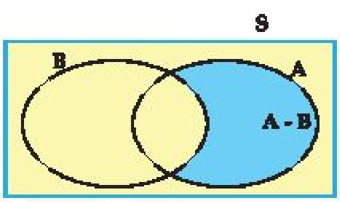
Another solution : $P(A') = 1 - P(A) = 1 - \frac{9}{20} = \frac{11}{20}$

C event B : read Al Ahram newspaper then : $P(B) = \frac{n(B)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

D event $A \cap B$ means reading both newspaper

$$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{8}{40} = \frac{1}{5}$$

Think : Does the event reading Al Akhbar newspaper mean to read Al Akhbar newspaper only? Explain your answer.



Notice that : The event of reading Al Akhbar newspaper is represented by venn opposite diagram by set A while the event of reading Al Akhbar only but not other newspaper is represented by the set A - B **and read as A difference B**

The difference between two events

If A , B are events of s, then A-B is the event of the occurrence of A and the non-occurrence of B, i.e., the occurrence of the event A only. Note that : $(A - B) \cup (A \cap B) = A$

Drill **In the previous example Find :**

1. the probability that the student reads Al - Akhbar newspaper only.
2. the probability that the student reads Al - Ahram newspaper only.
3. the probability that the student reads Al - Akhbar only or Al - Ahram only.

General Exercises

1. A set of cards numbered from 1 to 30 and well mixed. If a card is randomely drawn. Find the probability that the card drawn is carrying :
   - A. a number multiple of 6.
   - B. a number multiple of 8.
   - C. a number multiple of 6 and 8 together.
   - D. a number multiple of 6 or 8.
2. If A and B are two mutually exclusive events from the space sample of a random experiment such that the probability of occurrence of events B is three times the probability of occurrence of event A. The probability of occurrence of one at least of the two events is 0.64. Find the probability of occurrence of each of the two events A and B .
3. If A and B are two events from the sample space of a random experiment If $P(A) = 0.5$, $P(A \cup B) = 0.8$ and $P(B) = X$. then calculate the value of X If :
   - A. A, and B are two mutually exclusive events.
   - B. $P(A \cap B) = 0.1$
4. For an irregular dice the probability of the appearence of the numbers 1, 2, 3, 4 and 5 are equal and the probability of the appearance of the number 6 is 3 times the probability of the appearance of the number 1, if the cube is rolled once calculate the probaility of :
   - A. the appearence of number 6
   - B. the appearence of a prime odd number
5. Threeplayers A, B and C join in the competition of weight lifting. If the probability that the first player wins is equal to twice the probability of the second player to win and the probability that the player B wins is equal to the probability that the player C wins. Find the probability that player B or C wins, taking into consideration that one player will win.
6. S is a sample space of a random experiment were its outcomes are equal if A and V are two events from S. If the number of outcomes that leads to the occurrence of the event Ais equal to 13, and the number of all the possible outcomes of the random experiment is equal to 24 and $P(A \cup B) = \frac{5}{6}$, $P(B) = \frac{5}{12}$ Find :
   - A. the probability of occurrence of the event A.
   - B. the probability of occurrence of the event A and B.

7. 45 students participated in some sports activity, 27 of them are members in the school football team, 15 in basketball team and 9 in both football and basketball team. A student is randomly selected. Represent this situation using a venn diagram, then find the probability that the selected student is :

   A. a member in the football team.
   B. a member in the basketball team.
   C. a member in the basketball team and football team.
   D. a member does not participate in any team.

Activity

In a survey of 6000 birth cases in a provience selected randomly. Researchers paid much attention to find a relation between mother's age when she gives brith and the place where she lives. the following table shows the number of births in urban and rural villages:

Mother age	Place of living	
	Urban	rural villages
Less than 20 years	120	240
From 20 years to less than 22 years	240	360
From 22 years to less than 30 years	1740	1440
From 30 and more	1500	360

1. What do you infer from the table ?
2. If the event A expresses the mother who gave birth and lives in the urban area and the event B expresses the mother who gave birth and lives in the rural area and whose age is not more than 22 years, Find :
   A. P (A)
   B. P (B)
3. Represent the sets A and B using the venn diagram, then Find:
   A. $P(A \cap B)$
   B. $P(A \cup B)$
   C. $P(A - B)$
   D. $P(A \cup B)`$
4. Predict the number of births if the mother lives in the rural area and aged 30 years or more. take into consideration that the number of births is 9000 in the provience.
5. Write a report about over population rate and its side effects for the national income, showing the vital role the mass media should do to reduce the growth of population.

Unit test

First **Complete :**

1. If the probability of occurance of an event A is 65%, Then the probability that event does not occure equals ....

2. If P (A) = P (A`), then P (A) = ...

3. If A, and B are two matually exclusive events and P (A) $= \frac{1}{3}$, P (A ∪ B) = $\frac{7}{12}$ then P (B) = ...

4. If A and B are two events from the sample space of the random experiment and P (A) = 0.7 , P (A - B) = 0.5 , then P (A ∩ B) = ...

Second

1. A box contains 20 balls which have the same shape size and weight and well mixed. 8 of them are red, 7 are white and the rest of the balls are green, A ball in drawn randomly. Find the probability that the drawn ball is .... :

   A red **B** white or green **C** not white

2. Abag contains 30 identical cards mixed well A card is randomly drawn: Find the probability the number on the card is :

   A divisible by 3. **B** divisible by 5.

   C divisible by 3 and 5. **D** divisible by 3 or 5.

3. If A and B are two events from the sample space of a random experiment and P (A) = 0.8 , P (B) = 0.7 , P (A ∩ B) = 0.6 then find :

   A the probability of non occurance of the events A and B together.

   B the probability of non occurance of at least one of the events.

   D the probability of non occurance of one of the events but not the other.

Geometry

Unit (4): The Circle

Drivers are to be familiar with traffic signs well and to distinguish between them.

Search in the different knowledge resources (traffic department - library - internet) for traffic signs.

4 - 1

Basic Definitions and Concepts

What you'll learn

- The basic concepts related to the circle.
- The concept of axis of symmetry in the circle.

Key terms

- Circle
- Surface of a circle
- Radius
- chord
- Diameter
- Axis of symmetry in a circle

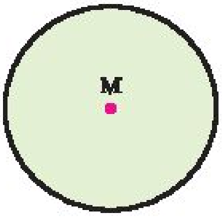
Think and Discuss

Yousef used the program, **Google Earth** , on his computer to study the geography of Egypt. Yousef noticed some green, circular areas next to the desert areas so, he asked his father about them.

The father Said: You learn that a drop of water means the source of life. Therefore, we should minimize the consumption of water in order to irrigate the land by the central irrigation method (sprinkle irrigation) in which, the wheels of the irrigation machine circle around a fixex point which draws those circles.

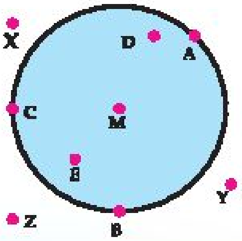
1. How can you draw the circle of a football field?
2. What is your role in minimizing the consumption of water?

The circle: is the set of points of a plane which are at constant distance from a fixed point in the same plane. The fixed point is called the centre of the circle and the constant distance is called the radius length.



The circle is usually denoted by its center. So we say, circle M to mean the circle which its center is point M, as in the figure opposite.

When drawing circle M in a plane, it divides the points of the plane in to three sets of points as in the figure, and they are :



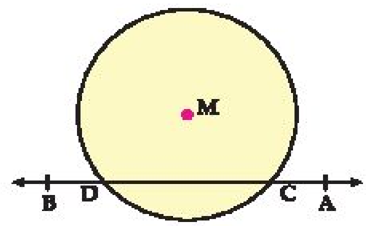
1. The set of points inside the circle like points: M, D, E, ...
2. The set of points on the circle like points: A, B, C, ...
3. The set of points outside the circle like points: X, Y, Z, ...

Surface of the circle :

set of points of the circle ∪ the set of points inside the circle

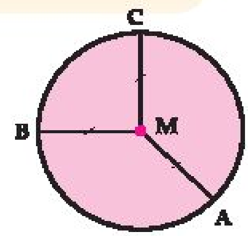
Drill

Inthe figure opposite, complete :



1. $\overleftrightarrow{AB}$ ∩ circle M = ...
2. $\overleftrightarrow{AB}$ ∩ surface of circle M = ...
3. M ∉ circle M, M ∈ ....

Radius of a circle : is a line segment with one endpoint at the center and the other endpoint on the circle.



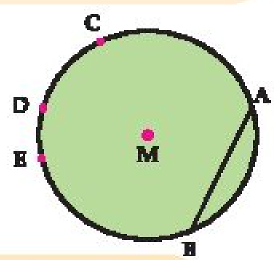
In the figure opposite $\overline{MA}$, $\overline{MB}$, $\overline{MC}$ are radii for circle M where :

MA = MB = MC = radius length of the circle (r)

Two circles are congruent if their radii are equal in length.

The chord : is a line segment whose end points are any two points on the circle.

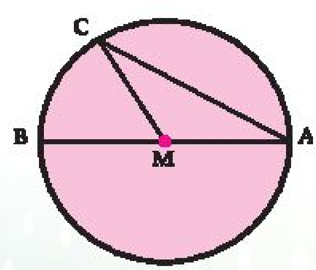
Drill



In the figure opposite :
Draw all the chords of the circle which pass through the pairs of points A, B, C, D, E.

Diameter : is the chord passing through the center of the circle.

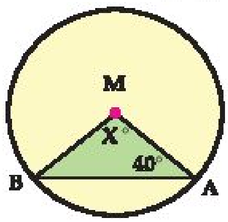
Drill



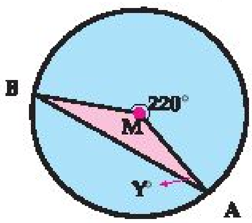
1. Which chord in the following figure is a diameter in circle M ?
2. What are the number of diameters in any circle ?
3. To prove that the diameter of a circle is its largest chord in length, complete :
   In the triangle A M C : AM + MC > ....
   In circle M : CM = BM (radii)
   Thus : AM + ... > ....   ∴ AB > ....

4. If the radius length of a circle = r then the diameter length = ... and the perimeter of the circle = ... , area of the circle = ...

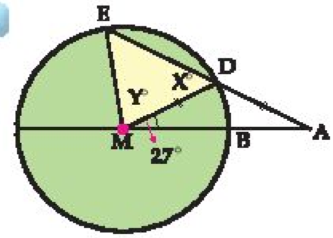
5. **In each of the following figures find the value of the used symbol in measuring :**

a


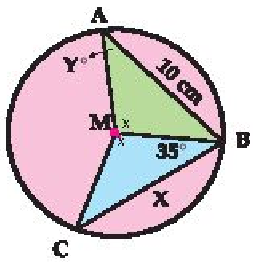
X = ...

b


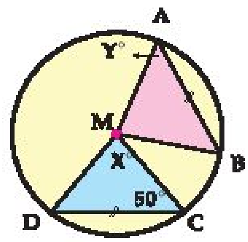
Y = ...

c


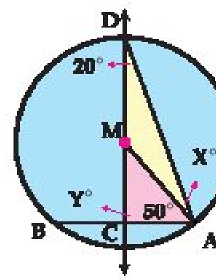
X = ..., Y = ...

d


X = ..., Y = ...

e


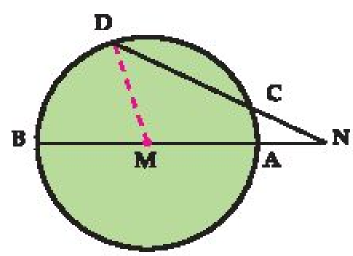
X = ..., Y = ...

f


X = ..., Y = ...

Example 1

In the figure opposite : $\overline{AB}$ is a diameter in circle M. $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$. ***Prove that :*** NB > ND.



Solution

Draw a radius $\overline{MD}$ in Δ N M D :

M N + M D > N D

∵ M B = M D (radii)

∴ M N + M B > N D

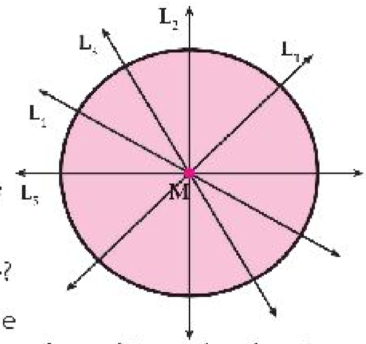
∴ N B > N D (Q.E.D.)

Drill

In the previous example, prove that : N C > N A.

Symmetry in the circle

Activity 1



1. Draw circle M on a transparent paper using compasses.
2. Draw the straight line L_1 passing through the center of the circle and dividing it in to two arcs.
3. Fold the paper around the straight line L_1, what do you notice?
4. Draw another straight line L_2 passing through the center of the circle and, then fold the paper around it repeat this step a number of times by drawing the straight lines L_3, L_4, ..., ... what do you notice in each case ?

From the previous activity we deduce that :

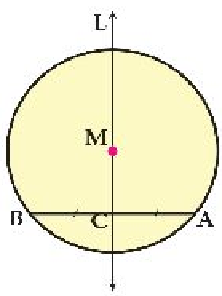
> ***Any straight line passing through the center of a circle is an axis of symmetry of it.***

Think : What is the number of axes of symmetry in the circle?

Activity 2

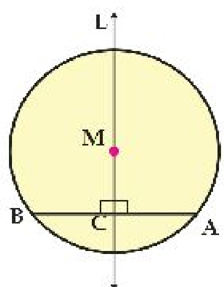
Study each of the following figures (as given in the drawing). What do you deduce?

1



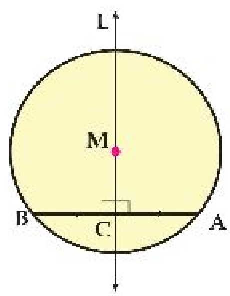
Deduction : ....

2



Deduction : ....

3



Deduction : ....

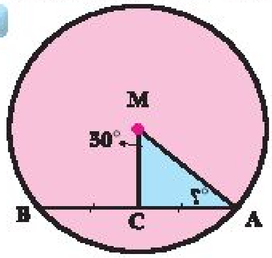
Important Corollaries

From 1	the straight line passing through the center of the circle and the midpoint of any chord of it is perpendicular to this chord.
From 2	the straight line passing through the center of a circle and perpendicular to any chord of it bisects this chord.
From 3	the perpendicular bisector of any chord of a circle passes through the center of the circle.

Drill

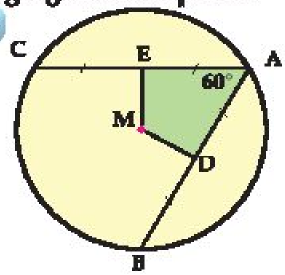
1 **M circle in each of the following figures complete :**

A



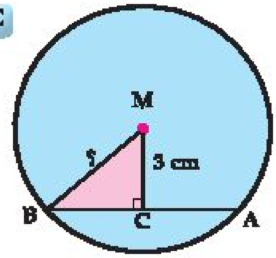
m (∠ M A C) = ...

B



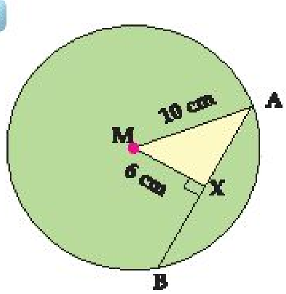
m (∠ D M E) = ...

C



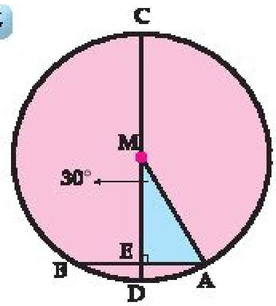
If A B = 8 cm,
then M B = ...

D



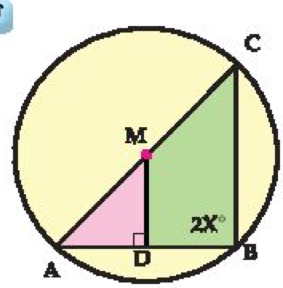
A B = ...

E



If A B = 10 cm,
then C D = ...

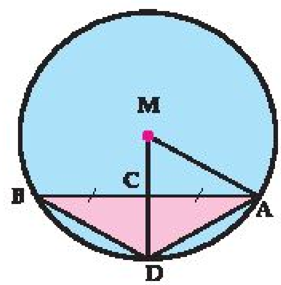
F



X = ...

2 **In the figure opposite :** M circle with radius length 13 cm, $\overline{AB}$ is a chord of length 24 cm, C is the midpoint of $\overline{AB}$, $\overrightarrow{MC}$ ∩ circle M = {D}.

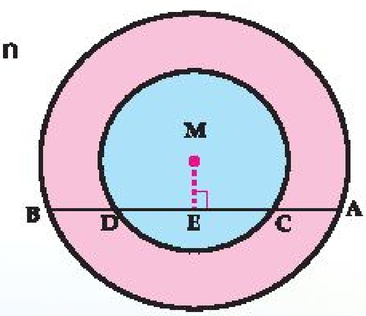
Find the area of the triangle A D B.



Example 2

In the figure opposite: two concentric circles M, $\overline{AB}$ is a chord in the larger circle intersecting the smaller circle at C and D :

Prove that : AC = BD.



Solution

Given : $\overline{AB}$ ∩ the smaller circle = {C, D}

R. T. P : A C = B D

Construction : Draw $\overline{ME} \perp \overline{AB}$ to intersect it at E .

Proof : **In the larger circle** $\overleftrightarrow{ME} \perp \overline{AB}$ $\therefore$ EA = EB **(1) (corollary)**

In the smaller circle $\overleftrightarrow{ME} \perp \overline{CD}$ $\therefore$ EC = ED **(2) (corollary)**

By subtracting (2) from (1), we get:

EA - EC = EB - ED $\therefore$ AC = BD **(Q . E . D.)**

Drill

In the figures opposite :
What are the line segments that are equal in length? Explain your answer.

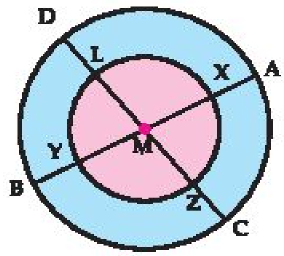


fig (2)

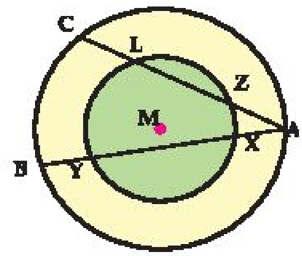
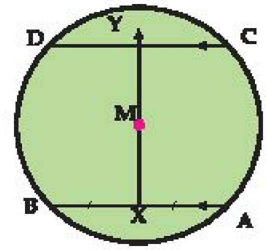


fig (1)

Example 3

In the figure opposite : M circle, $\overline{AB}$ // $\overline{CD}$, X is the midpoint of $\overline{AB}$ $\overrightarrow{XM}$ is drawn to intersect $\overline{CD}$ at Y. ***Prove that*** Y is the midpoint of $\overline{CD}$



Solution

Given : $\overline{AB}$ // $\overline{CD}$, AX = BX

R . T . P : CY = DY

Proof : $\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overleftrightarrow{MX} \perp \overline{AB}$

$\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{XY}$ intersects them $\therefore$ m ($\angle$ DYX) = m ($\angle$ AXY) = 90° alternating angles

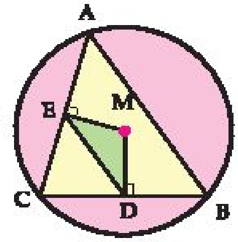
$\therefore$ $\overleftrightarrow{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$ **(Q.E.D)**

Drill

$\overline{AB}$ and $\overline{CD}$ are two parallel chords in circle M. AB = 12 cm , CD = 16 cm. Find the distance between those two chords if the radius length of circle M equals 10 cm. Are there any other answers? Explain your answer.

Think If $\overline{AB}$ and $\overline{CD}$ are two parallel chords in a circle where AB > CD, which chord is closer to the center of the circle ? Explain your answer.

Example 4



In the figure opposite : ABC triangle is an inscribed triangle inside a cricle with center M, $\overline{MD} \perp \overline{BC}$, $\overline{ME} \perp \overline{AC}$.

Prove that : First : $\overline{ED} \parallel \overline{AB}$

Second : Perimeter $\Delta CDE = \frac{1}{2}$ Perimeter ΔABC

Solution

Given : $\overline{MD} \perp \overline{BC}$ and $\overline{ME} \perp \overline{AC}$

R . T . P : First: $\overline{ED} \parallel \overline{AB}$

Second : Perimeter $\Delta CDE = \frac{1}{2}$ Perimeter ΔABC

Proof :

First: $\because \overline{MD} \perp \overline{BC}$ $\therefore$ D is the midpoint of $\overline{BC}$ **(1)**

$\because \overline{ME} \perp \overline{AC}$ $\therefore$ E is the midpoint of $\overline{AC}$ **(2)**

in ΔABC, D is the midpoint of $\overline{BC}$ and E is the midpoint of $\overline{AC}$

$\therefore \overline{DE} \parallel \overline{AB}$ **(Q.E.D 1)**

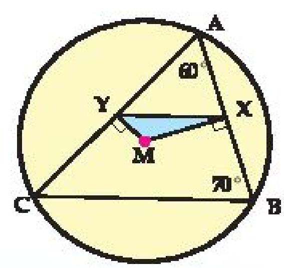
$DE = \frac{1}{2} AB$ **(3)**

Second : **From (1), (2), (3) :**

$\therefore$ Perimeter $\Delta CDE = CD + CE + ED$ $= \frac{1}{2} CB + \frac{1}{2} AC + \frac{1}{2} AB$

$= \frac{1}{2} (CB + AC + AB) = \frac{1}{2}$ Perimeter ΔABC

Drill



In the figure opposite : In circle M, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$,

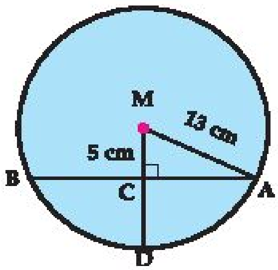
m ($\angle$ A) = 60°, m ($\angle$ B) = 70°.

Find : the measures of the angles of the triangle M X Y.

Exercises (4-1)

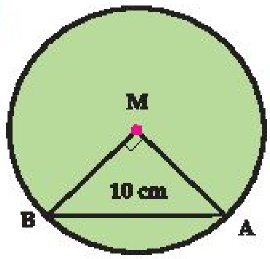
1 **M is a circle in each of the following figures. complete :**

A



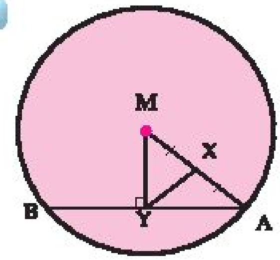
A B = ...
C D = ...

B



m (∠ A) = ...
M A = ...

C



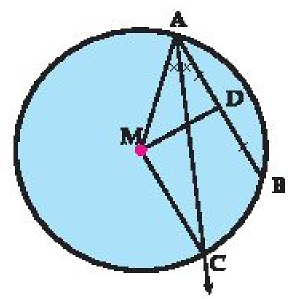
XY = 7 cm, ($\pi = \frac{22}{7}$)
Area of the circle =...... cm²

2 **In the figure opposite :** $\overline{AB}$ is a chord of circle M,

$\overrightarrow{AC}$ bisects ∠ B A M and intersects circle M at C.

If D is the midpoint of $\overline{AB}$,

Prove that : $\overline{DM} \perp \overline{CM}$.

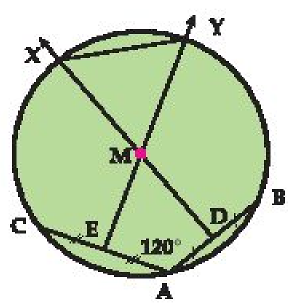


3 **In the figure opposite :** $\overline{AB}$ and $\overline{AC}$ are two chords in circle M that include an angle measuring 120°,

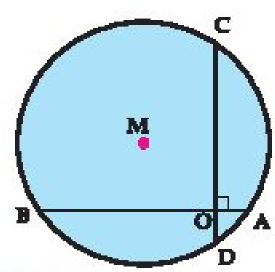
D, and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

$\overrightarrow{DM}$ and $\overrightarrow{EM}$ are drawn to intersect the circle at X and Y respectively.

Prove that : the triangle X Y M is an equilateral triangle.



4 **In the figure opposite :** Circle M has a radius length of 7 cm, $\overline{AB}$ and $\overline{CD}$ are two perpendicular and intersecting chords at point O. If AB = 12 cm and C D = 10 cm, ***Find*** the length of $\overline{MO}$



4 - 2

Positions of a point, a straight Line and a circle with respect to a circle.

What you'll learn

- Identifying the position of a point with respect to a circle.
- Position of a straight line with respect to a circle.
- Relation of the tangent with the radius of a circle.
- Position of a circle with respect to another circle.
- Relation of the line of centers with the common chord and the common tangent.

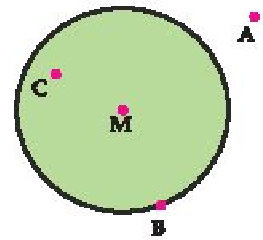
Key terms

- Point is outside a circle.
- Point is on the circle
- Point is inside a circle.
- Two distant circles.
- Two intersecting circle.
- Two circles touching
- Common tangent
- Line of centers
- Common chord

First: Position of a point with respect to a circle.

Think and Discuss

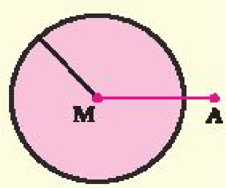
In the figure opposite, circle M divides the points of the plane in to three sets of points.



1. How can you determine the position of the points: A, B, and C with respect to circle M ?
2. What is the relation between (MA, r), (MB , r) and (MC , r) ?

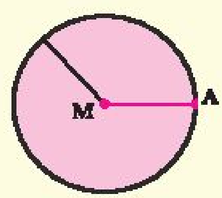
If M circle with radius length r and A was a point on the circle plane, then:

1 A is outside the circle



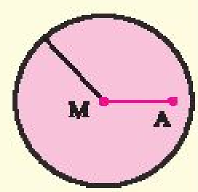
So : MA > r
and vise versa

2 A is on the circle



So : MA = r
and vise versa

3 A is inside the circle



So : MA < r
and vise versa

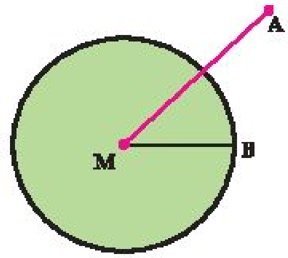
Drill

If M circle with radius length = 4 cm and A is a point in its plane, Complete :

1. *IF:* MA = 4 cm, then A is .... circle M, because ....
....
2. *IF:* MA = $2\sqrt{3}$ cm, then A is ....circle M, because ....
....
3. *IF:* MA = $3\sqrt{2}$ cm, then A is .... circle M, because ....
....
4. *IF:* MA = zero, then A is...............circle M and represented by ....

Example 1

If circle M with radius length 5 cm, A is a point in its plane and MA = 2x - 3 cm. ***Find*** the values of X, if A is located outside the circle.



Solution

$\because$ Point A is located outside the circle M $\therefore$ M A > 5 So : 2X - 3 > 5 i.e. 2X > 8 $\therefore$ X > 4

Drill

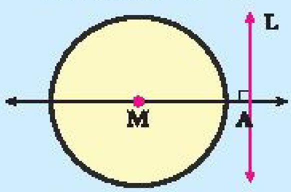
From the previous example, find the value of X in the following cases :

1. MA = 2x + 1, point A on the circle.
2. MA = 8x – 27, point A inside the circle.

Second: Position of a straight line with respect to a circle :

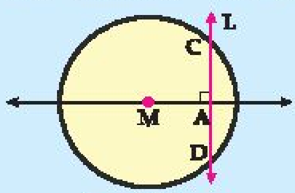
If M circle with radius length of r, L is a straight line on its plane, $\overleftrightarrow{MA} \perp$ L where $\overleftrightarrow{MA} \cap L = \{A\}$, Then:

1 the straight line L is located outside the circle M
L ∩ circle M = ϕ



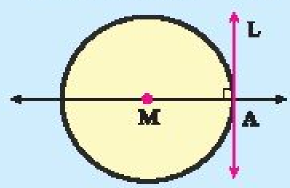
So : MA > r
and vise verse

2 the straight line L is a secant to the circle M
L ∩ circle M = {C, D}



So : MA < r
and vise verse

3 the straight line L is tangent to circle M
L ∩ the circle = {A}



So : MA = r
and vise verse

Think : In each of the following cases, Find L ∩ surface of circle M.

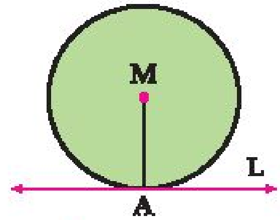
Drill

If M circle with radius length 7 cm and $\overline{MA} \perp$ L where A ∈ L . Complete the following:

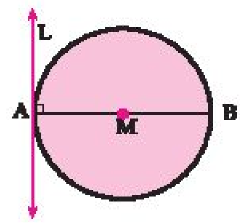
1. ***If*** MA = 4 $\sqrt{3}$ cm Then the straight line L ....
2. ***If*** MA = 3 $\sqrt{7}$ cm Then the straight line L ....
3. ***If*** 2 MA - 5 = 9 Then the straight line L ....
4. ***If*** the straight line L intersects circle M and MA = 3X -5 Then X ∈ ....
5. ***If*** the straight line L tangent to circle M and MA = $X^2 - 2$ Then X ∈ ....

Important facts

1. A tangent to a circle is perpendicular to the radius at its point of tangency.



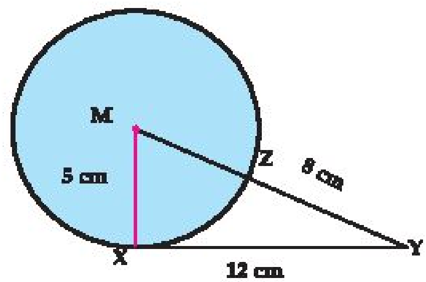
2. If a straight line is perpendicular to a diameter of a circle at one of its endpoints, then it is a tangent to the circle.



Think

1. **How many tangents can be drawn to circle M ?**
   First : from a point on the circle.
   Second: from a point outside the circle.
2. What is the relation between the two drawn tangents to the circle from the two end points of any diameter in it ?

Example 2



In the figure opposite: M circle with radius length of 5 cm, XY = 12 cm, $\overline{MY} \cap$ circle M = {Z} and ZY = 8 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to circle M at X.

Solution

$\because$ $\overline{MY} \cap$ circle M = {Z} $\therefore$ MY = M Z + ZY

$\because$ M Z = M X = 5 cm (radii) $\therefore$ MY = 5 + 8 = 13 cm

$\because$ $(MY)^2 = (13)^2 = 169$, $(MX)^2 = (5)^2 = 25$, $(XY)^2 = (12)^2 = 144$

$\therefore$ $(MX)^2 + (XY)^2 = 25 + 144 = 169 = (MY)^2$

$\therefore$ m ($\angle$ M XY) = 90° **(The converse of the pythagorean theorem)**

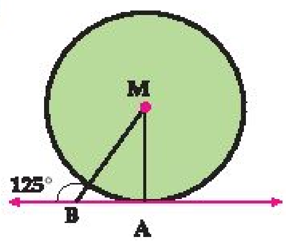
$\because$ $\overleftrightarrow{XY} \perp \overline{MX}$

$\therefore$ $\overleftrightarrow{XY}$ is a tangent to the circle at X. **(Q.E.D)**

Drill

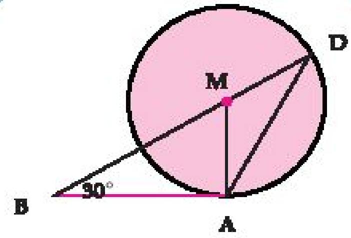
1 **M circle is in each of the following figures and $\overleftrightarrow{AB}$ is a tangent : Complete :**

A



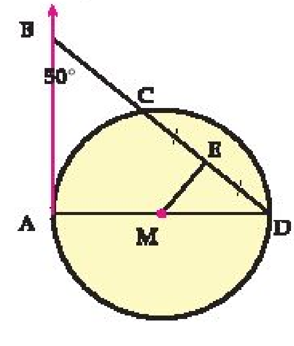
m (∠ A M B) = ...

B



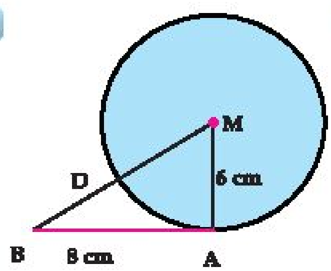
m (∠ A D B) = ...

C



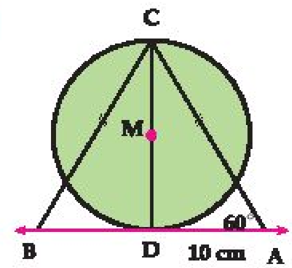
m (∠ A M E) = ...

D



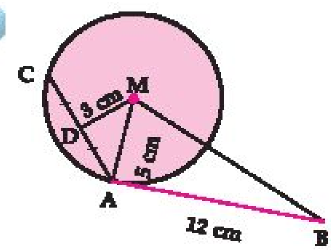
D B = ... cm

E



Perimeter Δ A B C = ... cm

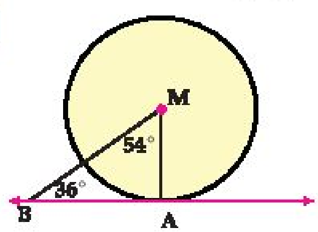
F



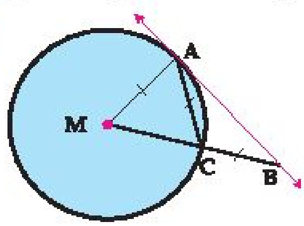
Perimeter of the figure A B M D = ... cm

2 **In each of the following figures, explain why $\overleftrightarrow{AB}$ is a tangent to circle M :**

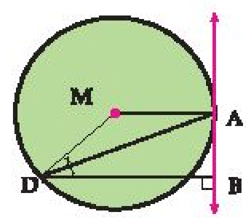
A



B

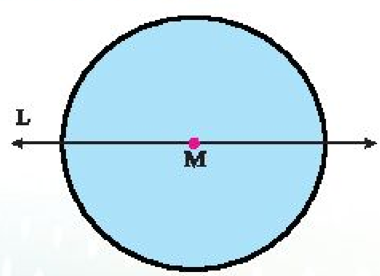


C



Third : Position of a circle with respect to another circle.

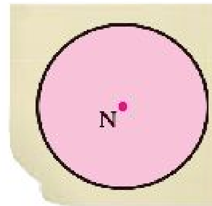
Activity



1 Draw a circle with center M and with an appropriate radius length = r_1 cm.

2 Draw one of the axes of symmetry of circle M. Let it be the straight line L as in the figure opposite.

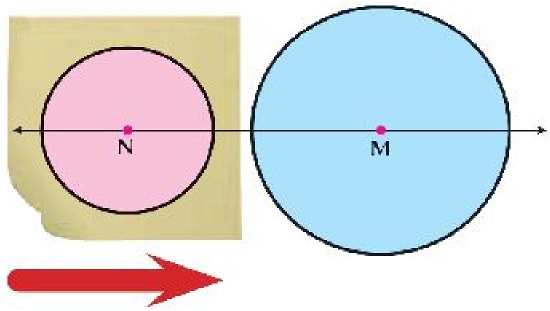
3 On a transparent paper draw a circle with center N and with an appropriate radius length $= r_2$ cm where $r_2 < r_1$.



4 Put the transparent paper where point N belongs to the straight line L.

Notice that : the straight line $L = \overleftrightarrow{MN}$ is called $\overleftrightarrow{MN}$ the line of centers of the two circles M and N and it is an axis of symmetry for both of them.

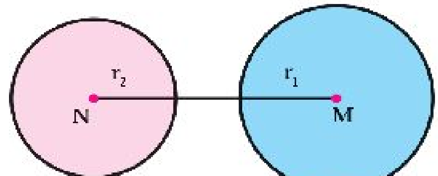
5 Move the transparent paper towords circle M where N remains $\in$ L to see different positions of the two circles. Measure the two circles in relation to each other. Mesure length of $\overline{MN}$ in each case.
What is the relation between the length of $\overline{MN}$ **(the distance between the centers of the two circles M and N)**, $r_1 + r_2$ or $r_1 - r_2$ in each position.



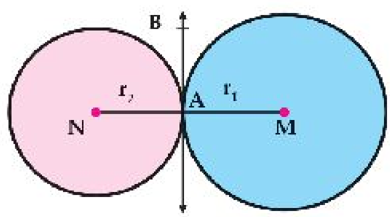
Drill

If M and N are two circles on the plane, their two radii are r_1 and r_2 respectively where $r_1 > r_2$. Complete:

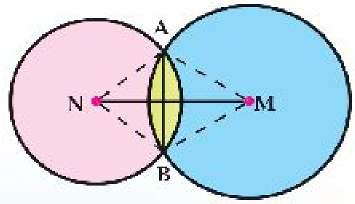
1 If $MN > r_1 + r_2$, then $M \cap N =$ ....,
surface of circle $M \cap$ surface of circle N
= .... and the two circles are distant.



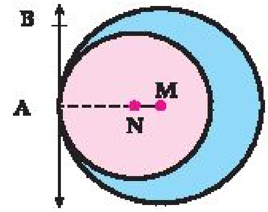
2 If $MN = r_1 + r_2$, then $M \cap N =$ ....,
surface of circle $M \cap$ surface of circle N = ...
and the two circles are touching externally.



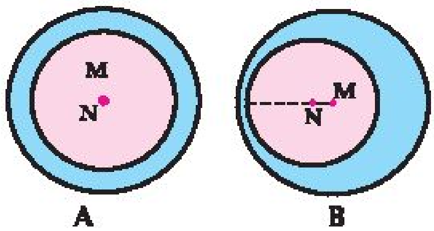
3 If $r_1 - r_2 < MN < r_1 + r_2$,
then $M \cap N =$ ....,
surface of circle $M \cap$ surface of circle N = the surface of the yellow area and the two circles are intersecting.



4. If: $MN = r_1 - r_2$, then $M \cap N$ = ..., surface of circle M ∩ surface of circle N = ... and the two circles are touching internally.



5. If: $MN < r_1 - r_2$ and then $M \cap N$ =.........., surface of circle M ∩ surface of circle N = ... and the two circles are intersecting as in figure .... when M N = zero, the two circles are concentric. as in figure ....



Corollaries

1. The line of centers of two touching circles passes through a point of tangency and is perpendicular to the common tangent.
2. The line of centers of two intersecting circles is perpendicular to the common chord and bisects it.

Example 3

Two circles M and N with radii length of 9 cm and 4 cm respectively. Show the position of each of them with respect to the other in the following cases:

A MN = 13 cm	B M N = 5 cm	C M N = 3 cm
D M N = zero	E M N = 10 cm	F M N = 15 cm

Solution

∵ $r_1 = 9$ cm, $r_2 = 4$ cm ∴ $r_1 + r_2 = 13$ cm and $r_1 - r_2 = 5$ cm

A M N = 13 cm ∴ $MN = r_1 + r_2$ ∴ the two circles are touching externally.

B M N = 5 cm ∴ $MN = r_1 - r_2$ ∴ the two circles are touching internally.

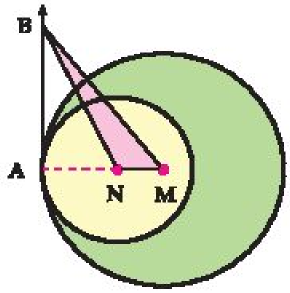
C M N = 3 cm ∴ $MN < r_1 - r_2$, $MN \neq 0$ ∴ circle N is inside circle M.

D M N = zero ∴ the two circles are concentric.

E M N = 10 cm ∴ $r_1 - r_2 < MN < r_1 + r_2$ ∴ the two circles are intersecting.

F M N = 15 cm ∴ $MN > r_1 + r_2$ ∴ the two circles are distant.

Example 4



M and N are two circles with radii length of 10 cm and 6 cm respectively and are both touching internally at A, $\overleftrightarrow{AB}$ is a common tangent for both at A. If the area of the triangle B M N = 24 cm^2 ***Find*** the length of $\overline{AB}$.

Solution

∵ The two circles are touching internally at A ∴ $A \in \overleftrightarrow{MN}$, $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$

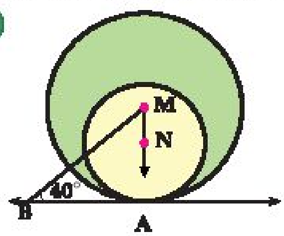
then the length of $\overline{AB}$ is the height of the triangle B M N whose base is $\overline{MN}$

where : M N = 10 - 6 = 4 cm (why ?)

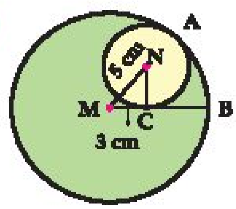
Area Δ B M N = $\frac{1}{2}$ × M N × A B ∴ 24 = $\frac{1}{2}$ × 4 × A B ∴ A B = 12 cm

Drill

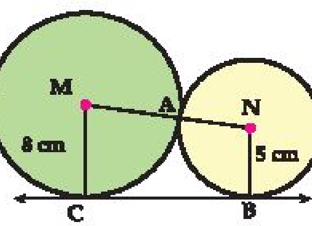
In each of the following figures the circles are touching two - by - two. Use the information of each figure and complete :

1. 

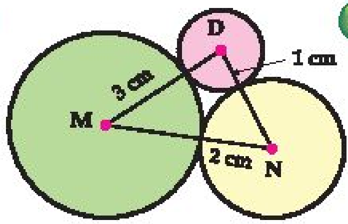
m (∠ B M N) = ... °

2. 

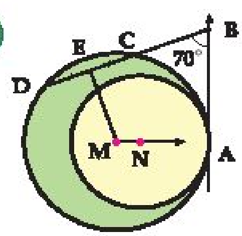
B C = ... cm

3. 

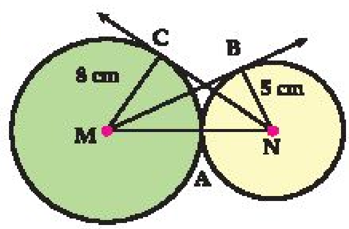
B C = ... cm

4. 

m (∠M D N) = ... °

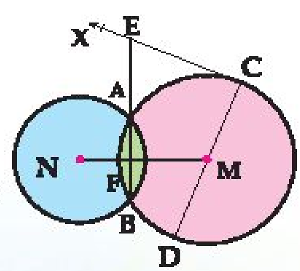
5. 

m (∠ E M N) = ...°

6. 

M B = ... cm,
N C = ... cm

Example 5



M and N are two intersecting circles at A and B, $\overline{CD}$ is a diameter in circle M and $\overrightarrow{CX}$ is a tangent to the circle M at C where $\overrightarrow{CX} \cap \overrightarrow{BA}$ = {E}, $\overleftrightarrow{MN} \cap \overline{AB}$ = {F}. ***Prove that:*** m (∠ DMN) = m (∠ CEB).

Solution

Given : circle M ∩ circle N = {A, B}, $\overline{CD}$ is a diameter in circle M and $\overrightarrow{CX}$ is a tangent to circle M.

R. T. P: Prove that m (∠ D M N) = m (∠ C E B).

Proof : ∵ the line of centers is perpendicular to the common chord.

∴ $\overleftrightarrow{MN} \perp \overline{AB}$ i.e m(∠ A F M) = 90°

∵ $\overline{CD}$ is a diameter in circle M and $\overrightarrow{CX}$ is a tangent at C

∴ $\overrightarrow{CX} \perp \overline{CD}$ i.e m (∠ E C D) = 90°

∴ m (∠ C E F) + m(∠ C M F) = 360° - (90° + 90°) = 180° **(why ?)**

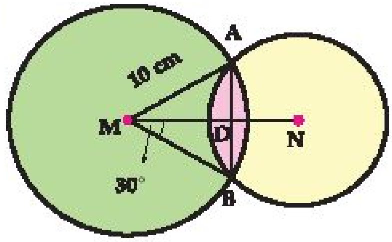
∵ m (∠ D M F) + m (∠ C M F) = 180°

∴ m (∠ D M N) = m (∠ C E F) **(Q.E.D)**

Drill

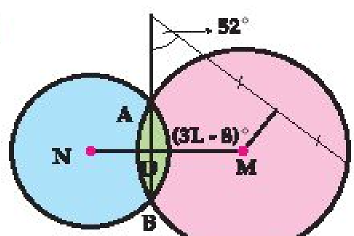
1 **In each of the following figures M and N are two intersecting circles at A and B Complete :**

A



A B = ... cm

B



L = ...

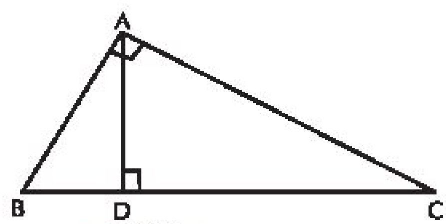
Notice that:

A B C is a right angled triangle at A. If $\overline{AD} \perp \overline{BC}$ then :

$(AB)^2 = BD \times BC$ ***(Euclidean theorem)***

, $(AD)^2 = DB \times DC$ ***(Corollary)***

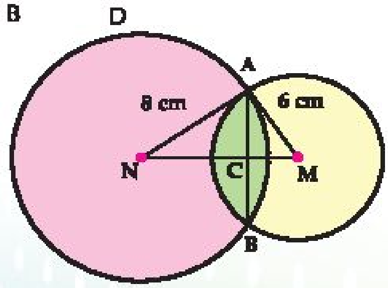
, $AD \times BC = AB \times AC$ ***Why ?***



2 **In the figure opposite :** M and N are two intersecting circles at A, B

$\overline{MN} \cap \overline{AB}$ = {C}, A M = 6 cm, A N = 8 cm and $\overline{MA} \perp \overline{AN}$.

Find the length of $\overline{AB}$



Exercises (4-2)

1 **Complete to make the following statements correct :**

A If the radius length of the circle is 8 cm, the straight line L is distant from its center by 4 cm, then L is ....

B If the surface of circle M ∩ surface of circle N = {A} then the two circles M and N are ....

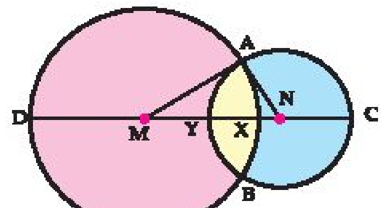
C M and N are two intersecting circles. The two radii length are 3 cm and 4 cm respectively then : M N ∈ ....

D If the area of the circle M = 16 π cm^2, A is a point on its plane where M A = 8 cm, then A is .... circle M.

E circle M with radius length of 6 cm, if the straight line L is outside the circle then the distance of the center of the circle from the straight line L ∈ ....

F A circle with diameter length (2X + 5)cm, the straight line L is distant from its center by (X + 2) cm then the straight is ....

2 **In the figure opposite :** M and N are two intersecting circles at A and B, the two radii length are 8 cm and 6 cm respectively and XY = 4 cm. Study the figure, then answer the following questions:



A ***Complete :*** Y M = ... cm , C X = ... cm and C D = ... cm

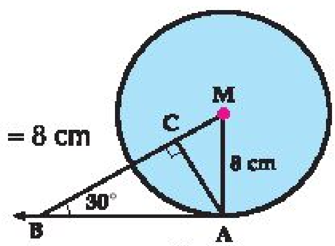
B Is the perimeter of the triangle A N M = the length of $\overline{CD}$? Explain your answer.

C What is the measure of angle N A M ?

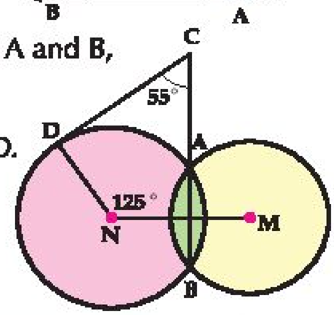
D Find the area of the triangle N A M.

E What is the length of the common chord $\overline{AB}$?

3 **In the figure opposite:** $\overrightarrow{AB}$ is a tangent to the circle M at A and MA = 8 cm m (∠ A B M) = 30°. ***Find*** the length of each : $\overline{AB}$ and $\overline{AC}$



4 **In the figure opposite :** M and N are two intersecting circles at A and B, C ∈ $\overrightarrow{BA}$, D ∈ the circle at N and M (∠ MND) = 125° m (∠ B C D) = 55°. ***Prove that*** $\overleftrightarrow{CD}$ is a tangent to circle N at D.



5 $\overline{AB}$ is a diameter in circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$, are two tangents of the circle M, $\overrightarrow{CM}$ intersects the circle M at X and Y and intersects $\overleftrightarrow{BD}$ at E. ***Prove that*** C X = Y E.

6 M and N are two intersecting circles at A and B MA = 12 cm, NA = 9 cm, and MN = 15 cm. ***Find*** the length of $\overline{AB}$.

4 - 3

Identifying the circle

Think and Discuss

- Why is a compass used in drawing a circle?
- What is the axis of the straight segment ?
- Is the center of the circle located on the axis of any chord in it?
- How can you draw (identify) a circle on a plane?

A circle can be drawn (identified) with given terms:

1. Center of the circle.
2. Radius length of the circle.

What you'll learn

- How to draw a circle passing through a given point.
- How to draw a circle passing through two given points.
- How to draw a circle passing through three given points.

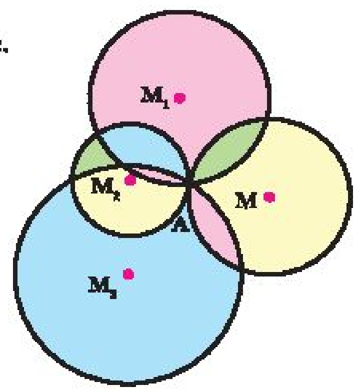
Key terms

- Circumcircle

First: Drawing a circle passing through a given point :

Given : A is a given point on the plane.

R.T.P: Draw a circle passing through point A.



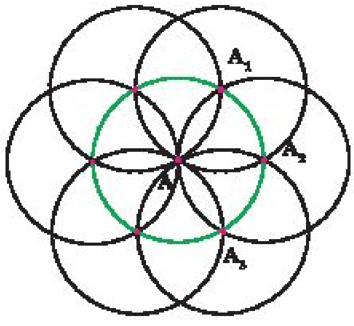
Construction :

1. Take any choosen point as M on the same plane.
2. State the tip of the compass at M and with an opening equalling MA draw the circle M. The circle M passes through point A.
3. State the tip of the compass at another point M_1 and with an opening equalling M_1A draw circle M_1. The circle M_1 passes through point A.
4. Repeat the previous work and note :

For each choosen point (center of the circle) it is possible to draw a circle passing through point A.

- What are the number of points on the plane? What are the number of circles that can be drawn and pass through point A ?
- If the radii of these circles are equal in length, where are their centers located ?

From the previous we deduce that:



1. An infinite number of circles can be drawn passing through a given point as A.
2. If the radii of these circles are equal in length then their centers are located on a congruent circle and its center is point A.

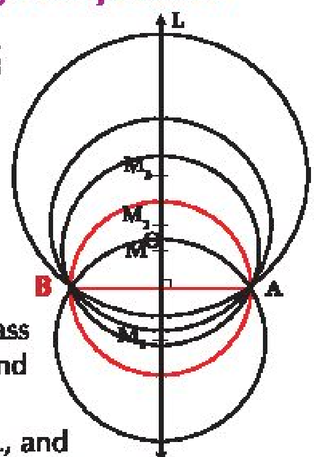
Drill

If L is a straight line on the plane; A is a given point where $A \in L$. Use the geometric tools and draw a circle passing through point A, with radius length 2 cm. How many circles can be drawn? (do not erase the arces).

Second: Drawing a circle passing through two given points:

Given: A and B are two given points in the plane.

R.T.P: Draw circle M passing through the two points A and B i.e $\overline{AB}$ is a chord in circle M.



Construction:

1. Draw the straight segment $\overline{AB}$.
2. Draw the straight line L, the axis of $\overline{AB}$ where $L \cap \overline{AB} = \{F\}$.
   (the center of the circle is on the axis of the chord $\overline{AB}$).
3. Take any chosen point M where $M \in L$, state the tip of the compass at M and with an opening equalling M A, draw the circle M to find that it passes through point B.
4. State the tip of the compass at another point as M_1 where $M_1 \in L$, and with an opening equalling M_1, A, draw the circle M_1 where it passes through point B.
5. Repeat the previous work and note :

For each chosen point E on the axis of $\overline{AB}$ (center of the circle), it is possible to draw a circle passing through the two points A and B.

- What is the number of points of the straight line L? What is the number of circles that can be drawn andpass through the two points A and B ?
- What is the radius length of the smallest circle that can be drawn to pass through the two points A and B ?
- Can two circles intersect at more than two points ?

From the previous, we deduce that :

1. An infinite number of circles can be drawn to pass through two given points like A and B.
2. The radius length of the smallest circle can be drawn in order to pass through the two points A and B is equal to $\frac{1}{2}\ \overline{AB}$.
3. Two circles can not be intersected in more than two points.

Drill

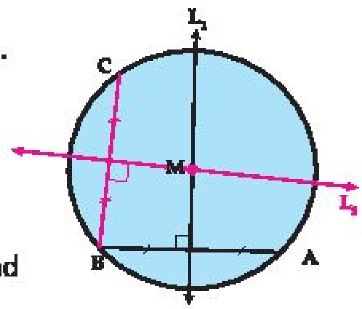
Using your geometric tools and draw $\overline{AB}$ with length 4 cm then draw on one figure :

1. A circle passing through the two points A and B and its diameter length is 5 cm. What are the possible solutions?
2. A circle passing through the two points A and B and its radius length is 2 cm. What are the possible solutions?
3. A circle passing through the two points A and B and its diameter length is 3 cm. What are the possible solutions?

Third : Drawing a circle passing through three given points:

Given: A, B and C are three given points on the plane.

R.T.P: Draw circle M passing through the three points A, B and C.

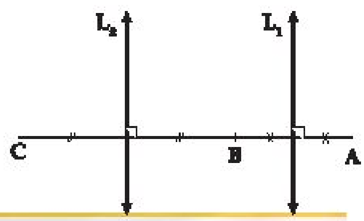


Construction:

1. Draw the straight line L_1 axis of $\overline{AB}$ thus $M \in L_1$.
2. Draw the straight line L_2 axis of $\overline{BC}$ thus $M \in L_2$.
3. If $L_1 \cap L_2 = \{M\}$, state the tip of the compass at point M and with an opening equalling MA. Draw the circle M. You will find it passing through the two points B and C.
4. If $L_1 \cap L_2 = \phi$, can you identify the position of point M ? Explain your answer.

Notice that :

If A, B, and C are collinear then $L_1 // L_2$ and $L_1 \cap L_2 = \phi$
A circle cannot be drawn passing through the three points A, B, and C.



From the previous, we deduce that :

There is one and only one circle which passes through three noncollinear points.

Drill

Using the geometric tools and draw the triangle A B C in which AB = 4 cm, BC = 5 cm and CA = 6 cm, Draw circle passing through the points A, B and C. What is the kind of triangle ABC with respect to the measures of its angles ? Where is the center of the circle located with respect to the triangle?

Corollaries

Corollary (1) *The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.*

It is said to be that a triangle is inscribed in a circle if its vertices are on the circle.

Corollary (2) *The perpendicular bisectors of the sides of a triangle intersect at a point which is the center of the circumcircle of the triangle.*

Exercises (4-3)

1. **Draw the triangle X Y Z in Which X Y = 5 cm, Y Z = 3 cm Z X = 7 cm then draw the circumscribed circle about the triangle X Y Z .**
   - A. What is the kind of the triangle XY Z with respect to its angles measures?
   - B. Where is the center of the circle with respect to this triangle ?
2. Draw the right angled triangle ABC at B where AB = 4 cm and BC = 3 cm, then draw the circumscribed circle about this triangle. Where is the center of the circle with respect to the sides of this triangle?
3. Draw the equilateral triangle A B C of a side length 4 cm. Draw the circumcirle of the triangle A B C .
   - A. Locate the position of the center of the circle with respect to : heights of the triangle medians of the triangle bisectors of the angles.
   - B. How many axes of symmetry are there in the equilateral triangle ?

4 - 4

The relation between the chords of a circle and its center

What you'll learn

- Deducing the relation between the chords of a circle and its center.
- How to solve problems related to the relation between the chords of a circle and its center.

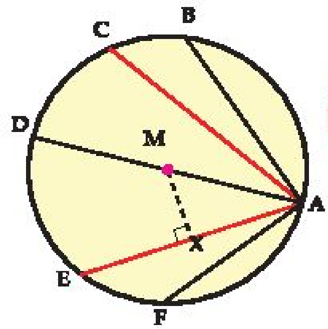
Key terms

- Equal chords.
- Congruent circles

Think and Discuss

In the figure opposite :

A is a point on circle M the chords $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$ were inscribed in it .



1. What is the relation between the length of the chord and its distance from the center of the circle ?
2. If the chords are equal in length, what can you conclude ?
3. If the chords are equidistant from the center of the circle, what do we expect ?

Notice that :

The distance of chords $\overline{AE}$, from the center of circle M equal M X where X is the midpoint of the chord $\overline{AE}$, in circle M which its radius length is r.

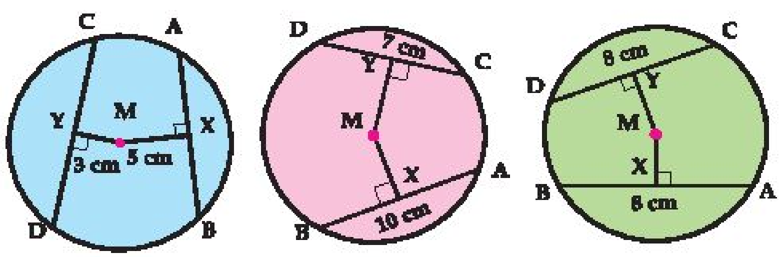
Thus : $(MX)^2 + (AX)^2 = (AM)^2 = r^2$ **(constant expression)**

i.e :

> ***The closer the chord is from the center of the circle, the longer its length is and vise versa.***

Drill

1 Complete by using the relation (> , < and =) :



A B .... C D M X .... MY M X .... MY

2 **In the figure opposite $MF < ME$,** ***Complete*** **:**

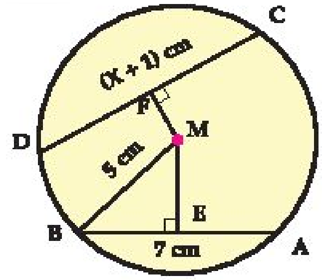
$\because$ $MF < ME$ $\therefore$ $CD >$ ………

$\therefore$ $X + 1 >$ ……… $X >$ ………

$\because$ $\overline{CD}$ is a chord in circle M $\therefore$ $CD \leqslant$ ………

$\therefore$ $X \leqslant$ ……… Thus ……… $< X \leqslant$ ………

i.e : $X \in$ ………



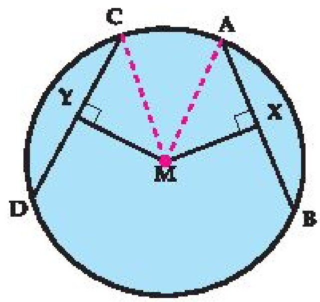
Theorem

If chords of a circle are equal in length, then thay are equidistant from the center.

Given: $AB = CD$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

R.T.P: Prove that $MX = MY$.

Construction: Draw $\overline{MA}$, $\overline{MC}$.



Proof: $\because$ $\overline{MX} \perp \overline{AB}$ $\therefore$ $AX = \frac{1}{2} AB$.

$\because$ $\overline{MY} \perp \overline{CD}$ $\therefore$ $CY = \frac{1}{2} CD$.

$\because$ $AB = CD$ $\therefore$ $AX = CY$.

$\because$ the two triangles AXM and CYM, both have :

$$\begin{cases} AM = CM \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \\ AX = CY \quad \text{(Proof)} \end{cases}$$

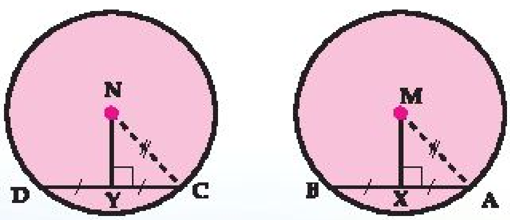
$\therefore$ $\Delta AXM \equiv \Delta CYM$ We get : $MX = MY$ **(Q.E.D.)**

Corollary

In congruent circles, chords which are equal in length, are equidistant from the centers

In the figure opposite :

The two circles M and N are congruent $AB = CD$, $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$, then : $MX = NY$.



Drill

Study the figure then complete :

A *If :*

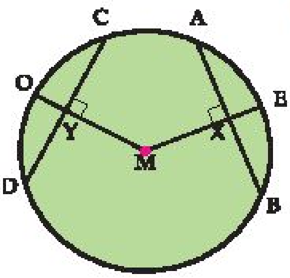
A B = C D,

then :

M X =.........

∵ M E =.........

∴ E X =.........



B *If :*

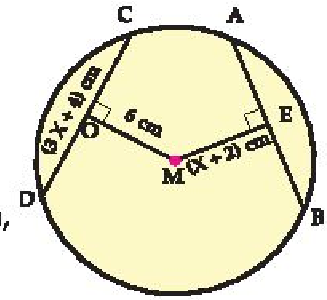
A B = C D,

then :

M E =.........

∴ X =......... cm,

C D =......... cm



C *If :*

AB = CD,

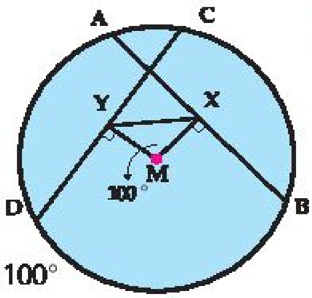
then :

M X =.........

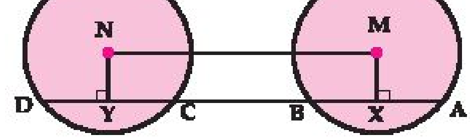
in Δ M XY:

∵ m (∠ X MY) = 100°

∴ m(∠ M XY) =.........°



D



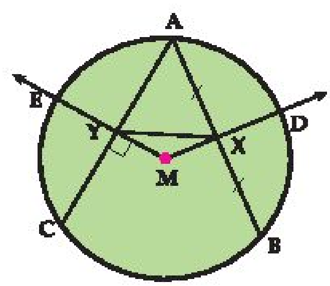
If : M and N are two congruent circles A B = C D

then : M X = ...

and the figure MXYN ....

Example 1

$\overline{AB}$ and $\overline{AC}$ are two equal chords in length in circle M and X is the midpoint of $\overline{AB}$, $\overrightarrow{MX}$ intersects the circle at D, $\overrightarrow{MY} \perp \overline{AC}$ intersects it at Y and intersects the circle at E.



Prove that : First : X D = Y E.

Second: m (∠ Y X B) = m (∠ XY C)

Given : A B = A C, X is the midpoint of $\overline{AB}$, $\overrightarrow{MY} \perp \overline{AC}$

R.T.P : prove that :

First : X D = Y E ***Second:*** m (∠ Y X B) = m (∠ XY C)

Proof : ∵ X is the midpoint of $\overline{AB}$ ∴ $\overrightarrow{MX} \perp \overline{AB}$.

∵ A B = A C, $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{AC}$ ∴ M X = MY

∵ M D = M E = r

∴ M D - M X = M E - MY ∴ X D = Y E **(Q.E.D 1)**

in Δ M XY ∵ M X = MY ∴ m (∠Y X M) = m (∠ XY M) (1)

∵ $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{AC}$ ∴ m (∠ M X B) = m (∠ MY C) = 90° (2)

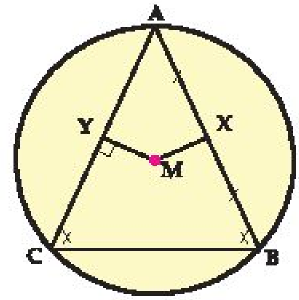
From (1) and (2) we get: m (∠Y X B) = m (∠ XY C) **(Q.E.D 2)**

Drill

In the figure opposite : Triangle A B C is inscribed in circle M, in which :

m (∠ B) = m (∠ C), X is the midpoint of $\overline{AB}$, $\overline{MY}$ ⊥ $\overline{AC}$.

Prove that : M X = MY



Converse of the theorem

In the same circle (or in congruent circles) chords which are equidistant from the center (s) are equal in length

Drill

Study the figure then complete :

1

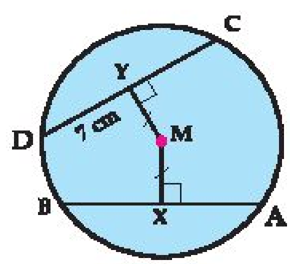
If :

M X = MY ,

Y D = 7 cm,

Then :

A B = ... cm



2

If :

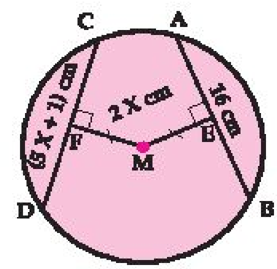
M E = M F,

Then :

C D = ...

∴ X = ...,

E M = ... cm , A M = ... cm



3

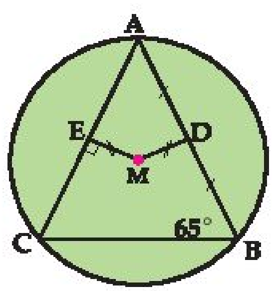
If :

M D = M E et

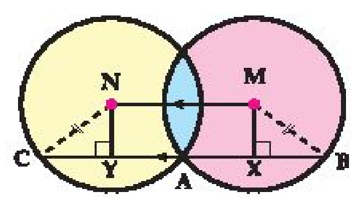
m (∠ B) = 65°,

Then :

m (∠ A) = ...°



4



∵ $\overleftrightarrow{MN}$ // $\overleftrightarrow{BC}$ ∴ M X = ...

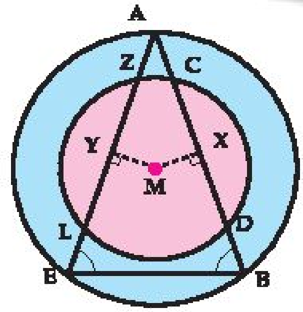
∵ the two circles M, and N .... ,

A ∈ $\overline{BC}$ ∴ A B = ...

Examples

2 Two concentric circles M, $\overline{AB}$ is a chord in the larger circle and intersects the smaller circle at C and D, $\overline{AE}$ is a chord in the larger circle and intersects the smaller circle at Z and L.

If m($\angle$ABE) = m($\angle$AEB), then ***prove that :*** CD = ZL.



Solution

Given: m ($\angle$ A B E) = m ($\angle$ A E B)

R.T.P: prove that C D = Z L

Construction: Draw $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

Proof : In $\triangle$ ABC : $\because$ m($\angle$ A B E) =m ($\angle$ A E B) $\therefore$ A B = A E.

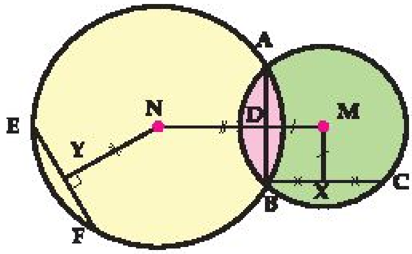
In the larger circle : AB = AE. (proof) $\therefore$ MX = MY (theorem)

$\because$ In the smaller circle MX = MY: (proof)

$\therefore$ CD = ZL ((Converse of the theorem)) **(Q.E.D.)**

3 **In the figure opposite :** M and N are two intersecting circles at A and B.,

$\overleftrightarrow{MN} \cap \overleftrightarrow{AB}$ = {D}, X is the midpoint of $\overline{BC}$, $\overline{NY} \perp \overline{EF}$, M X = M D, NY = N D. ***Prove that :*** B C = E F.



Solution

Given: X is the midpoint of $\overline{BC}$, $\overline{NY} \perp \overline{EF}$, MX = MD , and NY = ND.

R.T.P: BC = EF.

Proof : $\because$ $\overleftrightarrow{MN}$ is the line of centers, $\overline{AB}$ is a common chord for the two circles M and N

In circle M : $\because$ X is the midpoint of $\overline{BC}$

$\therefore$ $\overline{MX} \perp \overline{BC}$, $\overline{MD} \perp \overline{AB}$, MX = MD

$\because$ BC = AB (Converse of the theorem) **(1)**

In circle N : $\because$ $\overline{NY} \perp \overline{EF}$, $\overline{ND} \perp \overline{AB}$ and NY = ND

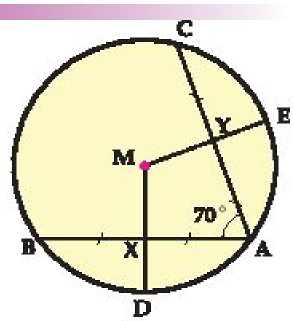
$\therefore$ EF= AB (Converse of the theorem) **(2)**

From (1) and (2) we get : BC = EF

Think If M and N are two congruent circles and intersecting at A and B. Is $\overleftrightarrow{AB}$ an axis to $\overline{MN}$? Explain your answer.

Exercises (4-4)

1 **In the figure opposite :** $\overline{AB}$, and $\overline{AC}$ are two chords equal in length in circle M and X is, the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, m ($\angle$ C A B) = 70°.



A ***Calculate*** m ($\angle$ DME). **B** ***Prove that :*** X D = Y E.

2 $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in circle M, X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$, m ($\angle$ M XY) = 30°

Prove that : **First :** MXY is an isosceles triangle. **Second:** AXY is an equilateral angle.

3 $\overline{AB}$ and $\overline{AC}$ are two chords in circle M, $\overline{MX} \perp \overline{AB}$, Y is the midpoint of $\overline{AC}$, m ($\angle$ ABC) = 75°, M X = MY.

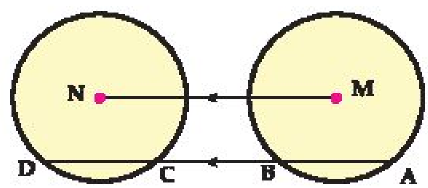
A ***Find*** m ($\angle$ B A C).

B ***Prove that :*** The perimeter of $\Delta AXY = \frac{1}{2}$ perimeter of ΔABC.

4 Two concentric circles M. $\overline{AB}$ and $\overline{CD}$ are two chords in the larger circle touching the smaller circle at X and Y respectively. ***Prove that :*** AB = AC

5 **In the figure opposite :** M and N are two congruent circles, $\overleftrightarrow{AB} \, // \, \overleftrightarrow{MN}$ was drawn and intersected circle M at A and B and intersected circle N at C and D.

Prove that : AC = BD.

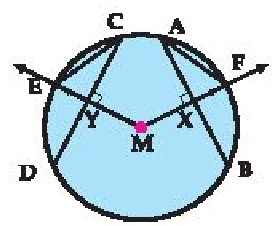


6 $\overline{AB}$ and $\overline{CD}$ are two chords in circle M,

$\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F,

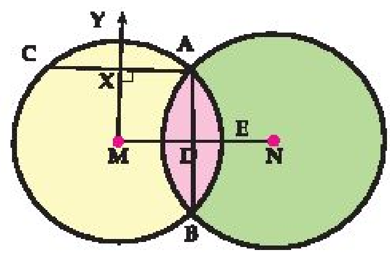
$\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E FX = EY .

Prove that : **First :** AB = CD **Second:** AF = CE.



7 **In the figure opposite :** the two circles M and N intersect at A and B. is drawn $\overrightarrow{MX} \perp \overline{AC}$ intersects $\overline{AC}$ at X and intersects circle M in Y, $\overline{MN}$ is drawn $\overline{AB}$ to intersect $\overline{AB}$ at D and circle M at E. If AC = AB,

Prove that : XY = DE .

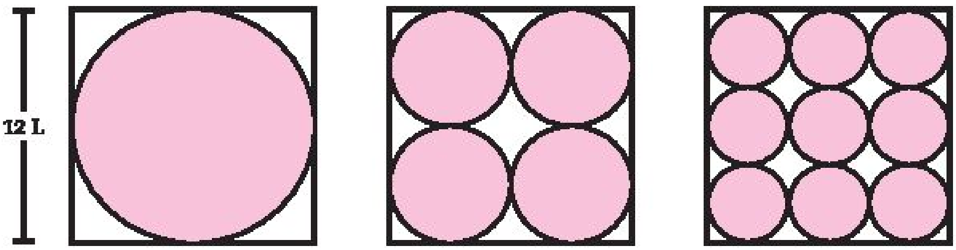


8 Two circles M and N touch internally at A, $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the larger circle and intersect the smaller circle at D and E respectively.

Prove that : AD = AE.

Geometric Patterns

1. A Bakery produces circular shaped pies. If the baker puts the pies in square shaped boxes, each of side length 12 L cm as shown in the following pattern.

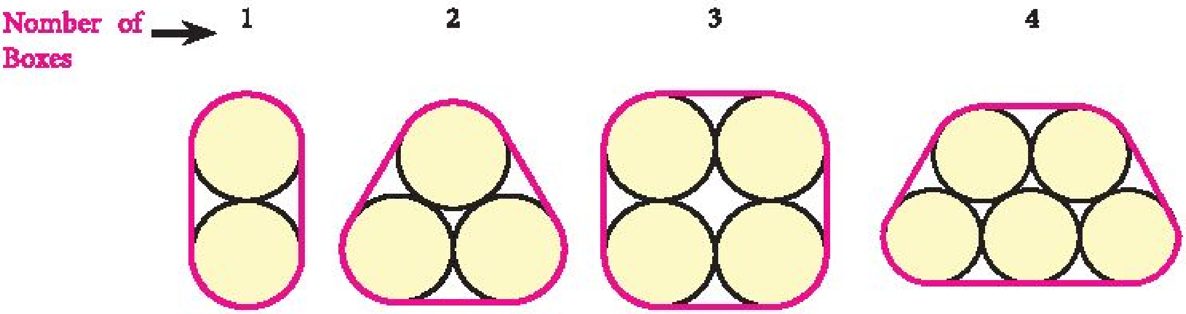


A. Calculate the area the pies take up in each box and record your observation.

B. What is the area that the pies take up in the fourth box in this pattern?

C. If all the pies are of the same size and equal in height. Are the prices of the boxes equal or different ? Explain.

2. A factory for producing jam puts the jam in cylindrical shaped cans of base radius length r cm. The cans were wrapped by plastic and a paper adhesive tape around it as shown in the following pattern :

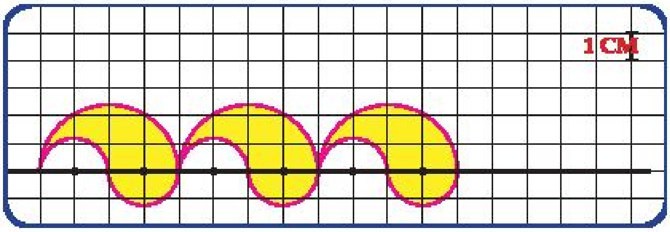


A. Find the length of the tape in each case. Is there a relation between the number of cans and the tape length?

B. What is the length of the tape circuling 6 cans ?

C. What is the length circuling 7 cans? Discuss the possible positions to join the cans and deduce the condition to continue the same pattern with the tape length.

3 **Study of the pattern opposite then draw the next unit for this pattern.**

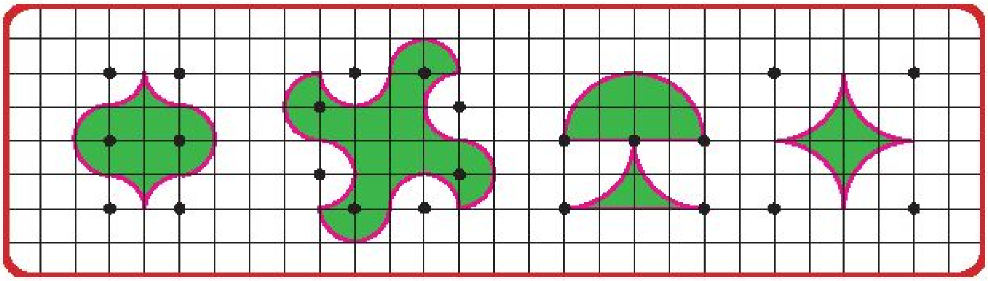


A What is the area of 10 units from this pattern ?

B What is the circumference of 7 units from this pattern?

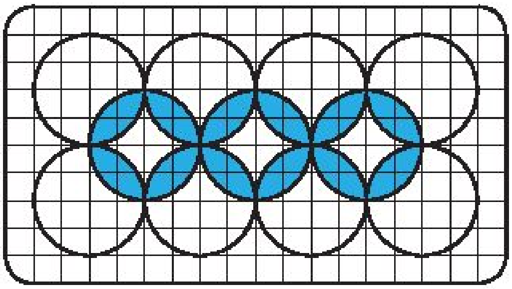
C How many units are needed to design a frame around a rectangular shaped photo with dimensions of 24 cm and 36 cm?

4 **Study of the following units, then find the area and circumference of each.**

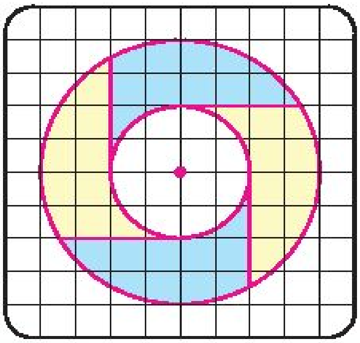


5 Technology : **Use your computer to draw the following shapes.**

A Tangency and intersecting congruent circles.



B Tangency and concentric circles.



Invent other models and use it in your study of the Arts education subject.

General Exercises

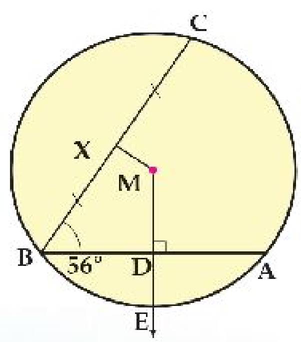
1 Complete to make the statement correct :

- **A** The chord of the circle is the drawn line segment between ....
- **B** The straight line passing vertically on the center of the circle on any chord in it ....
- **C** The line of two centers of two circles touching internally pass ....
- **D** The center of the circumscribed circle about the triangle is the intersect on point of ....
- **E** The chords of equal length in circle ....

2 Choose the correct answer :

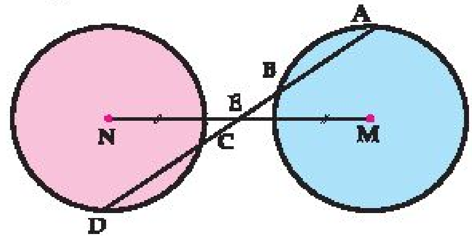
- **A** A tangent to a circle of diameter length 6 cm is at a distance of .... cm from its center. (6 **or** 12 **or** 3 **or** 2)
- **B** A circle can be drawn passing the vertices of a ....
  (Rhombus **or** rectangle **or** trapezoid **or** parallelogram)
- **C** $\overline{AB}$ is a diameter in circle M, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle, then $\overleftrightarrow{AC}$ .... $\overleftrightarrow{BD}$.
  (intersects **or** perpendicular to **or** parallel to **or** congruent on)
- **D** A circle with a circumference of 6π cm, and the straight line L is distant from its center by 3 cm, then the straight line L is ....
  (tangent to the circle **or** a secant **or** outside the circle **or** a diameter to the circle).
- **E** M and N are two intersecting circles, both their radii length are 3 cm and 5 cm, then: MN ∈ ....
  ( ]8 , ∞[**or**]2 , ∞[**or**]0, 2[**or**]2 , 8 [)

3 In the figure opposite: $\overline{AB}$ and $\overline{BC}$ are two chords in circle M which has radius length of 5 cm, $\overrightarrow{MD} \perp \overline{AB}$ intersects $\overline{AB}$ at D and intersects the circle M at E, X is the midpoint of $\overline{BC}$.
AB = 8 cm, m (∠ABC) = 56°



Find: **A** m (∠ DMX) **B** Length of $\overline{DE}$

4 **In the figure opposite:** M and N are two distant and congruent circles. E is the midpoint of $\overline{MN}$. Draw $\overleftrightarrow{AE}$ intersecting circle M at A and B intersects circle N at C and D.

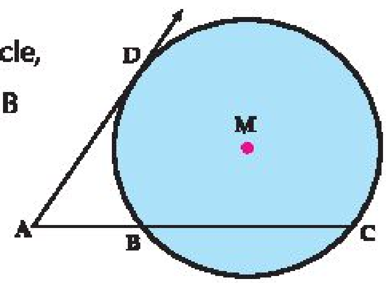


Prove that : **A** A B = C D.

B E is the midpoint of $\overline{AD}$.

5 **In the figure opposite :**

M circle with radius length of 5 cm, A is a point outside the circle, $\overrightarrow{AD}$ is a tangent to circle M at D, $\overrightarrow{AB}$ intersects the circle at B and C respectively where AB = 4 cm and AC = 12 cm.

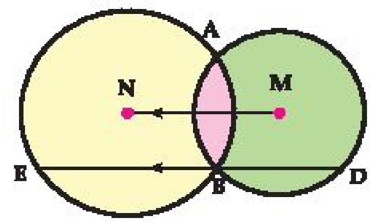


A Find the distance of the chord $\overline{BC}$ from the center of the circle.

B Calculate the length of $\overline{AD}$.

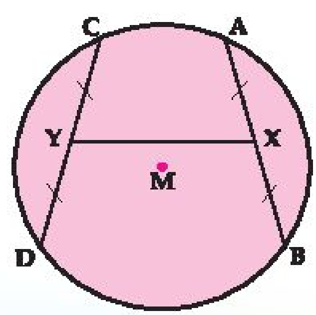
6 **In the figure opposite :**

M and N are two intersecting circles at A and B. Draw $\overleftrightarrow{BD}$ // $\overleftrightarrow{MN}$ intersecting the two circles at D and E respectively. ***Prove that :*** DE = 2 MN



7 **In the figure opposite :**

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in circle M. X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ where B and D are in one side from $\overleftrightarrow{XY}$.



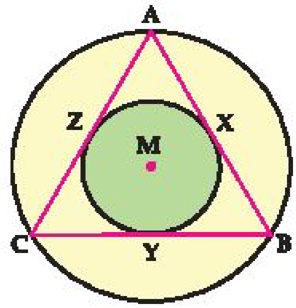
Prove that : m (∠ B XY) = m(∠ DY X).

Think : Does $\overline{AC}$ // $\overline{BD}$? Explain your answer

8 **In the figure opposite :**

Two concentric circles M. Their radii lengths are 4 cm and 2 cm. Draw the triangle ABC where their vertices are located on the larger circle and its sides are touching the smaller circle at X, Y and Z.

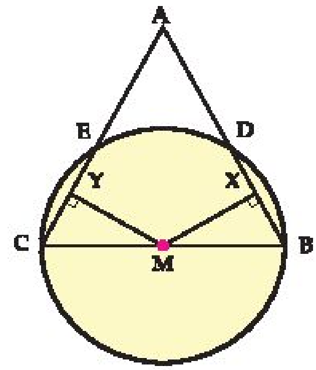
Prove that : The triangle ABC is equilateral and find its area.



9 **In the figure opposite :**

ABC is a triangle in which AB = AC. Circle M was drawn with diameter $\overline{BC}$ intersecting $\overline{AB}$ at D and $\overline{AC}$ at E, $\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CE}$

Prove that : BD = CE.



Unit test

1 **Complete to make the statement correct :**

A. Any three points that do not belong to one straight line include ....

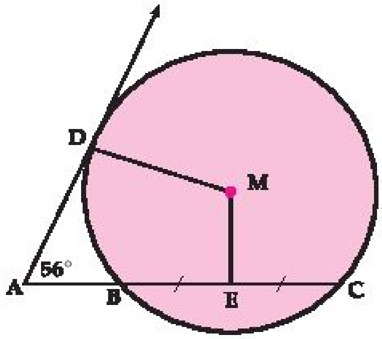
B. The axis of symmetry of the two circles M and N that are intersecting at A and B is ....

C. If AB = 7 cm, then the area of the smallest circle passing through the two points A and B = ... cm^2.

D. If M circle with circumference 8π cm, A is a point on the circle, then MA = ...

E. A chord with 8 cm length. The length of its radius is 5 cm, then it is distant from its center by .... cm.

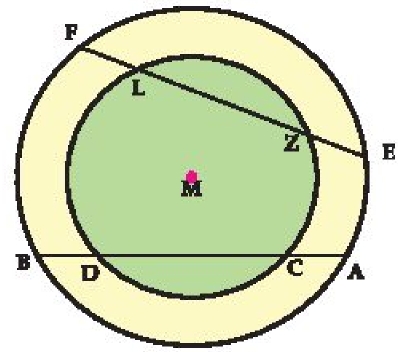
2 **In the figure opposite :**



$\overrightarrow{AD}$ is a tangent to the circle M, $\overrightarrow{AC}$ intersects the circle M at B and C, E is the midpoint of $\overline{BC}$, m (∠ A) = 56°.

Find m (∠ D M E).

3 **In the figure opposite :**

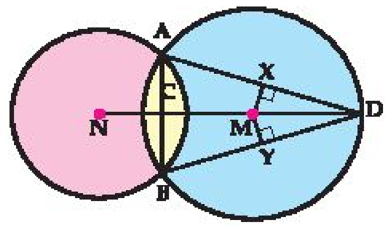


Two concentric circles M, $\overline{AB}$ is a chord in the larger circle and intersects the smaller circle at C and D. $\overline{EF}$ is a chord in the larger circle and intersects the smaller circle at Z and L where AB = EF.

Prove that : A. C D = Z L.

B. A D = Z F.

4 **In the figure opposite :**



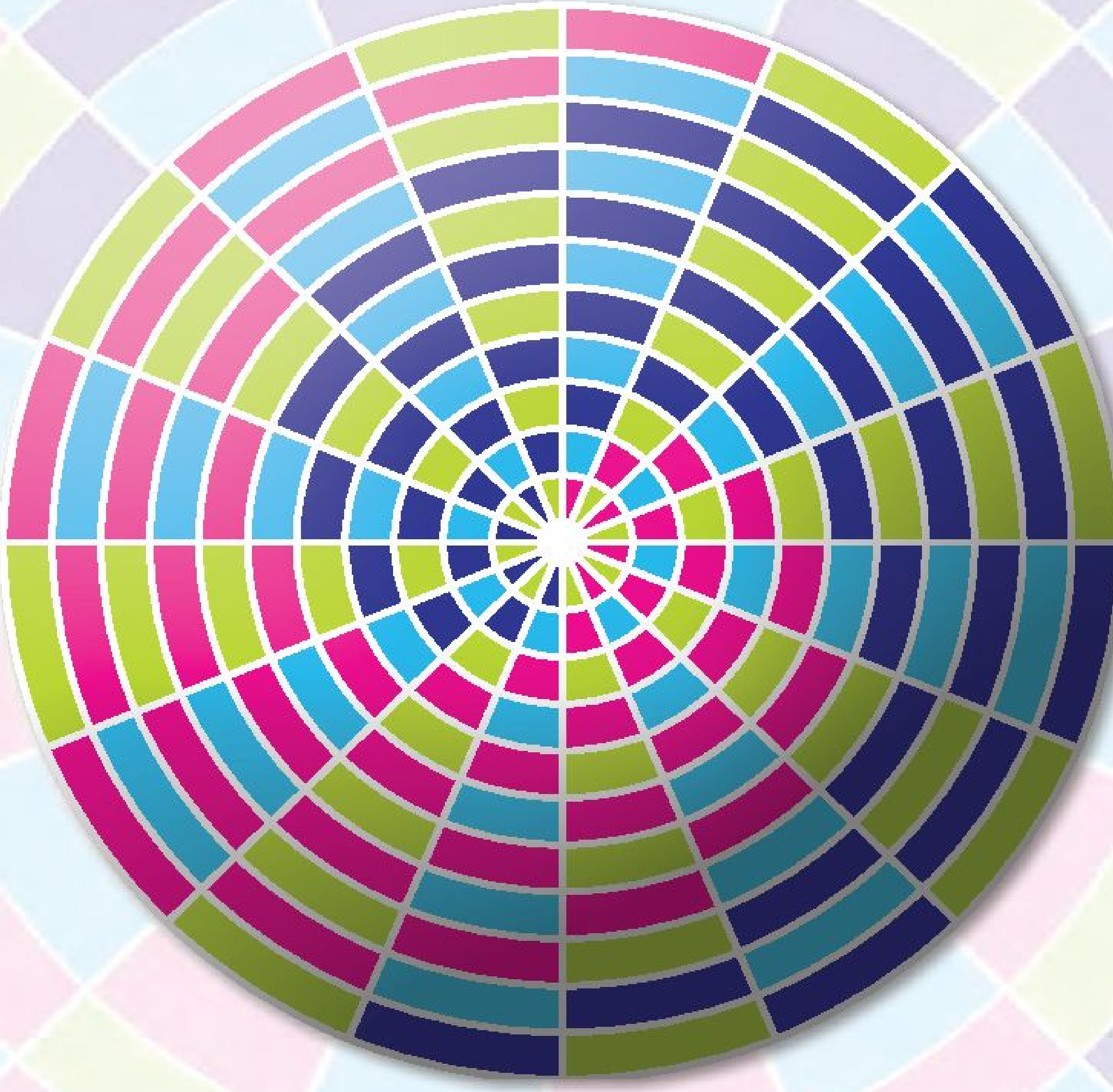
Circle M ∩ circle N = {A, B}, $\overleftrightarrow{AB} \cap \overleftrightarrow{MN}$ = {C},

D ∈ $\overleftrightarrow{MN}$, $\overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$.

Prove that : M X = MY.

Geometry

Unit 5: Angles and Arcs in the circle



5 - 1

Central Angles and Measuring Arcs

What you'll learn

- The concept of arc length.
- The concept of measuring an arc.
- How to find the relation between chords of a circle and its arcs.

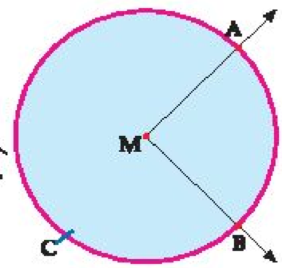
Key terms

- Central angle
- Inscribed angle
- Arc
- two adjacent arcs
- Measuring an arc
- Chord
- Tangent

Think and Discuss

In the opposite figure :

The two sides of $\angle$ AMB divide the cicle M into two arcs:

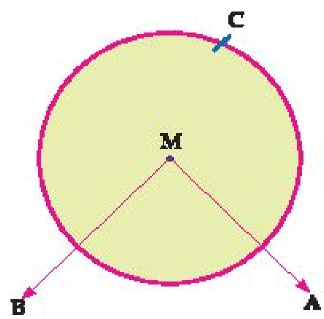


1. The minor arc AB and is denoted by $\overset{\frown}{AB}$.
2. The major arc ACB and is denoted by $\overset{\frown}{ACB}$.

- What is the position of the points of $\overset{\frown}{AB}$ with respect to $\angle$ AMB ?
- What is the position of points $\overset{\frown}{ACB}$ with respect to reflected angle of $\angle$AMB ?
- If $\angle$ AMB is a straight angle, **what do you notice ?**

Central Angle	It is the angle whose vertex is the center of the circle and the two sides are radii in the circle.

In the opposite figure we notice that :

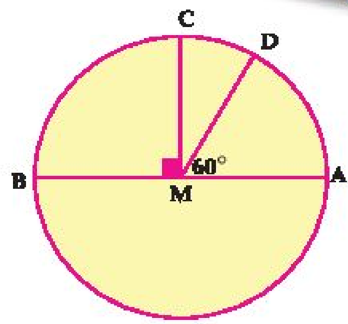


1. $\overset{\frown}{AB}$ is opposite to the central angle $\angle$AMB and $\overset{\frown}{ACB}$ is opposite to the central reflective angle $\angle$ AMB.
2. If $\angle$ AMB is a straight angle ($\overline{AB}$ is a diameter in circle M) then $\overset{\frown}{AB}$ is congruent to $\overset{\frown}{ACB}$ and each is called "a semicircle".

Measure of the arc	Is the measure of the central angle opposite to it.

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M, $\overline{MC} \perp \overline{AB}$ m (∠ AMD) = 60°



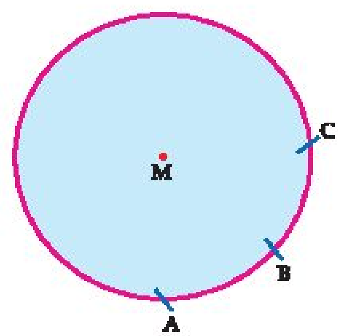
Notice that :

1. $m\ \overset{\frown}{AD}$ = m (∠ AMD) = 60°
2. $m\ \overset{\frown}{CB}$ = m (∠ CMB) = 90°
3. $m\ (\overset{\frown}{DC})$ = m (∠ DMC) = 30° ***(Why?)***
4. $m\ \overset{\frown}{AB}$ = m (∠ AMB) = 180°

i.e. Measure of the semicircle = 180 ° and measure of a circle = 360 °

Adjacent arcs	**are two arcs in the same circle that have only one point in common.**

Represent $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ in the opposite figure:



thus :

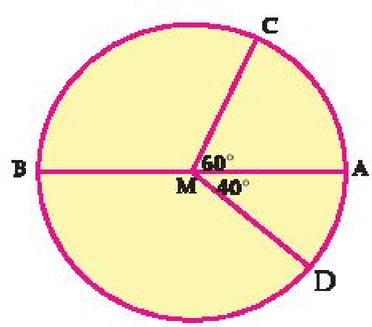
$m\ (\overset{\frown}{AB}) + m\ (\overset{\frown}{BC}) = m\ (\overset{\frown}{ABC})$

$, m\ (\overset{\frown}{AB}) = m\ (\overset{\frown}{ABC}) - m\ (\overset{\frown}{BC})$

Drill

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M, m (∠ AMC) = 60°, m (∠AMD)= 40° .



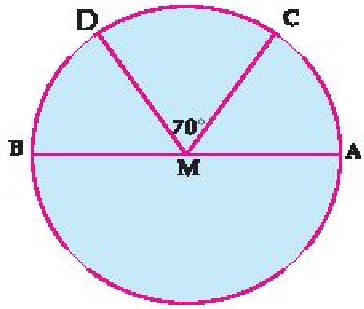
Complete :

1. $m\ (\overset{\frown}{AD})$ = ...° , $m\ (\overset{\frown}{AC})$ = ...°
2. $m\ (\overset{\frown}{CAD}) = m\ (\overset{\frown}{CA})$ + ...

   = ... + ... = ...°
3. $m\ (\overset{\frown}{BC}) = m\ (\overset{\frown}{ACB})$ - m (   ) = 180° - .... = ...°

   (Why?)
4. $m\ (\overset{\frown}{DCB})$ = measure of circle - m ($\frown$) = ... - .... = ...°

Example 1



$\overline{AB}$ is a diameter in the circle M, m ($\angle$ CMD) = 70° ,
m $(\overset{\frown}{AC})$: m $(\overset{\frown}{DB})$ = 5 : 6 , *find* m $(\overset{\frown}{ACD})$.

Solution

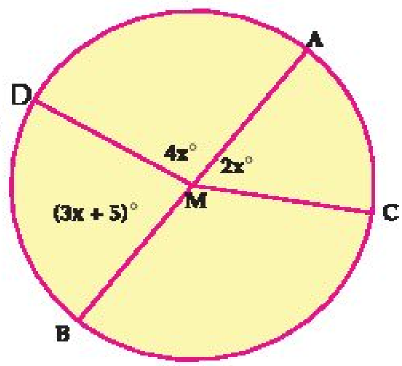
Suppose that m $(\overset{\frown}{AC}) = 5x$ $\quad \therefore$ m $(\overset{\frown}{DB}) = 6x$

$\because$ m $(\overset{\frown}{ADB})$ = m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{CD})$ + m $(\overset{\frown}{DB}) = 180°$

$\therefore$ $5x + 70° + 6x = 180°$ $\quad 11x = 110°$ $\therefore$ $x = 10°$, m $(\overset{\frown}{AC}) = 50°$

$\therefore$ m $(\overset{\frown}{ACD})$ = m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{CD}) = 50° + 70° = 120°$

Drill



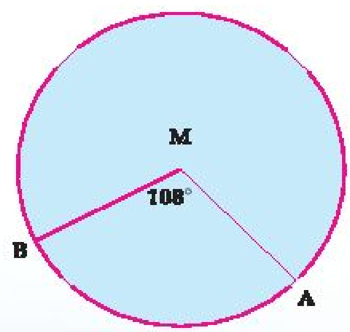
In the opposite figure : $\overline{AB}$ is a diameter of the circle M,
study the figure, then complete :

1. x = ...
2. m $(\overset{\frown}{AC})$ = ...°
3. m $(\overset{\frown}{AD})$ = ...
4. m $(\overset{\frown}{BC})$ = ... °
5. m $(\overset{\frown}{CAD})$ = ...
6. m $(\overset{\frown}{CBD})$ = ...
7. m $(\overset{\frown}{ACD})$ = ...
8. m $(\overset{\frown}{ADC})$ = ...

Arc length	**is a part of a circle's circumference proportional with its measure**

Where the arc length $= \dfrac{\text{The measure of the arc}}{\text{The measure of the circle}} \times$ **circumference of the circle**

Example 2



In the opposite figure :

M is a circle with radius length of 5 cm, m $(\overset{\frown}{AB}) = 108°$.

Find : the length of $\overset{\frown}{AB}$ $\quad (\pi = 3.14)$

Solution

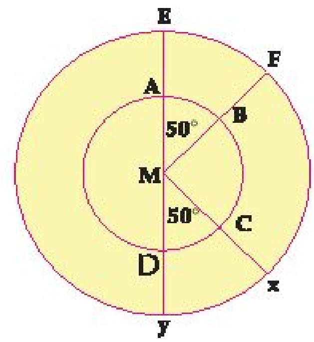
Arc length $= \dfrac{\text{Measure of the arc}}{\text{Measure of the circle}} \times$ circumference of the circle.

$= \dfrac{108}{360} \times 2 \times 3.14 \times 5 = 9.42$cm.

Drill

In the opposite figure : Two concentric circles, the radius length of the minor circle is 7 cm and the radius length of the major circle is 14 cm ($\pi = \frac{22}{7}$)

Complete : **In the minor circle :**



$m(\overset{\frown}{AB}) = m(\overset{\frown}{\ldots\ldots}) = \ldots\ldots\ldots^\circ$

length of $\overset{\frown}{AB} = \frac{50}{360} \times 2 \times \frac{22}{7} \times \ldots\ldots = \ldots\ldots$ cm

length of $\overset{\frown}{CD} = \ldots\ldots \times \ldots\ldots = \ldots\ldots$cm

$\therefore$ $\overset{\frown}{AB}$ (congruent to / not congruent to) $\overset{\frown}{CD}$

In the major circle :

$m(\overset{\frown}{EF}) = m(\overset{\frown}{\ldots\ldots}) = \ldots\ldots^\circ$, length of $\overset{\frown}{EF} = \ldots\ldots \times \ldots\ldots = \ldots\ldots$ cm

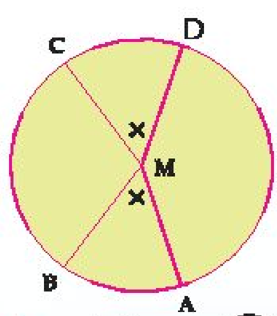
length of $\overset{\frown}{XY} = \ldots\ldots \times \ldots\ldots = \ldots\ldots$cm

$\therefore$ $\overset{\frown}{EF}$ (congruent to / not congruent to) $\overset{\frown}{XY}$

- Is $\overset{\frown}{AB}$ congruent to $\overset{\frown}{EF}$? What do you deduce ?

Important corollaries :

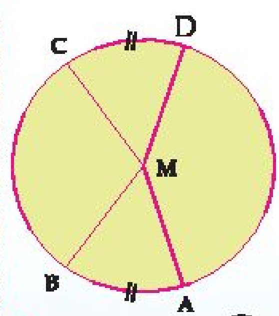
Corollary (1) *In the same circle (or in congruent circles), if the measures of arcs are equal, then the lengths of the arcs are equal, and conversely.*



In circle M

If: $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

then: the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

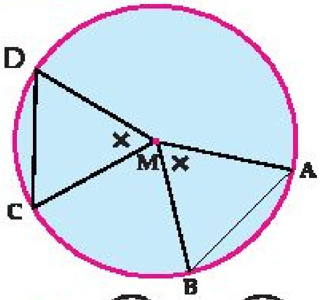


And conversely

If: the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

then : $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

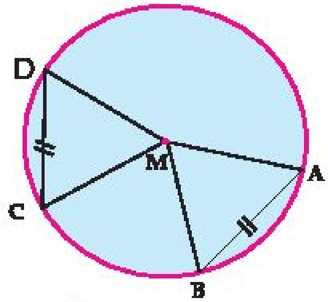
Corollary (2) *In the same circle (or in congruent circles), if the measures of arcs are equal, then their chords are equal in length, and conversely*



In circle M

If : $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

then : length of $\overline{AB}$ = length of $\overline{CD}$



And conversely

If : AB = CD

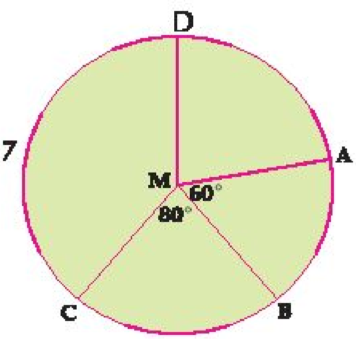
then : $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$

Drill

In the opposite figure :

$m(\overset{\frown}{AB}) = 60^\circ$ et $m(\overset{\frown}{BC}) = 80^\circ$, $m(\overset{\frown}{AD}) : m(\overset{\frown}{DC}) = 4 : 7$

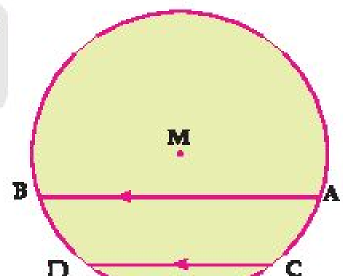
1. Mention the arcs equal in measure.
2. Mention the arcs equal in length.
3. Draw the chords equal in length.



Corollary (3) *If two parallel chords are drawn in a circle, then the measures of the two arcs between them are equal.*

If $\overline{AB}$ and $\overline{CD}$ are two chords in circle M, $\overline{AB} \parallel \overline{CD}$

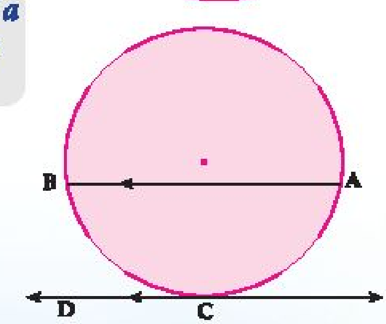
then : $m(\overset{\frown}{AC}) = m(\overset{\frown}{BD})$.



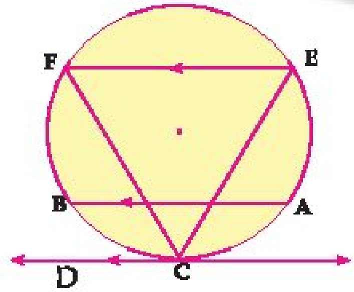
Corollary (4) *If a chord is parallel to a tangent of a circle, then the measures of the two arcs between them are equal.*

If $\overline{AB}$ is a chord of circle M, $\overleftrightarrow{CD}$ is a tangent at c,

$\overline{AB} \parallel \overleftrightarrow{CD}$ **then** $m(\overset{\frown}{AC}) = m(\overset{\frown}{BC})$.



Drill



In the opposite figure :

M is a circle , $\overleftrightarrow{CD}$ is a tangent to the circle at C , $\overline{AB}$ and $\overline{EF}$ are two chords of the circle where : $\overline{AB} // \overline{EF} // \overleftrightarrow{CD}$

Complete the following to prove that CE = CF

Solution

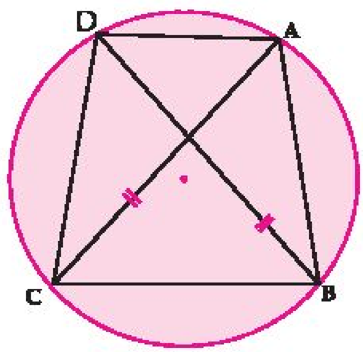
∵ $\overline{AB} // \overline{EF}$ ∴ m $\overset{\frown}{(......)}$ = m $\overset{\frown}{(......)}$ (1)

∵ The tangent $\overleftrightarrow{CD} // \overline{AB}$ ∴ m $\overset{\frown}{(......)}$ = m $\overset{\frown}{(......)}$ (2)

By adding the two sides of (1) and (2)

∴ m $(\overset{\frown}{EC})$ = m $\overset{\frown}{(......)}$ ∴ CE = ...

Example 3



In the opposite figure :

ABCD is a quadrilateral inscribed in a circle in which AC = BD, AB = (3x - 5) cm , CD = (x + 3) cm.

Find with proof the length of $\overline{AB}$.

Solution

Given : ABCD is a quadrilateral inscribed in a circle, AC = BD , AB = (3x - 5) cm , CD= (X + 3) cm

R.T.P.: Find the length of $\overline{AB}$.

Proof : ∵ AC = BD given ∴ m $(\overset{\frown}{ABC})$ = m $(\overset{\frown}{BCD})$

∴ m $(\overset{\frown}{ABC})$ - m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{BCD})$ - m $(\overset{\frown}{BC})$

∴ m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{DC})$ ∴ AB = CD

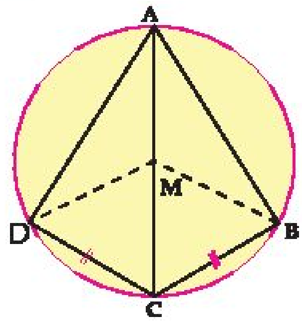
∵ AB = CD ∴ 3x - 5 = x + 3

2x = 8 ∴ x = 4

∵ AB = 3x - 5 ∴ AB = 3 × 4 - 5 = 7cm

Example 4

In the opposite figure :



ABCD is a quadrilateral inscribed in a circle M, $\overline{AC}$ is a diameter in the circle, CB = CD.

Prove that : $m(\overset{\frown}{AB}) = m(\overset{\frown}{AD})$

Solution

Given : $\overline{AC}$ is a diameter in a circle, CB = CD

R.T.P.: $m(\overset{\frown}{AB}) = m(\overset{\frown}{AD})$

Proof: $\because$ CB = CD $\quad\therefore m(\overset{\frown}{CB}) = m(\overset{\frown}{CD})$ (1)

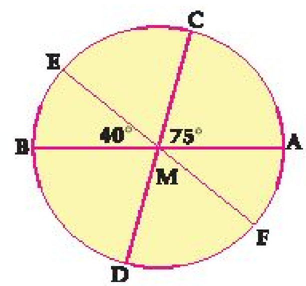
$\because$ $\overline{AC}$ is a diameter in the circle

$\therefore m(\overset{\frown}{AB}) = 180^\circ - m(\overset{\frown}{CB})$, $m(\overset{\frown}{AD}) = 180^\circ - m(\overset{\frown}{CD})$ (2)

from (1) **and** (2) **we get :** $m(\overset{\frown}{AB})) = m(\overset{\frown}{AD})$

Exercises 5-1

1 **In the opposite figure :**



$\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ are diameters of the circle M ***Complete :***

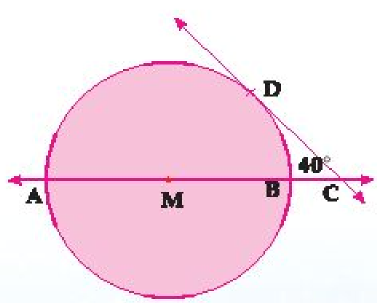
A $m(\overset{\frown}{AC})$ = ...   **B** $m(\overset{\frown}{ACE})$ = ...

C $m(\overset{\frown}{ACD})$ = ...   **D** $m(\overset{\frown}{AFE})$ = ...

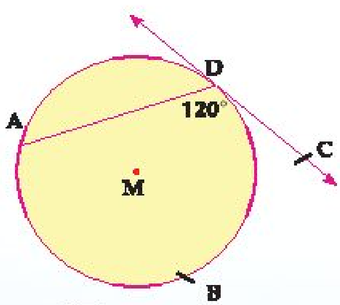
2 **In each of the following figures:**

$\overleftrightarrow{CD}$ is a tangent to the circle M at D, ***Complete:***

A



B

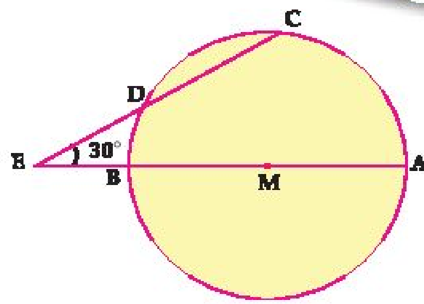


$m(\overset{\frown}{DB})$ = ... $m(\overset{\frown}{AD})$ = ... $m(\overset{\frown}{ABD})$ = ...

3 **In the opposite figure :**

$\overline{AB}$ is a diameter in a circle M, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$,
$m(\angle AEC) = 30°$, $m(\overset{\frown}{AC}) = 80°$.

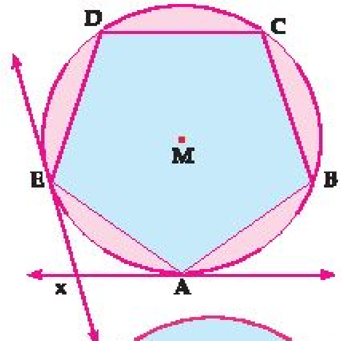
Find: $m(\overset{\frown}{CD})$



4 **In the opposite figure :**

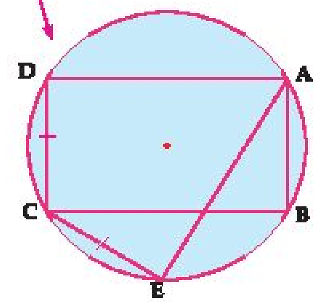
ABCDE is a regular pentagon inscribed in a circle M,
$\overleftrightarrow{AX}$ is a tangent to the circle at A, $\overleftrightarrow{EF}$ is a tangent to the circle at E
where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$.

Find: **A** $m(\overset{\frown}{AE})$ **B** $m(\angle AXE)$.



5 **In the opposite figure :**

ABCD is a rectangle inscribed in a circle.
Draw the chord $\overline{CE}$ where CE = CD.
Prove that : AE = BC.

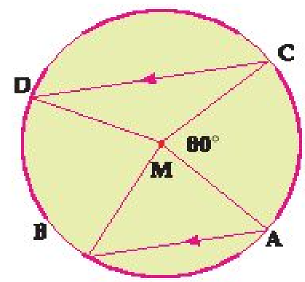


6 **In the opposite figure :**

M is a circle with radius length of 15 cm, $\overline{AB}$ and $\overline{CD}$ are two parallel chords of the circle, $m(\overset{\frown}{AC}) = 80°$, length of $\overset{\frown}{AC}$ = length of $\overset{\frown}{AB}$.

Find:

A $m(\angle MAB)$ **B** $m(\overset{\frown}{CD})$ **C** length $\overset{\frown}{CD}$

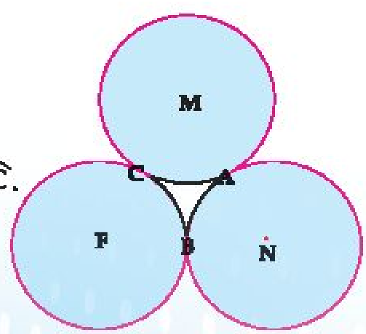


7 **In the opposite figure :**

M, N and F are three congruent circles and touching at A, B and C. The radius length of each is 10 cm,

A **Prove that :** length of $\overset{\frown}{AB}$ = length of $\overset{\frown}{BC}$ = length of $\overset{\frown}{AC}$.

B Perimeter of figure ABC.



5 - 2

The relation between the inscribed and central angles subtended by the same arc

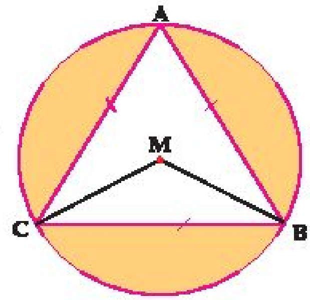
What you'll learn

- ☆ How to infer the relation between the measures of the inscribed and central angles subtended by the same arc

Key terms

- ☆ Inscribed angle.
- ☆ Central angle.

Think and Discuss



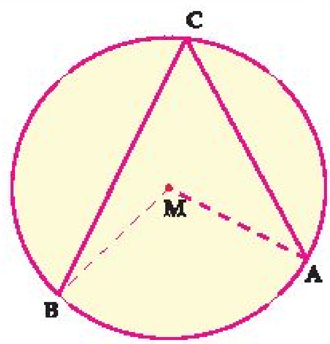
In the opposite figure :

The circle M passes through the vertices of the equilateral triangle ABC.

- ◆ What is the measure of central $\angle$ BMC? **Explain your answer.**
- ◆ What is the vertices of $\angle$ BAC? Does the vertices of the angle belong to the set of points of circle M ?
- ◆ What are the two sides of $\angle$ BAC ?
- ◆ If $\angle$ BMC is central with arc $\overset{\frown}{BC}$. How do you describe $\angle$ BAC ?
- ◆ Compare between m($\angle$ BAC) and m ($\angle$ BMC). **What do you notice ?**

Inscribed angle	**An angle the vertex of it lies on the circle and its sides contain two chords of the circle**

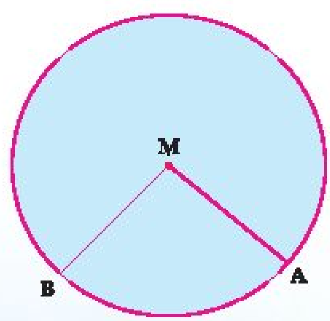
In the opposite figure : Notice that :



1. $\angle$ ACB is an inscribed angle and $\overset{\frown}{AB}$ is the arc opposite to it.
2. For each inscribed angle, there is one central angle subtended by the same arc.

Think

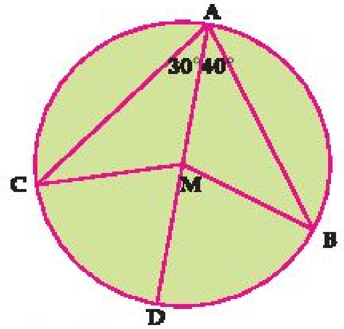
In the opposite figure :



What is the number of inscribed angles subtended with the central $\angle$ AMB at $\overset{\frown}{AB}$?

(Clarify your answer with a drawing)

Activity **In the opposite figure :**



$\overline{AD}$ is a diameter in circle M. Study the figure, then answer the following questions :

1. Mention two pairs of equal angles in measure.
2. If m (∠ BAD) = 40° , find m (∠ BMD) .
3. If m (∠ CAD) = 30° , find m (∠ CMD) .
4. Compare between m (∠ BAC) , m (∠ BMC). **What do you conclude?**

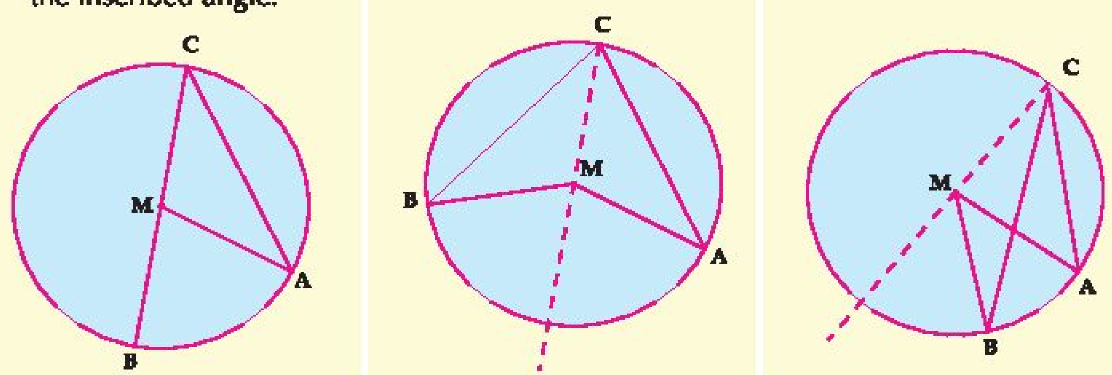
Theorem ***The measure of the inscribed angle is half the measure of the central angle, subtended by the same arc.***

Given : ∠ ACB is an inscribed angle, ∠ AMB is a central angle.

R.T.P. : Prove that m (∠ ACB) = $\frac{1}{2}$ m (∠ AMB).

Proof : **There are three cases to prove this theorem.**

1. If M belongs to one of the two sides of the inscribed angle.
2. If M is a point inside the inscribed angle.
3. If M is a point outside the inscribed angle.



First case: If M belongs to one of the two sides of the inscribed angle.

∵ ∠AMB is outside Δ AMC

∴ m (∠AMB) = m (∠ A) + m (∠C) (1)

∵ AM = CM **(radii lengths)** ∴ m (∠A) = m (∠C) (2)

From (1) **and** (2) **we get :** m (∠AMB = 2 m (∠C)

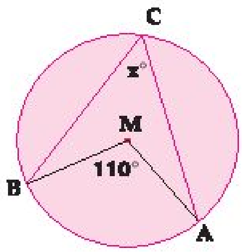
∴ m (∠ACB) = $\frac{1}{2}$ m (∠AMB) ***(Q.E.D)***

Activity

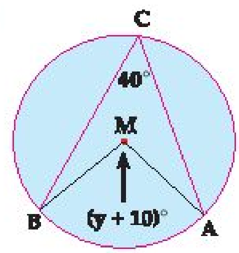
Prove that the theorem in the other two cases are correct and save your work in the portfolio.

Drill

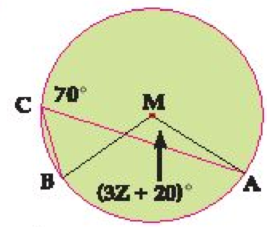
M is a circle. In each of the following figures, find the value of the symbol used in measuring:

1


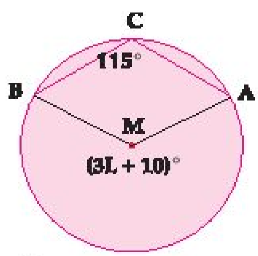
x = ...

2


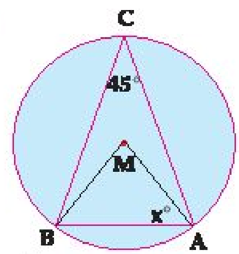
y = ...

3


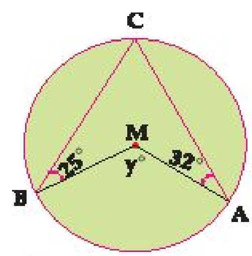
Z = ...

4


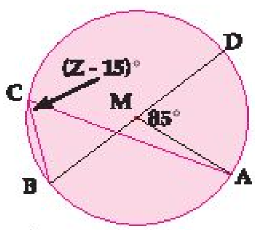
L = ...

5


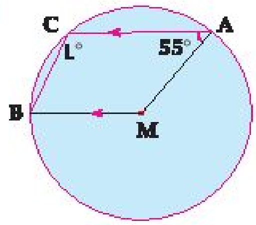
x = ...

6


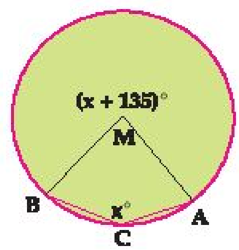
y = ...

7


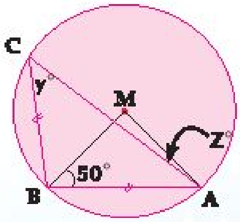
Z = ...

8


L = ...

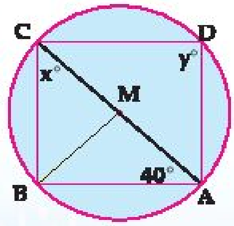
9


x = ...

10


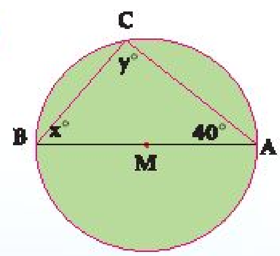
y = ...

Z = ...

11


x = ...

y = ...

12


x = ...

y = ...

Example 1

A is a point outside the circle M, $\overrightarrow{AB}$ is a tangent to the circle at B, $\overrightarrow{AM}$ intersects the circle M at C and D respectively m (∠ A) = 40° *Find*. with proof m(∠ BDC).

Solution

Given : $\overrightarrow{AB}$ is a tangent to the circle at B, m(∠ A) = 40° ,$\overrightarrow{AM}$ intersects the circle M at C and D.

R.T.P. : m (∠ BDC)

Construction: Draw the radius $\overline{BM}$.

Proof : ∵ $\overrightarrow{AB}$ is tangent to the circle at B, $\overline{BM}$ is a radius.

∴ m (∠ ABM) = 90°

In Δ ABM :

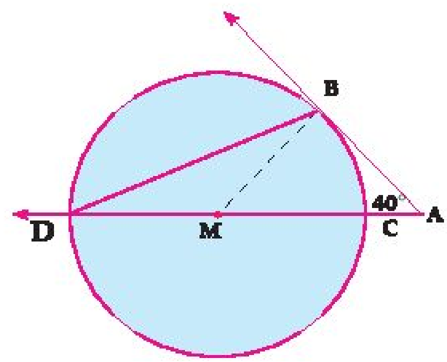
∵ m (∠ A) = 40° , m(∠ ABM) = 90°

∴ m (∠ BMC) = 180° - (40° + 90°) = 50°

∵ Inscribed ∠ BDC and central ∠ BMC are both subtended at $\overset{\frown}{BC}$.

∴ m (∠ BDC) = $\frac{1}{2}$ m (∠ BMC)

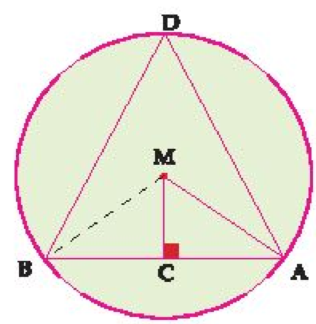
∴ m (∠ BDC) = $\frac{1}{2}$ × 50 = 25 ° ***(Q.E.D.)***



Drill

In the opposite figure : $\overline{AB}$ **is a chord of circle M,** $\overline{MC}$ ⊥ $\overline{AB}$.

Prove that : m (∠ AMC) = m (∠ ADB)



Solution

Draw $\overline{BM}$, ***Complete :*** In Δ MAB :

∵ MA= MB , $\overline{MC}$ ⊥ $\overline{AB}$

∴ m (∠ AMC) = m (∠ ....) = $\frac{1}{2}$ m (∠ ....) (1)

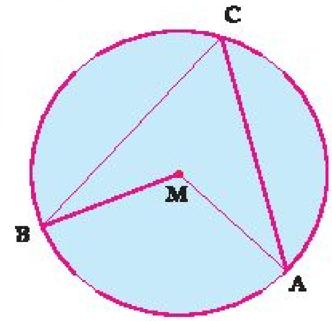
∵ inscribed ∠ ADB and central ∠ .... are subtended at $\overset{\frown}{........}$

∴ m (∠ ....) = $\frac{1}{2}$ m (∠ ....) (2)

From (1) and (2) we get : m (∠ AMC) = m (∠ ....) .

Corollary (1)

The measure of an inscribed angle is half the measure of the subtended arc.



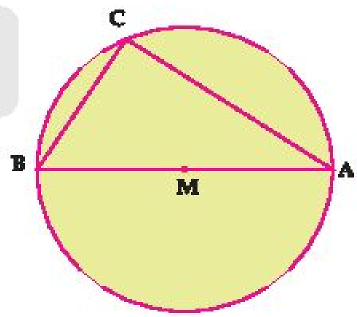
In the opposite figure :

$m(\angle C) = \frac{1}{2} m(\angle AMB)$, $m(\angle AMB) = m(\overset{\frown}{AB})$

$\therefore m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$

Corollary (2)

The inscribed angle drawn in a semicircle is a right angle.



i.e. : If the arc opposite to the inscribed angle equals the semicircle **then:** $m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$

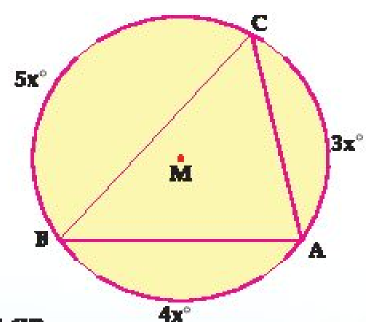
$\because m(\overset{\frown}{AB}) = 180^\circ$ $\therefore m(\angle C) = 90^\circ$

Think

- **What** is the type of the inscribed angle opposite to an arc less than a semicircle ? Why ?
- **What** is the type of the inscribed angle opposite to an arc greater than the semicircle ? Why ?
- **Is** the inscribed right angle inscribed in a semicircle ? **Explain your answer?**

Example 2

In the opposite figure : ABC is an inscribed triangle in circle M, $m(\overset{\frown}{AB}) : m(\overset{\frown}{BC}) : m(\overset{\frown}{AC}) = 4 : 5 : 3$. *find* $m(\angle ACB)$:



Solution

Suppose that : $m(\overset{\frown}{AB}) = 4x^\circ$, $m(\overset{\frown}{BC}) = 5x^\circ$, $m(\overset{\frown}{AC}) = 3x^\circ$

$\therefore 4x + 5x + 3x = 360^\circ$

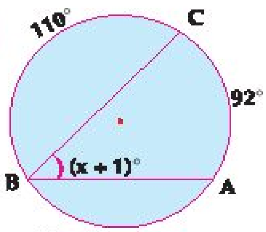
$12x = 360^\circ$ $\therefore x = 30^\circ$

$\therefore m(\overset{\frown}{AB}) = 4 \times 30 = 120^\circ$ and opposite to the inscribed $\angle ACB$.

$\because m(\angle ACB) = \frac{1}{2} m(\overset{\frown}{AB})$ $\therefore m(\angle ACB) = \frac{1}{2} \times 120^\circ = 60^\circ$ ***Q.E.D.***

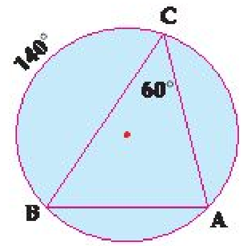
Drill

Study each of the following figures, then complete :

1. 

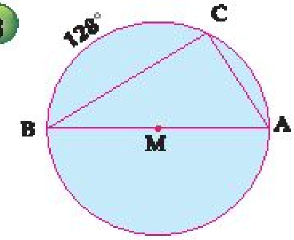
$x =$ ....

$m(\overset{\frown}{AB}) =$ ....

2. 

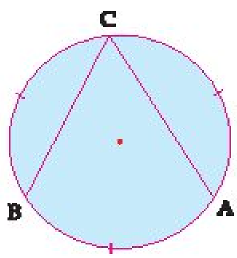
$m(\angle A) =$ ....

$m(\overset{\frown}{AC}) =$ ....

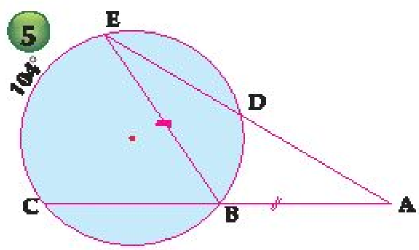
3. 

$m(\angle C) =$ ....

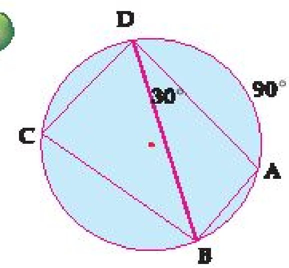
$m(\angle B) =$ ....

4. 

$m(\angle C) =$ ....

5. 

$m(\angle EAC) =$ ....

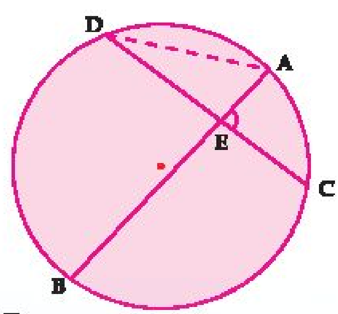
6. 

$m(\angle DCB) =$ ....

Example 3

Well known problem (1)

If two chords intersect at a point inside a circle, then the measure of the included angle equals half of the sum of the measure of the two opposite arcs.



Solution

Given : $\overline{AB} \cap \overline{CD} = \{E\}$

R.T.P : $m(\angle AEC) = \frac{1}{2}[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})]$

Construction: Draw $\overline{AD}$

Proof : $\because$ $\angle AEC$ is outside the ΔAED .

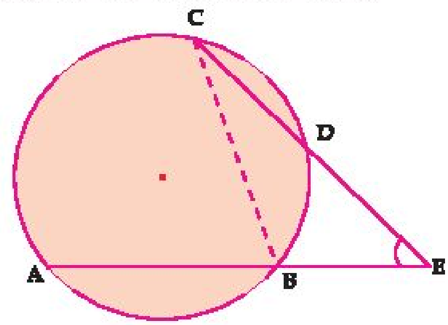
$\because$ $m(\angle AEC) = m(\angle D) + m(\angle A) = \frac{1}{2} m(\overset{\frown}{AC}) + \frac{1}{2} m(\overset{\frown}{BD})$

$= \frac{1}{2}[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})]$. ***Q.E.D.***

Example 4

Well known problem (2)

If two rays carrying two chords in a circle are intersecting outside it, then the measure of their intersecting angle equals half of the measure of the major arc subtracted from it half of the measure of the minor arc in which both are included by the two sides of this angle.



Solution

Given : $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

R.T.P : $m(\angle E) = \frac{1}{2}[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})]$

Construction : Draw $\overline{BC}$.

Proof : $\because \angle ABC$ is exterior to ΔBEC.

$\therefore m(\angle ABC) = m(\angle E) + m(\angle BCD)$

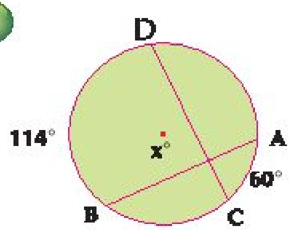
$\therefore m(\angle E) = m(\angle ABC) - m(\angle BCD)$

$= \frac{1}{2} m(\overset{\frown}{AC}) - \frac{1}{2} m(\overset{\frown}{BD})$

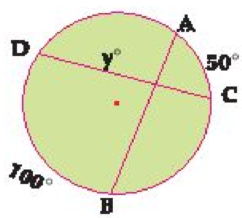
$= \frac{1}{2}[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})]$ ***Q.E.D***

Drill

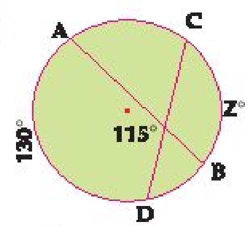
In each of the following figures, find the value of the symbol used in measuring:

1. 

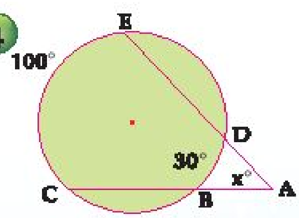
X = ...

2. 

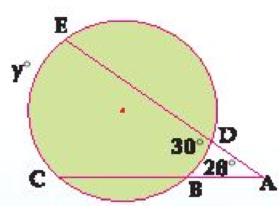
Y = ...

3. 

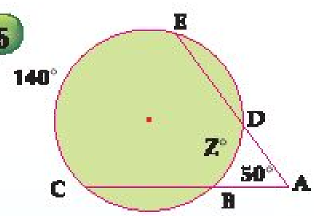
Z = ...

4. 

X = ...

5. 

Y = ...

6. 

Z = ...

Example 5 **In the opposite figure :**

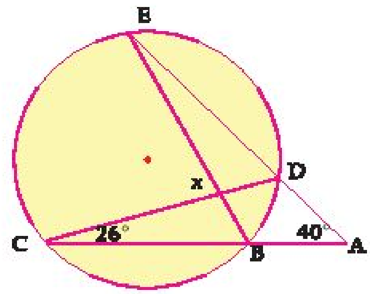
$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, m ($\angle$A) = 40°, $\overline{DC} \cap \overline{BE} = \{X\}$, m ($\angle$BCD) = 26°

Find : **A** m $(\overset{\frown}{CE})$ **B** m ($\angle$EXC) .

Solution

Given : $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, m ($\angle$A) = 40°, $\overline{DC} \cap \overline{BE} = \{X\}$, m ($\angle$BCD) = 26°

R.T.P. : **A** m $(\overset{\frown}{CE})$ **B** m ($\angle$EXC) .



Proof : ∵ m ($\angle$BCD) = 26°

∴ m $(\overset{\frown}{BD})$ = 2m ($\angle$BCD) = 52°

∵ $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

∴ m ($\angle$A) = $\frac{1}{2}$ [m$(\overset{\frown}{CE})$ - m $(\overset{\frown}{BD})$]

∴ 40 = $\frac{1}{2}$ [m$(\overset{\frown}{CE})$ - 52]

m$(\overset{\frown}{CE})$ = 80 + 52 = 132° *(Q.E.D. 1)*

∵ $\overline{DC} \cap \overline{BE} = \{X\}$ ∴ m ($\angle$EXC) = $\frac{1}{2}$ [m$(\overset{\frown}{CE})$ + m $(\overset{\frown}{BD})$]

m ($\angle$EXC) = $\frac{1}{2}$ [132 + 52] = $\frac{1}{2}$ × 184 = 92°

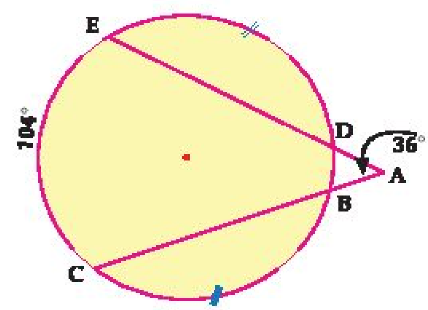
(Q.E.D. 2)

Drill

In the opposite figure :

m ($\angle$A) = 36°, m$(\overset{\frown}{EC})$= 104°, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DE})$

Find : **A** m $(\overset{\frown}{BD})$ **B** m $(\overset{\frown}{DE})$.



Solution

Complete: ∵ $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

∴ m ($\angle$A) = $\frac{1}{2}$ [..]

∴ 36 = $\frac{1}{2}$ [..]

∴ m $(\overset{\frown}{BD})$ = ... *(Q.E.D. 1)*

∵ m $(\overset{\frown}{DE})$ +m $(\overset{\frown}{BC})$ = 360° - (.......... + ...) = ...

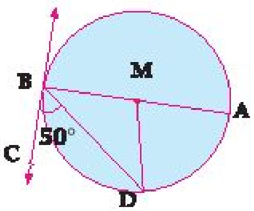
∵ m $(\overset{\frown}{DE})$ = m $(\overset{\frown}{BC})$ ∴ 2 m $(\overset{\frown}{DE})$ =

∴ m $(\overset{\frown}{DE})$ = ... *(Q.E.D. 2)*

Exercises 5-2

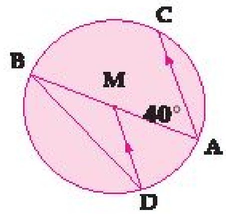
1 **M is a circle. In each of the following figures, study each figure, then complete:**

A



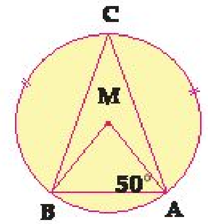
m ($\angle$AMD) = ... °

B



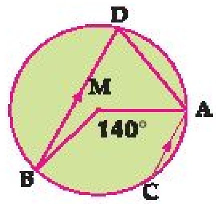
m ($\angle$BDM) = ... °

C



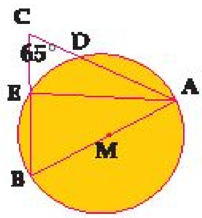
m ($\angle$CAM) = ... °

D



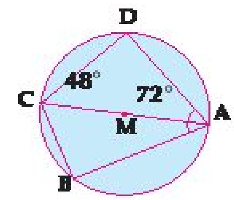
m ($\angle$CAD) = ... °

E



m ($\angle$CAE) = ... °

F

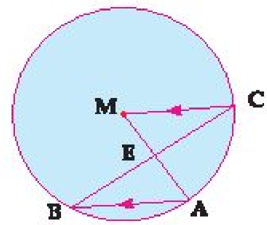


m ($\angle$BAC) = ... °

2 **In the opposite figure :**

$\overline{AB}$ is a chord in circle M, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM}$ = {E},

Prove that : BE > AE.

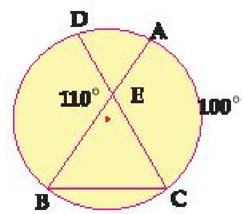


3 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle, $\overline{AB} \cap \overline{CD}$ = {E}

m ($\angle$DEB) = 110°, m ($\overset{\frown}{AC}$) = 100°.

Find : m ($\angle$DCB)

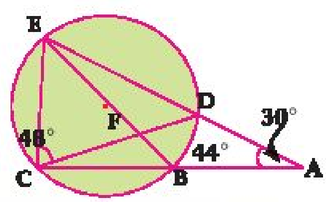


4 **In the opposite figure :**

$\overrightarrow{CB} \cap \overrightarrow{ED}$ = {A}, $\overline{BE} \cap \overline{CD}$ = {F}, if :

m ($\angle$A) = 30°, m ($\overset{\frown}{BD}$) = 44°, m ($\angle$DCE) = 48°

Find : **A** m ($\overset{\frown}{CE}$) **B** m ($\overset{\frown}{BC}$)

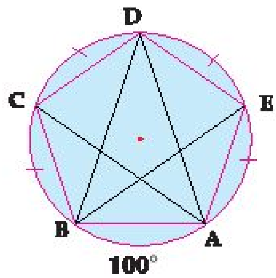


5 $\overline{AB}$, $\overline{AC}$ are two chords in the circle, X and Y are the two midpoints of $\overset{\frown}{AB}$ and $\overset{\frown}{AC}$ respectively, $\overline{XY}$ was drawn and intersected $\overline{AB}$ at D and $\overline{AC}$ at E prove that AD = AE

5 - 3

Inscribed Angles Subtended by the Same Arc

Think and Discuss



In the opposite figure: m $(\overset{\frown}{AB})$ = 100°

- Do the inscribed angles ∠AEB , ∠ADB et ∠ACB include the same arc?
- *Find* m (∠AEB), m (∠ADB) , m (∠ACB).

  What do you notice ?
- Do the inscribed angles that include equal arcs in measure are equal in measue? **Explain your answer.**

What you'll learn

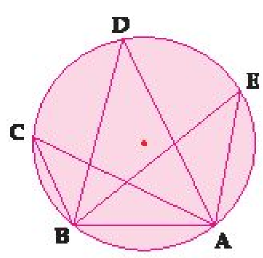
- How to infer the relation between the inscribed angles that include equal arcs in measure

Theorem 2

In the same circle, the measures of all inscribed angles subtended by the same arc are equal in measure.

Given : ∠C, ∠D and ∠E are common inscribed angles at $\overset{\frown}{AB}$

R.T.P : m (∠C) = m (∠D) = m (∠E)



Proof: ∵ m (∠C) = $\frac{1}{2}$ m $(\overset{\frown}{AB})$

, m(∠D) = $\frac{1}{2}$ m $(\overset{\frown}{AB})$

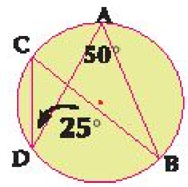
, m (∠E) = $\frac{1}{2}$ m $(\overset{\frown}{AB})$

∴ m (∠C) = m (∠D) = m (∠E) *(Q.E.D.)*

Drill

Study each of the following figures, then complete :

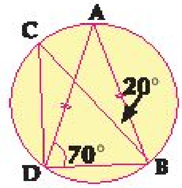
1



m (∠C) = ... °

m (∠ B) = ... °

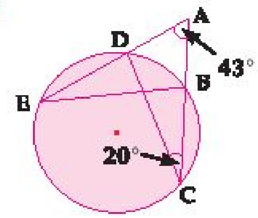
2



m (∠C) = ... °

m (∠ B D C) = ... °

3

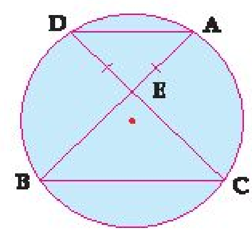


m (∠ BED) = ... °

m (∠ ABE) = ... °

Example 1

In the opposite figure :



$\overline{AB} \cap \overline{CD} = \{E\}$, EA = ED

Prove that : EB = EC .

Solution

In Δ AED ∵ EA = ED ∴ m (∠D) = m (∠A) (1)

∵ ∠ABC , ∠ADC are both inscribed and include $\overset{\frown}{AC}$ ∴ m (∠B) = m (∠D) (2)

∵ ∠DCB , ∠ DAB are both inscribed and include $\overset{\frown}{BD}$ ∴ m (∠C) = m (∠A) (3)

From (1), (2) and (3) we deduce that : m (∠ B) = m (∠C)

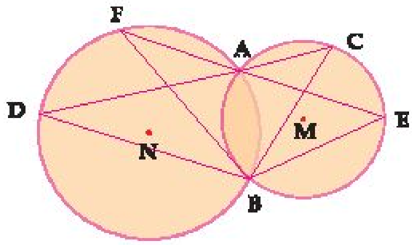
In Δ EBC: ∵ m (∠B) = m (∠C) ∴ EB = EC *(Q.E.D.)*

Drill

In the opposite figure :

M and N are two intersecting circles at A and B. $\overleftrightarrow{AC}$ intersects the circle M at C and intersects the circle N at D, $\overleftrightarrow{AE}$ intersects the circle M at E, and the circle N at F .

Prove that : m (∠ EBC) = m (∠ FBD)

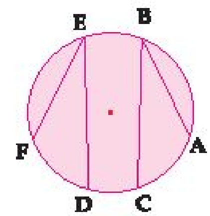


Corollary ***In the same circle or in congruent circles, the measures of the inscribed angles subtended by arcs of equal measures are equal***

Notice that :

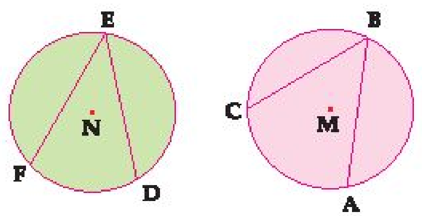
1. In the circle M if : m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{DF})$

   then : m (∠ B) = m (∠ E)



2. For any two circles M and N, if : m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{DF})$

   then : m(∠ B) = m (∠ E)



3 The converse of the previous corollary is true:

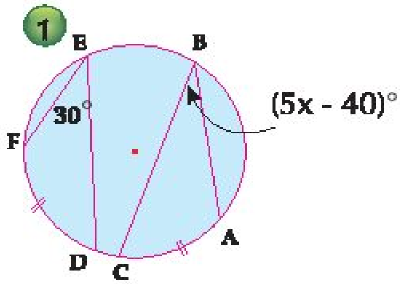
i.e. : In the same circle or in congruent circles, the inscribed angles of equal measures subtend arcs of equal measures.

Think :

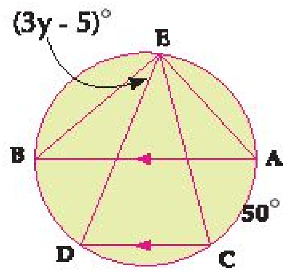
Are each two chords not intersecting inside a circle and subtended by two congruent arcs are parallel? Explain your answer.

Drill

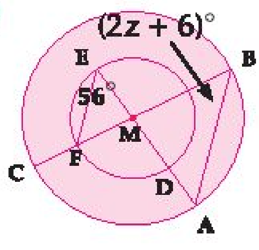
In each of the following figures, find the value of the symbol used in measuring :

1


X = ...

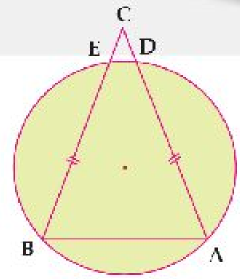
2


Y = ...

3


Z = ...

Example 3



In the opposite figure :

$\overline{AD}$ and $\overline{BE}$ are two equal chords in length in the circle ,

$\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$. **Prove that :** CD = CE .

Solution

AD = BE

Prove that : CD = CE

$\because$ AD = BE $\qquad\qquad$ $\therefore$ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{BE})$

by adding m $(\overset{\frown}{DE})$ to each of the two sides, we get : m $(\overset{\frown}{ADE})$ = m $(\overset{\frown}{BED})$

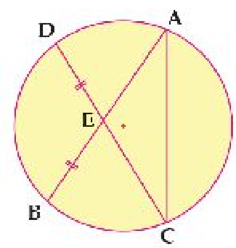
$\therefore$ m $(\angle B)$ = m $(\angle A)$

in Δ ABC $\qquad$ $\because$ m $(\angle A)$ = m $(\angle B)$ $\qquad$ $\therefore$ A C = BC $\qquad$ (1)

$\because$ AD = BE $\qquad$ (2)

By subtracting the two sides of (2) **from** (1) **we get :** CD = CE

Drill

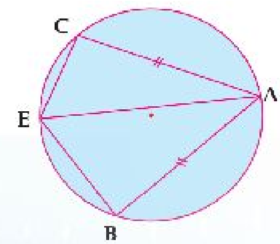


In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle, $\overline{AB} \cap \overline{CD} = \{E\}$.

Prove that : the triangle ACE is an isosceles triangle.

Drill



In the opposite figure :

A B = A C , E $\in \overset{\frown}{BC}$

Prove that : m $(\angle AEB)$ = m $(\angle AEC)$

Think

What is the number of bisectors of $\angle$ BEC? Explain your answer.

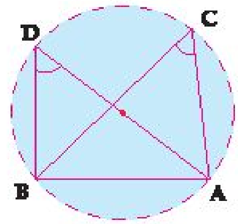
The converse of theorm 2

If two angles subtended to the same base and on the same side of it, have the same measure, then their vertices are on an arc of a circle and the base is a chord in it.

In the opposite figure, notice that :

$\angle$ C , $\angle$ D are both drawn on the base $\overline{AB}$, and on one side of it , m ($\angle$ C) = m ($\angle$ D)

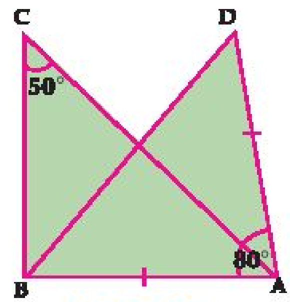
Then : The points A , B , C and D lie on one circle where $\overline{AB}$ is a chord in it.



Example 4

In the opposite figure : AB = AD , m ($\angle$ A) = 80° , m ($\angle$ C) = 50°

Prove that : The points A, B, C and D have one circle passing through them.



Solution

In Δ ABD $\quad\because$ A B = AD , m ($\angle$ A) = 80°

$\therefore$ m ($\angle$ D) = m ($\angle$ A B D) = $\frac{180° - 80°}{2}$ = 50°

$\because$ m ($\angle$ D) = m ($\angle$ C) = 50°

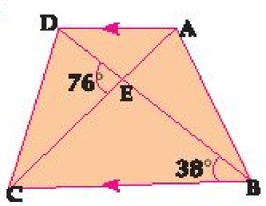
They are both drawn angles on one base $\overline{AB}$ and on one side of it .

$\therefore$ The points A , B , C and D have one circle passing through them **(Q.E.D.)**

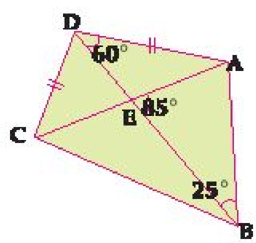
Drill

Which of the following figures can have a circle passing through the points A, B, C and D ? Mention the reason.

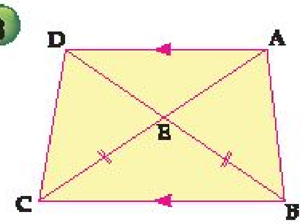
1



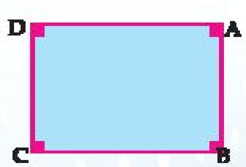
2



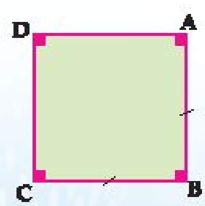
3



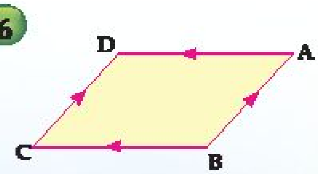
4



5

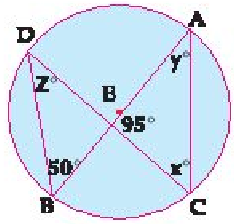


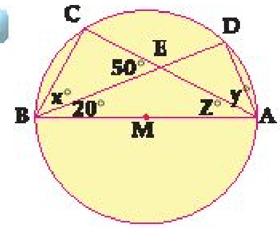
6

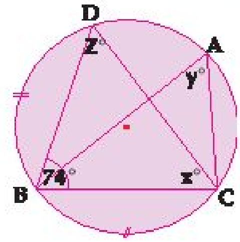


Exercises (5 – 3)

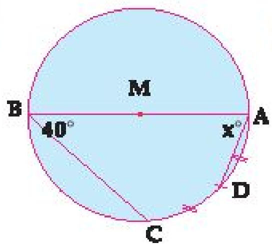
1 In each of the following figures, find the value of the symbol used in measuring :

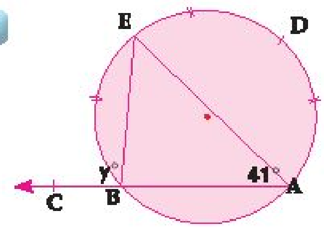
A


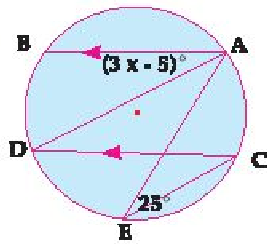
B


C


2 In each of the following figures, find the value of the symbol used in measuring.

A


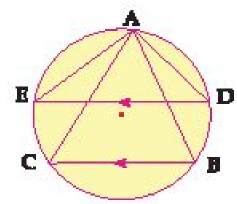
B


C


3 In the opposite figure :

ABC is an inscribed triangle inside a circle , $\overline{DE} \parallel \overline{BC}$.

Prove that : m ($\angle$ DAC) = m ($\angle$ BAE) .



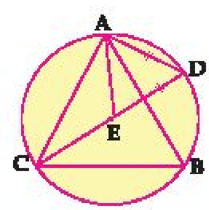
4 $\overline{AB}$ is a diameter in circle M , m ($\angle$ ABC) = 40° , D $\in \overset{\frown}{BC}$.

Find m ($\angle$CDB)

5 ABC is an equilateral triangle drawn inside a circle,

D $\in \overset{\frown}{AB}$, E $\in \overline{DC}$ where AD = DE.

Prove that : The triangle ADE is equilateral.



6 ABC is an isosceles triangle which has AB =AC, D is the midpoint of $\overline{BC}$, draw $\overrightarrow{BE} \perp \overline{AC}$ where $\overrightarrow{BE} \cap \overline{AC}$ = {E}. **Prove that :** the points A , B , D and E have one circle passing through them.

5 - 4

Cyclic Quadrilaterals

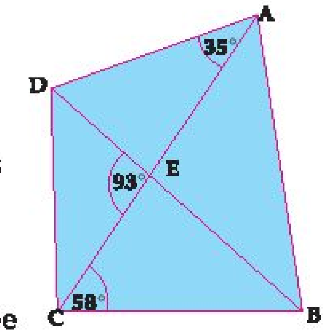
What you'll learn

- The concept of the cyclic quadrilateral.
- Identifying when the shape is cyclic quadrilateral.

Key terms

- Cyclic quadrilateral.

Think and Discuss



In the opposite figure :

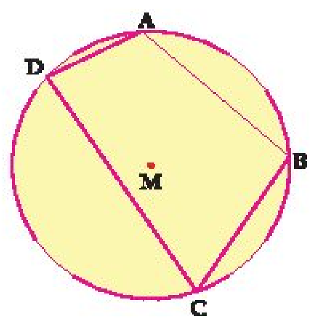
ABCD is a quadrilateral, its diagonals intersect at E,
m (∠ ACB) = 58° , m(∠ CAD) = 35°,
m (∠ CED) = 93°.

Can a circle be drawn passing through the vertices of the quadrilateral ABCD ?
Explain your answer?

Cyclic quadrilateral	**is a quadrilateral figure whose four vertices belong to one circle.**

Notice :

1. **The figure ABCD** is a cyclic quadrilateral because its vertices A, B, C and D belong to the circle M.

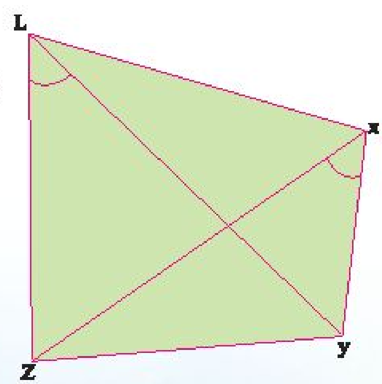


2. **The figure XYZL is a cyclic quadrilateral because:**

m (∠ YXZ) = m (∠ YLZ)

They are two drawn angles on the base $\overline{YZ}$ and in one direction of it, A circle can be drawn passing through the points X, Y, Z and L.

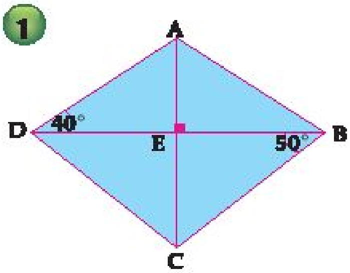
i.e. The vertices of figure XYZL belong to one circle.



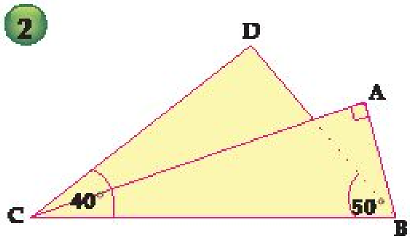
Drill

Which of the following figures is a cyclic quadrilateral ? Explain your answer .

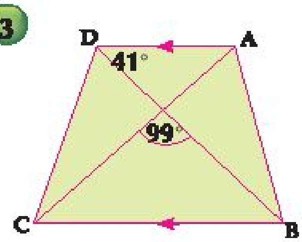
1



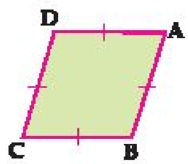
2



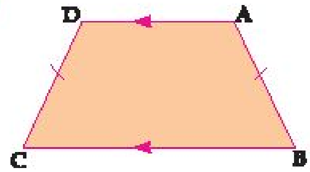
3



4



5

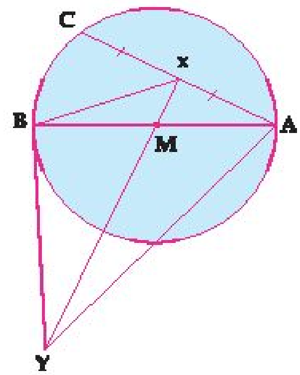


Example 1

In the opposite figure :

$\overline{AB}$ is a diameter in circle M, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersecting the tangent of the circle at B in Y .

Prove that : the figure AXBY is a cyclic quadrilateral.



Solution

Given : $\overline{AB}$ is a diameter in the circle M where AX = CX , $\overrightarrow{BY}$ is a tangent to the circle at B.

R.T.P. : AXBY is a cyclic quadrilateral.

Proof : $\because$ X is the midpoint of $\overline{AC}$ $\therefore$ $\overline{MX} \perp \overline{AC}$, m ($\angle$ AXY) = 90°

$\because$ $\overline{AB}$ is a diameter and, $\overrightarrow{BY}$ is a tangent at B $\therefore$ $\overrightarrow{BY} \perp \overline{AB}$, m ($\angle$ ABY) = 90°

$\because$ m ($\angle$ AXY) = m ($\angle$ ABY) = 90°

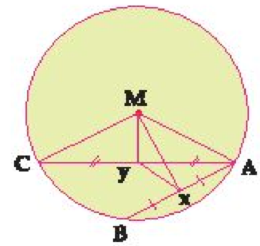
They are two drawn angles on the base $\overline{AY}$ and in one direction of it.

$\therefore$ Figure AXBY is a cyclic quadrilateral.

Think *In the previous example, where is the center of the circle passing through the vertices of the figure AXBY ? located? Explain your answer.*

Drill

In the opposite figure :



A circle with center M. X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : First: AXYM is a cyclic quadrilateral.

Second: m ($\angle$MXY) = m ($\angle$MCY)

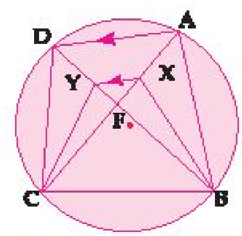
Third : $\overline{AM}$ is a diameter in the circle passing through the points A, X, Y and M

Example 2

ABCD is a cyclic quadrilateral with diagouals intersecting at F, X $\in \overline{AF}$ and Y $\in \overline{DF}$ where $\overline{XY}$ // $\overline{AD}$.

Prove that : First: BXYC is cyclic quadrilateral. **Second:** m ($\angle$XBY) = m ($\angle$XCY)

Solution



Given: ABCD is a quadrilateral inscribed inside a circle, $\overline{XY}$ // $\overline{AD}$

R.T.P.: **Prove that : First:** BXYC is cyclic quadrilateral.

Second: m ($\angle$XBY) = m ($\angle$XCY)

Proof: $\because$ $\overline{XY}$//$\overline{AD}$ $\therefore$ m ($\angle$CAD) = m ($\angle$CXY) *Corresponding*

$\because$ m ($\angle$CAD) = m ($\angle$CBD) *both are inscribed and common in* $\overset{\frown}{CD}$

$\therefore$ m ($\angle$CXY) = m ($\angle$CBY)

and they are two inscribed angles on the base $\overline{CY}$ and in one direction of it.

$\therefore$ BXYC is a cyclic quadrilateral *(Q.E.D 1)*

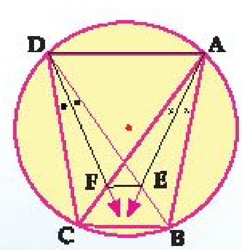
$\because$ BXYC is a cyclic quadrilateral *(Proof)*

$\therefore$ m ($\angle$XBY) = m ($\angle$XCY)

because they are both inscribed angles common at $\overset{\frown}{CD}$. *(Q.E.D 2)*

Drill

In the opposite figure



In the opposite figure : ABCD is a cyclic quadrilateral which has :

$\overrightarrow{AE}$ bisects $\angle$BAC and $\overrightarrow{DF}$ bisects $\angle$ BDC ,

Prove that : First: AEFD is a cyclic quadrilateral

Second: $\overline{EF}$ // $\overline{BC}$.

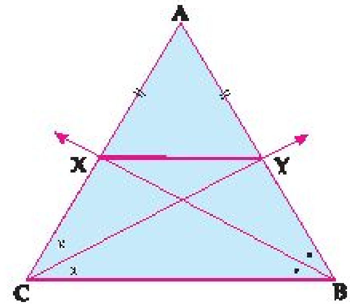
Exercises (5-4)

1 **In the opposite figure :**

ABC is a triangle in which has AB = AC and $\overrightarrow{BX}$ bisects ∠B and intersect $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects ∠C and intersect $\overline{AB}$ at Y

Prove that : **First:** BCXY is a cyclic quadrilateral.

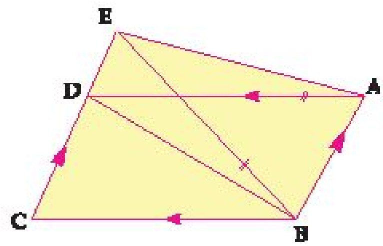
Second: $\overleftrightarrow{XY} // \overleftrightarrow{BC}$.



2 **In the opposite figure :**

ABCD is a parallelogram E ∈ $\overrightarrow{CD}$ where BE = AD

Prove that : ABDE is a cyclic quadrilateral.

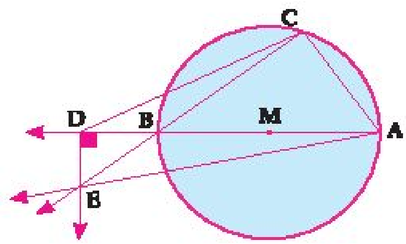


3 **In the opposite figure :**

$\overline{AB}$ is a diameter at circle M, D ∈ $\overrightarrow{AB}$, D ∉ $\overline{AB}$,

draw $\overrightarrow{DE} \perp \overrightarrow{AB}$, C ∈ $\overset{\frown}{AB}$ and $\overrightarrow{CB} \cap \overrightarrow{DE}$ = {E}

Prove that : ACDE is a cyclic quadrilateral.



4 ABCD is a square , $\overrightarrow{AX}$ bisects ∠BAC and intersects $\overline{BD}$ at X and $\overrightarrow{DY}$ bisects ∠CDB and intersects $\overline{AC}$ at Y .

Prove that : **First:** AXYD is a cyclic quadrilateral

Second: m ∠ (AYX) = 45°

5 ABC is a triangle inscribed in circle, X ∈ $\overset{\frown}{AB}$, Y ∈ $\overset{\frown}{AC}$ where m ($\overset{\frown}{AX}$) = m ($\overset{\frown}{AY}$),

$\overline{CX} \cap \overline{AB}$ = {D}, $\overline{BY} \cap \overline{AC}$ = {E} .

Prove that : **First:** BCED is a cyclic quadrilateral

Second: m (∠ DEB) = m (∠ XAB) .

5 - 5 Properties of Cyclic Quadrilaterals

What you'll learn

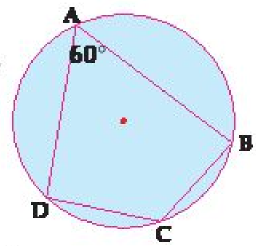
- ☆ Properties of the cyclic quadrilateral shape.
- ☆ How to solve problems on the Properties of the cyclic quadrilateral shape.

Key terms

- ☆ Cyclic quadrilateral.

Think and Discuss

In the opposite figure :



$m(\angle A) = 60°$, then $m(\overset{\frown}{BCD}) = ...°$

- If $m(\overset{\frown}{BAD}) = ...°$
- If $m(\angle BCD) = ...°$
- If $m(\angle B) = 80°$ then $m(\angle D) = ...$
- What do you notice on the sum of the two opposite angles in the cyclic quadrilateral ?

Theorem 3 ***In a cyclic quadrilateral, each two opposite angles are supplementary.***

Given: ABCD is a cyclic quadrilateral.

R.T.P: **Prove that :** (1) $m(\angle A) + m(\angle C) = 180°$

(2) $m(\angle B) + m(\angle D) = 180°$

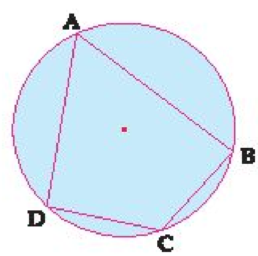
Proof: $\because m(\angle A) = \frac{1}{2} m(\overset{\frown}{BCD})$

, $m(\angle C) = \frac{1}{2} m(\overset{\frown}{BAD})$

$\therefore m(\angle A) + m(\angle C)$

$= \frac{1}{2} [m(\overset{\frown}{BCD}) + m(\overset{\frown}{BAD})]$

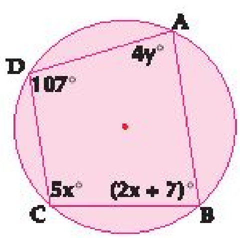
$= \frac{1}{2} \times 360° = 180°$



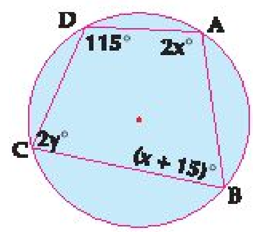
Similarly : $m(\angle B) + m(\angle D) = 180°$

(Q.E.D.)

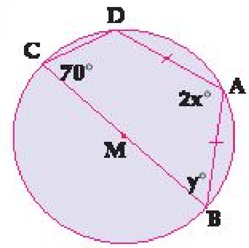
Drill **In each of the following figures, find the value of the symbol used in measuring :**

1


X = ... ,Y = ...

2


X = ... ,Y = ...

3


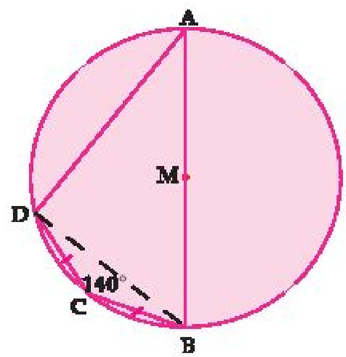
X = ... ,Y = ...

Example 1

ABCD is a quadrilateral inscribed in circle M where M ∈ $\overline{AB}$, CB = CD , m ∠(BCD) = 140°

Find : First : m (∠A) Second : m (∠D)

Solution



∵ ABCD is a cyclic quadrilateral

∴ m (∠A) + m (∠C) = 180° *(theorem)*

∴ m (∠A) = 180° - 140° = 40° *(Q.E.D first)*

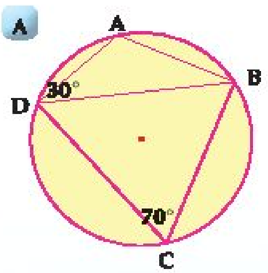
Draw $\overline{BD}$, in Δ BCD ∵ CB = CD

∴ m (∠CDB) = m (∠CBD) = $\frac{180 - 140}{2}$ = 20°

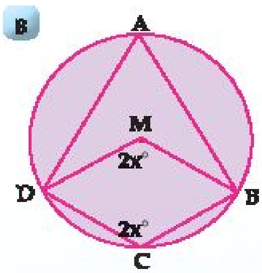
∵ $\overline{AB}$ is a diameter in circle M ∴ m (∠ADB) = 90°

∴ m (∠ADC) = 90° + 20° = 110° *(Q.E.D second)*

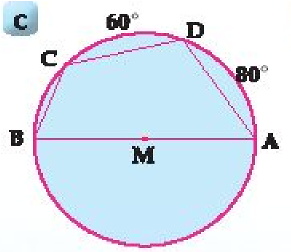
Drill **With the assistance of the given figures, find with proof :**

A


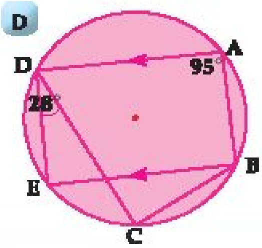
m (∠ABD)

B


m (∠A)

C


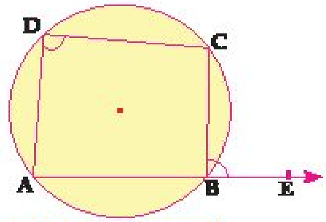
measures of figure's angles ABCD

D


measures of figure's angles ABCD

Corollary *The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.*

In the opposite figure :



ABCD is a cyclic quadrilateral, $E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

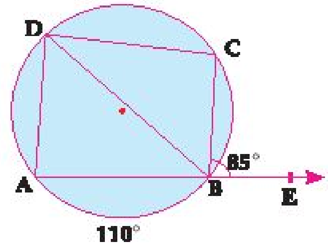
$\therefore$ $\angle$EBC is an angle outside the cyclic quadrilateral ABCD,

$\angle$ D is the inner angle opposite to it.

Thus : m ($\angle$EBC) = m ($\angle$D) *(The supplements of one angle is equal in measure)*

Example 2

In the opposite figure :



$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, m $(\widehat{AB})$ = 110°, m ($\angle$CBE) = 85°

Find m ($\angle$BDC) .

Solution

$\because$ m $(\widehat{AB})$ = 110° , $\angle$ ADB is an inscribed angle with arc $\widehat{AB}$

$\therefore$ m ($\angle$ ADB) = $\frac{1}{2}$ m $(\widehat{AB})$ = 55° .

$\because$ $\angle$CBE is exterior angle at a vertex of the cyclic quadrilateral ABCD

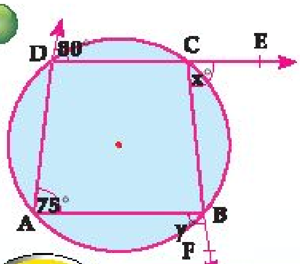
$\therefore$ m ($\angle$CBE) = m ($\angle$CDA) = 85° *(Corollary)*

$\therefore$ m ($\angle$BDC) = 85° - 55° = 30° *(Q.E.E.)*

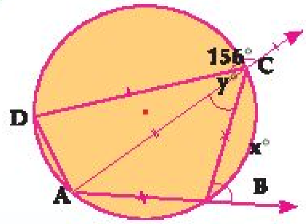
Drill

In each of the following figures, find the value of the symbol used in measuring.

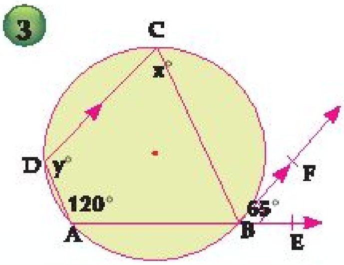
1



2



3



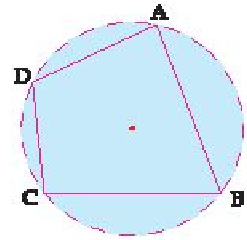
The converse of theorem 3 *If two opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.*

In the opposite figure :

If $m(\angle A) + m(\angle C) = 180°$

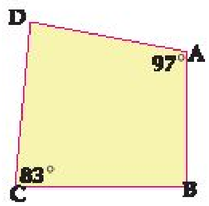
or : $m(\angle B) + m(\angle D) = 180°$

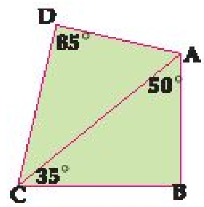
So, ABCD is a cyclic quadrilateral.

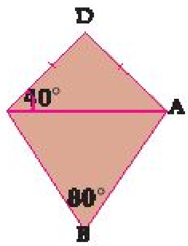


Drill

In each of the following figures, prove that ABCD is a cyclic quadrilateral :

1. 

2. 

3. 

Corollary

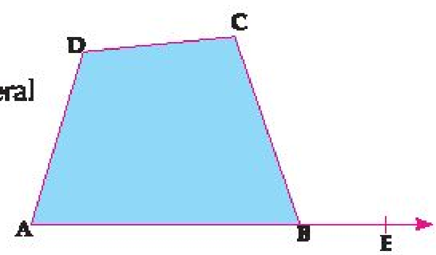
If the measure of the exterior angle at a vertex of a quadrilateral figure is equal to the measure of the interior angle at the opposite vertex, then the figure is a cyclic quadrilateral.

In the opposite figure :

ABCD is a quadrilateral , $E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

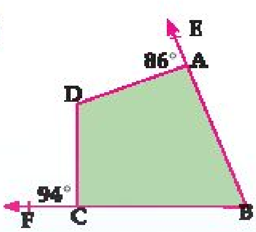
∴ ∠EBC is an exterior angle at a vertex of the quadrilateral ABCD and, ∠ D is the inner angle opposite to it.

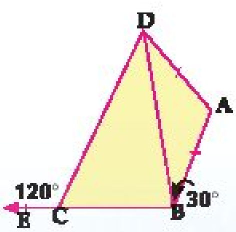
If $m(\angle EBC) = m(\angle D)$ then ABCD is a cyclic quadrilateral.

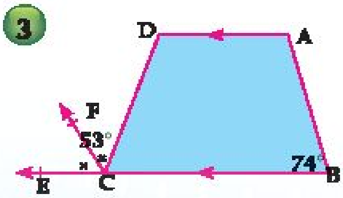


Drill

Prove that each of the following figures is a cyclic quadrilateral:

1. 

2. 

3. 

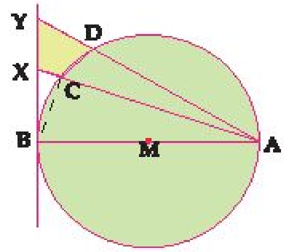
Example 3

In the opposite figure :

$\overline{AB}$ is a diameter in circle M , $\overline{AC}$ and $\overline{AD}$ are two chords in it and in one side from $\overline{AB}$.

A tangent to the circle was drawn from B and intersected $\overrightarrow{AC}$ at X and $\overrightarrow{AD}$ at Y .

Prove that : XYDC is a cyclic quadrilateral.



Solution

Draw $\overline{BC}$ ∵ $\overline{AB}$ is a diameter

∴ m (∠ACB) = 90° and ∠ ABC is complement to ∠BAX (1)

∵ $\overline{AB}$ is a diameter and $\overrightarrow{BY}$ is tangent to the circle at B.

∴ m (∠ABX) = 90° and ∠AXB is complement to ∠BAX (2)

From (1) and (2)

∴ m (∠ABC) = m (∠AXB)

∵ ∠ YDC is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠YDC) = m (∠ ABC) = m (∠ AXB)

∵ ∠ AXB is an exterior angle at the vertex of the quadrilateral XYDC and ∠ YDC is opposite to it.

∴ XYDC is a cyclic quadrilateral.

Think

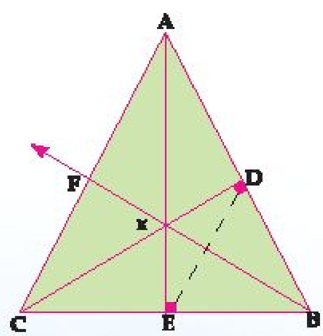
State the cases of the quadrilateral to be cyclic. Mention all the possible cases.

Drill

In the opposite figure, prove that :

The perpindicular line segments on the sides of the triangle from the opposite vertices intersect at one point.

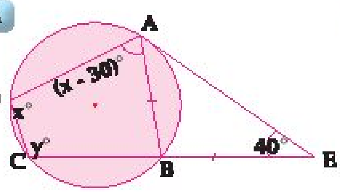
What is the number of cyclic quadrilaterals in the opposite figure? and what are they?



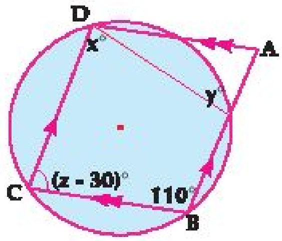
Exercises (5-5)

1 **In each of the following figures, find the value of the symbol used in measuring.**

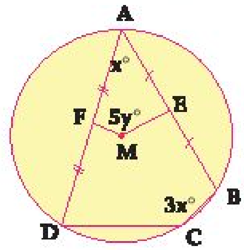
A



B



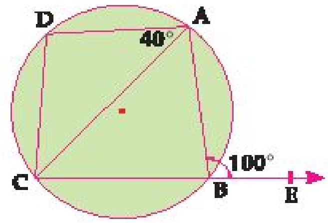
C



2 **In the opposite figure :**

m (∠ ABE) = 100° , m (∠CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$.



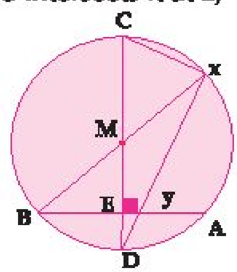
3 **In the opposite figure :**

$\overline{AB}$ is a chord in circle M and $\overline{CD}$ is a perpindicular diameter on $\overline{AB}$ and intersects it at E,

$\overrightarrow{BM}$ intersects the circle at X and $\overline{XD}$ ∩ $\overline{AB}$ = {Y}

Prove that : First : XYEC is a cyclic quadrilateral.

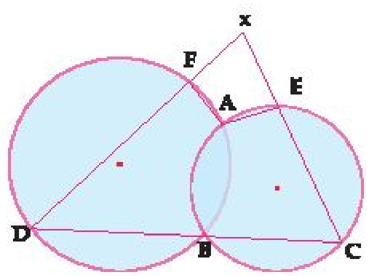
Second: m (∠ DYB) = m (∠ DBX)



4 **In the opposite figure :**

Two intersecting circles at A and B, $\overline{CD}$ passes through point B and intersect the two circles at C and D, $\overrightarrow{CE}$ ∩ $\overrightarrow{DF}$ = {X} .

Prove that : AFXE is a cyclic quadrilateral.



5 ABC is an inscribed triangle in a circle which has AB > AC and D ∈ $\overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects ∠A and intersects $\overline{BC}$ at E and intersects the circle at F.

Prove that : BDEF is a syclic quadrilateral.

5 - 6

The relation between the tangents of a circle

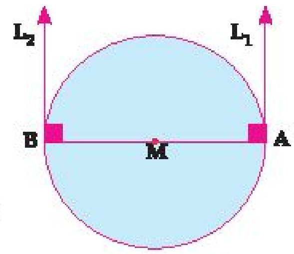
What you'll learn

- How to infer the relation between the two tangent segments drawn from a point outside the circle.
- The concept of a circle inscribed in a polygon.
- How to infer the relation lon the relation between the tangents of a circle.

Key terms

- Chord of tangency.
- A circle inscribed in a polygon.
- Common tangents.

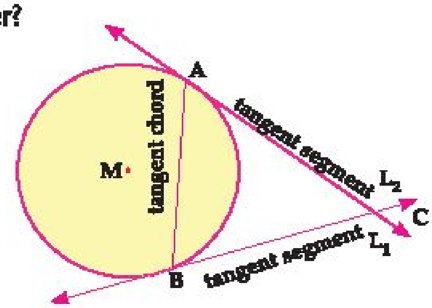
Think and Discuss



You know that the two tangents drawn at the two ends of a diameter in a circle are parallel.

What is the relation between the two tangents drawn at the two ends of a chord of a circle that does not pass through its center?

In the opposite figure :

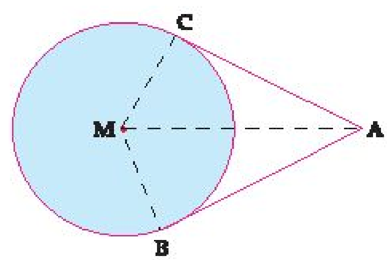


Notice that :

If $\overline{AB}$ is a chord in circle M, then the two tangents L_1 and L_2 intersect at the point C.

Both $\overline{CA}$ and $\overline{CB}$ are called a tangent line segment and $\overline{AB}$ is called a chord of tangency.

Theorem 4 *The two tangent - segments drawn to a circle from a point outside it are equal in length.*



Given : A is a point outside the circle M, $\overline{AB}$ and $\overline{AC}$ are two tangent segments of the circle at B and C.

R.T.P: Prove that : AB = AC

onstruction :

Draw $\overline{MB}$, $\overline{MC}$ and $\overline{MA}$

Proof : $\because$ $\overline{AB}$ is a tangent segment to circle M

$\therefore$ m ($\angle$ABM) = 90°

$\because$ $\overline{AC}$ is a tangent segment to circle M

$\therefore$ m ($\angle$ACM) = 90°

∵ The two triangles ABM and ACM have :

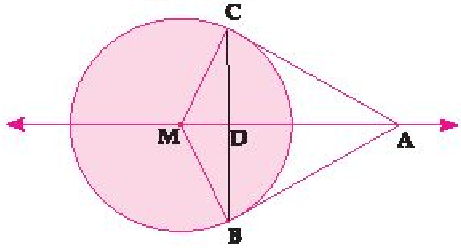
$m(\angle B) = m(\angle C) = 90^\circ$ *(Proof)*

MB = MC *(Lengths of radii)*

$\overline{AM}$ is a common side. ∴ $\triangle ABM \equiv \triangle ACM$

We get : $\overline{AB} \equiv \overline{AC}$ ∴ AB = AC *(Q.E.D.)*

Think **In the opposite figure :**

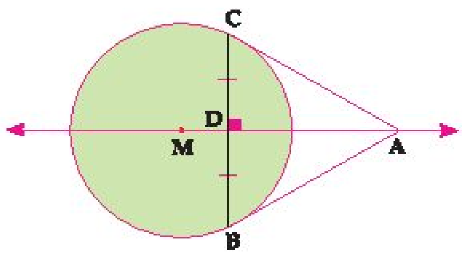


- Why is $\overleftrightarrow{MA}$ the axis of $\overline{BC}$?
- Why does $\overrightarrow{AM}$ bisect $\angle BAC$?
- Why does $\overrightarrow{MA}$ bisect $\angle BMC$?

Theorem corollaries:

Corollary 1 ***The straight line passing through the center of the circle and the intersection point of the two tangent is an axis of symmetry to the chord of tangency of those two tangents.***

In the opposite figure :



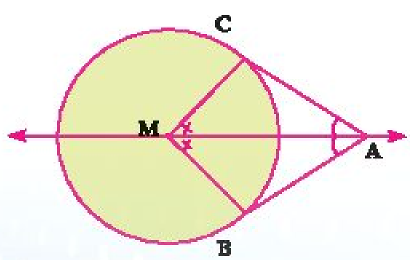
$\overline{AB}$ and $\overline{AC}$ are two tangents to circle M at B and C.

Then : $\overleftrightarrow{AM}$ is the axis of $\overline{BC}$

Thus : $\overleftrightarrow{AM} \perp \overline{BC}$, and BD = CD

Corollary 2 ***The straight line passing through the center of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangenvy.***

In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two tangents to the circle M at B and C.

Then : $\overrightarrow{AM}$ bisects $\angle A$

∴ $m(\angle BAM) = m(\angle CAM)$

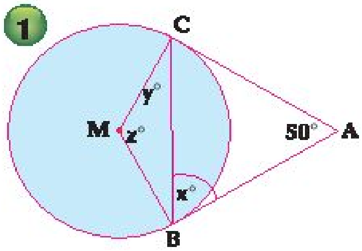
, $\overrightarrow{MA}$ bisects $\angle BMC$

∴ $m(\angle AMB) = m(\angle AMC)$

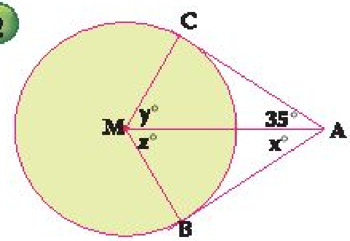
Drill

In each of the following figures, $\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle M. Find the value of the symbol used in measuring :

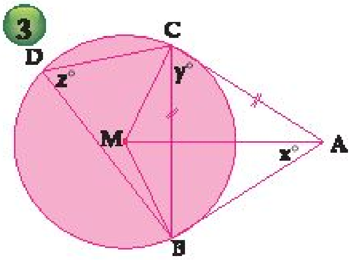
1



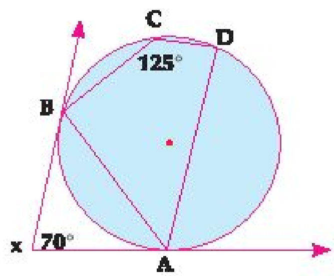
2



3



Example 1



In the opposite figure :

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B.

m (∠ AXB) = 70°, m (∠ DCB) = 125°

Prove that : First : $\overrightarrow{AB}$ bisects ∠ DAX . **Second:** $\overrightarrow{AD}$ // $\overrightarrow{XB}$.

Solution

Given: $\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle,
m (∠AXB) = 70° and m (∠ DCB) = 125° .

R.T.P.: **First :** $\overrightarrow{AB}$ bisects ∠DAX

Second: $\overrightarrow{AD}$ // $\overrightarrow{XB}$.

Proof : ∵ $\overline{XA}$ and $\overline{XB}$ are two tangent segments. ∴ XA = XB

in Δ XAB

∵ m (∠XAB) = m (∠XBA), m (∠X) = 70°

∴ m (∠XAB) = $\frac{180° - 70°}{2}$ = 55° (1)

∵ ABCD is a cyclic quadrilateral, m (∠C) = 125°

∴ m (∠ DAB) = 180° - 125° = 55° *(theorem)* (2)

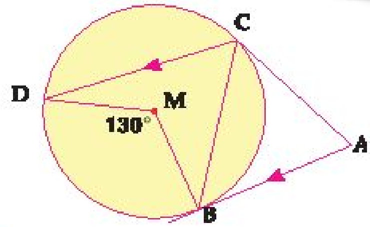
From (1) and (2) we get : m (∠XAB) = m (∠ DAB) = 55°

∴ $\overrightarrow{AB}$ bisects ∠DAX *(Q.E.D First)*

∵ m (∠XBA) = m (∠DAB) = 55° *alternating angle*

∴ $\overrightarrow{AD}$ // $\overrightarrow{XB}$ *(Q.E.D Second)*

Drill



In the opposite figure :

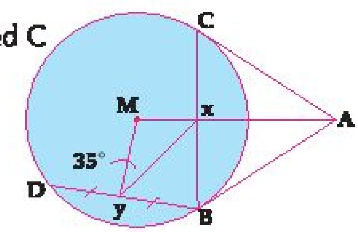
$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle M,

$\overline{AB} \parallel \overline{CD}$, m ($\angle$BMD) = 130° .

(1) **Prove that :** $\overrightarrow{CB}$ bisects $\angle$ ACD

(2) ***Find*** m ($\angle$A) .

Example 2



In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle M at B and C

$\overline{AM} \cap \overline{BC}$= {X}, Y is the midpoint of the chord $\overline{BD}$

m ($\angle$XYM) = 35° .

(A) **Prove that :** XBYM is a cyclic quadrilateral.

(B) ***Find*** m ($\angle$A) .

Solution

$\because$ $\overline{AB}$, and $\overline{AC}$ are two tangent segments to the circle M at B and C

$\therefore$ $\overleftrightarrow{AM}$ is the axis of $\overline{BC}$, m($\angle$BXM) = 90° (1)

$\because$ Y is the midpoint of the chord $\overline{BD}$

$\therefore$ m ($\angle$BYM) = 90° (2)

From (1) **and** (2)

$\therefore$ XBYM is a cyclic quadrilateral. *(Q.E.D 1)*

Draw $\overline{BM}$

$\because$ XBYM is a cyclic quadrilateral, m ($\angle$XYM) = 35° .

$\therefore$ m ($\angle$XBM) = m ($\angle$XYM) = 35°

$\because$ $\overline{AB}$ is a tangent segment and $\overline{BM}$ is a radius.

$\therefore$ m ($\angle$ ABM) = 90°

$\therefore$ m ($\angle$ ABC) = 90° - 35° = 55°

$\because$ AB = AC

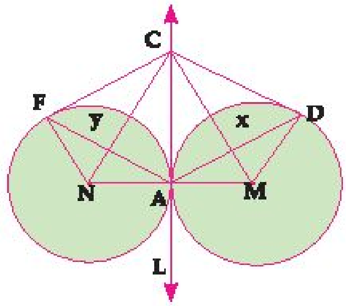
$\therefore$ m ($\angle$ABC) = m ($\angle$ACB) = 55°

$\therefore$ m ($\angle$A) = 180° - (55° + 55°) = 70° *(Q.E.D 2)*

Drill

In the opposite figure :

M and N are two circles touching externally at A. The line L is a common tangent for both of them at A, C ∈ L, Two other tangents were drawn from C to the two circles M and N touching them at D and E respectively $\overline{CM} \cap \overline{DA} = \{X\}$ and $\overline{CN} \cap \overline{AE} = \{Y\}$



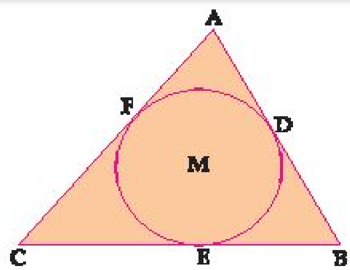
1. **What is the number of cyclic quadrilaterals in the opposite figure ? and what are they ?**
2. **Prove that : CD = CA = CE, and explain this geometrically.**

Definition **The inscribed circle of a polygon is the circle which touches all of its sides internally**

In the opposite figure :

M is the inscribed circle of the triangle ABC because it touches all of its sides internally at D, E and F .

i.e. : The triangle ABC is drawn outside the circle M.



Think ***Is the center of the inscribed circle for any triangle the intersection point of the bisectors of its interior angles ? Explain your answer.***

Example 3

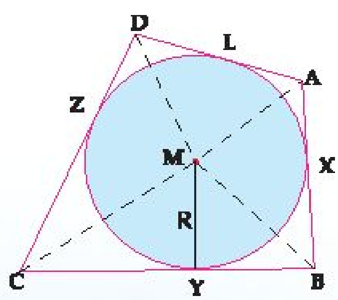
In the opposite figure :

M is an inscribed circle to the quadrilateral ABCD with radius length of 5 cm.

AB = 9cm and CD = 12 cm.

Find the perimeter of ABCD, then calculate its area.

Solution



∵ The circle M is an inscribed circle to the quadrilateral ABCD

∴ The circle M touches the sides of ABCD at X , Y , Z and L

∵ $\overline{AX}$ and $\overline{AL}$ are two tangent segments to the circle M

∴ AX = AL

$\because$ $\overline{BX}$ and $\overline{BY}$ are two tangent segments to circle M $\therefore$ BX = BY

Similiarly, CZ= CY $\therefore$ DZ = DL

By addition, we get : (AX + BX) + (CZ + DZ) = AL + BY + CY + DL

$\therefore$ AB + CD = AD+ BC = $\frac{1}{2}$ the perimeter of ABCD

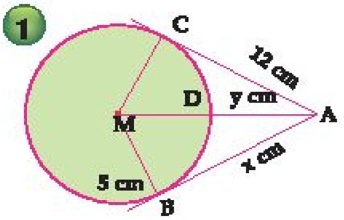
Perimeter of ABCD = 2(9 + 12) = 42cm ,

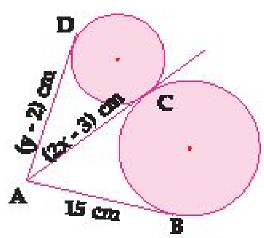
Area of ABCD $= \frac{1}{2} AB \times r + \frac{1}{2} BC \times r + \frac{1}{2} CD \times r + \frac{1}{2} AD \times r$

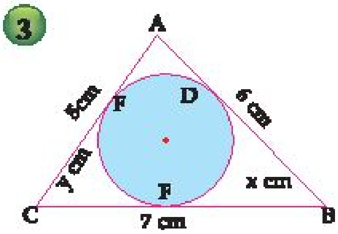
$= \frac{1}{2}$ perimeter $\times r = \frac{1}{2} \times 42 \times 5 = 105 cm^2$

Drill

Find the value of the symbol used in measuring :

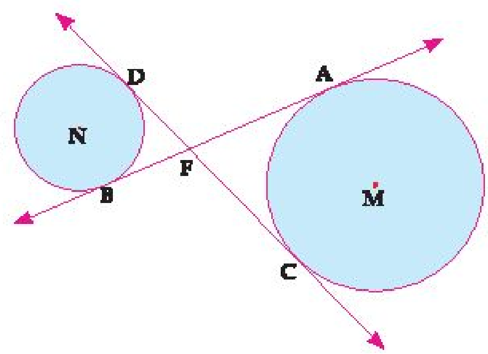
1 

2 

3 

Common tangents of two distant circles :

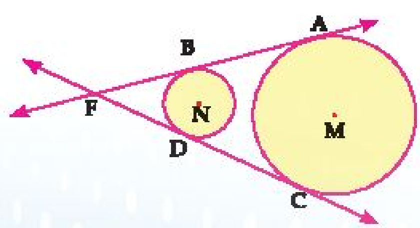
A $\overleftrightarrow{AB}$ is called a common internal tangent to the two circles M and N because the two circles M and N are located at two different sides of $\overleftrightarrow{AB}$, Also $\overleftrightarrow{CD}$ is an internal tangent to the two circles.



Notice that : $\overline{AB} \cap \overline{CD}$ = {E}

In the opposite figure : Prove that : AB = CD

B $\overleftrightarrow{AB}$ is called a common external tangent to the two circles M and N because the two circles M and N are are located in the same side of , $\overleftrightarrow{AB}$, also $\overleftrightarrow{CD}$ is an external tangent to the two circles.

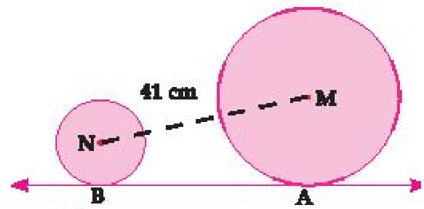


Notice that : $\overrightarrow{AB} \cap \overrightarrow{CD}$ = {F}

In the opposite figure : Prove that : AB = CD

Drill

In the opposite figure : $\overleftrightarrow{AB}$ is a common tangent to the two circles M and N externally at A and B respectively. Their two radii lengths are 17 cm and 8 cm respecticely. If MN = 41 cm, Find the length of $\overline{AB}$

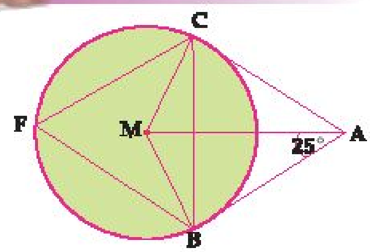


Exercises 5-6

1 **In the opposite figure :**

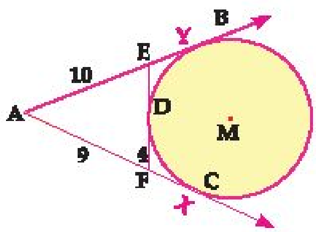
$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle M. m ($\angle$BAM) = 25° , E $\in$ $\overset{\frown}{BC}$ the major .

***Find* : First :** m ($\angle$ACB) **Second:** m ($\angle$BEC).

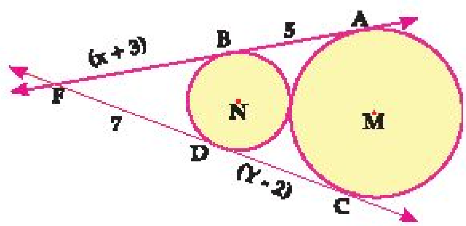


2 **In each of the following figures: Find the value of X and Y in cm.**

A



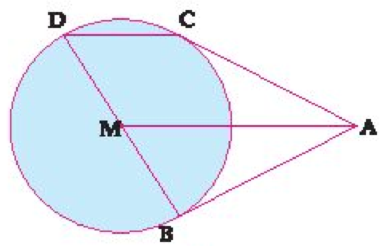
B



3 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle M and, $\overline{BD}$ is a diameter of the circle.

Prove that : $\overline{AM}$ // $\overline{CD}$

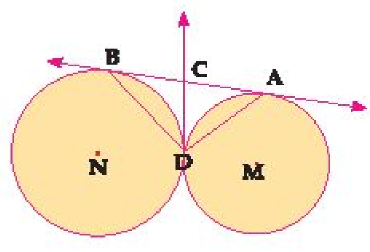


4 M and N are two circles touching externally at D and, $\overleftrightarrow{AB}$ is a common tangent to them at A and B, $\overrightarrow{DC}$ is a common tangent to the two circles at D.

Where $\overrightarrow{DC} \cap \overleftrightarrow{AB}$ = {C} .

Prove that : First : C in the midpoint of $\overline{AB}$.

Second: $\overline{AD} \perp \overline{BD}$.



5 $\overline{AB}$ is a diameter of the circle M, AB = 10 cm. C $\in$ circle M.

A tangent was drawn to the circle at C so, it intersected the two drawn tangents for it at A, B in X, Y respectively where XY = 13 cm

A **Prove that :** $\overline{XM} \perp \overline{YM}$. **B** ***Find*** the area of AXYB.

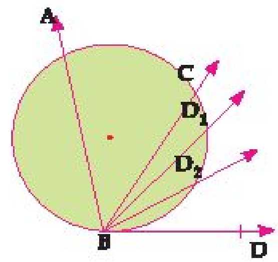
5 - 7

Angles of Tangency

Think and Discuss

In the opposite figure :

$\angle ABC$ is an inscribed angle with the two sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ and arc $\overset{\frown}{AC}$, $\overrightarrow{BD}$ is a tangent to the circle at B. If we imagine the revoltion of one of the sides of the inscribed angle, let it be $\overrightarrow{BC}$ moving away from $\overrightarrow{BA}$ so, it takes one of the positions $\overrightarrow{BC_1}$, $\overrightarrow{BC_2}$, ...



- **Does** the measure of the resulted inscribed angles increase such as $\angle ABC_1$ and $\angle ABC_2$, ...
- **Do the mesures of** m $(\overset{\frown}{AC_1})$ and m $(\overset{\frown}{AC_2})$ increase, ... ?
- If $\overrightarrow{BC}$ and $\overrightarrow{BD}$ are congruent, **what do you notice ?**

Notice that We get a larger inscribed angle in measure when $\overrightarrow{BC}$ and $\overrightarrow{BD}$ are about to be congruent $\angle$ ABD is called the angle of tangency it is a special case of the tangent angle :

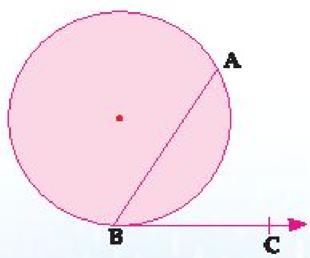
$m(\angle ABD) = \frac{1}{2} m(\overset{\frown}{ACD})$

Angle of Tangency	**The angle which is composed of the union of two rays, one is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.**

Thus :

The measure of the angle of tangency is half the measure of the arc between the two sides.

i.e. : $m(\angle ABC) = \frac{1}{2} m(\overset{\frown}{AB})$



What you'll learn

- The concept of the angle of tangency
- How to infer the relation between the angle of tangency and the inscribed angle subtended by the same arc.
- The relation between the angle of tangency and the centeral angle subtended by the same arc.
- How to solve problems on angels of tangency.

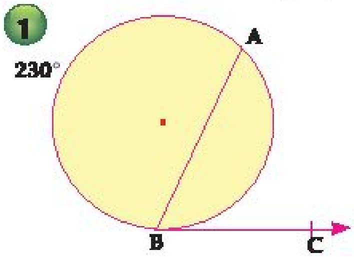
Key terms

- Angle of tangency.
- Inscribed angle.
- Central angle.

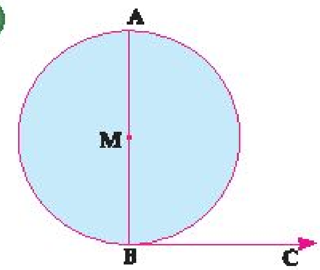
Drill

In each of the following figures, calculate m (∠ABC) .

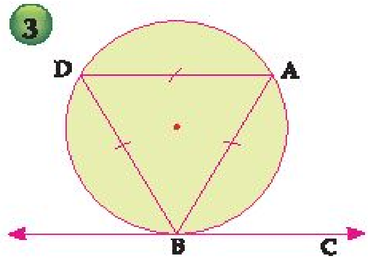
1



2



3



Theorem 5 ***The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.***

Given : ∠ABC is an angle of tangency and, ∠D is an inscribed angle.

R.T.P : **Prove that :** m (∠ABC) = m (∠D)

Proof : ∵ ∠ABC is an angle of tangency

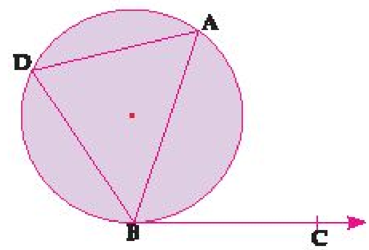
∴ m (∠ABC) = $\frac{1}{2}$ m $(\overset{\frown}{AB})$ (1)

∵ ∠D is an inscribed angle

∴ m (∠D) = $\frac{1}{2}$ m $(\overset{\frown}{AB})$ (2)

From (1) and (2) we get :

m (∠ABC) = m (∠ C)



Q.E.D.

Corollary ***The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.***

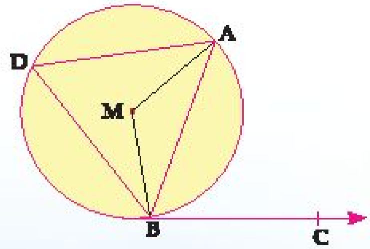
In the opposite figure :

$\overrightarrow{BC}$ is tangent to circle M, $\overline{AB}$ is a chord of tangency

∴ m (∠ ABC) = m (∠D) ***(theorem)***

∵ m (∠D) = $\frac{1}{2}$ m (∠AMB) ***(theorem)***

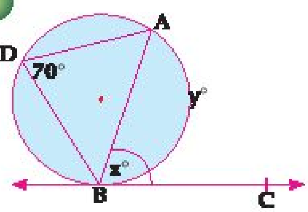
∴ m (∠ ABC) = $\frac{1}{2}$ m (∠ AMB)



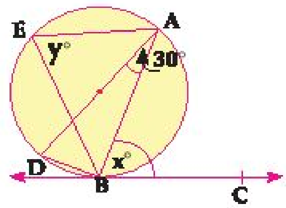
Drill **In each of the following figures :** $\overleftrightarrow{BC}$ is tangent to the circle.

Find the value of the symbol used in measuring.

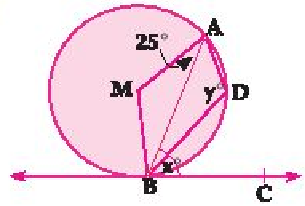
1



2

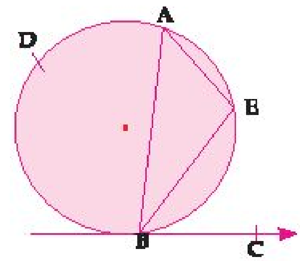


3



Important notice :

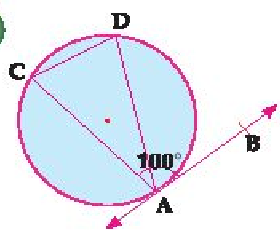
The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.



i.e. : ∠ ABC is supplementary to ∠ AEB .

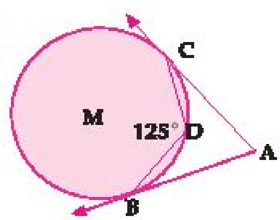
Drill **With the assistance of the given figures, complete :**

1



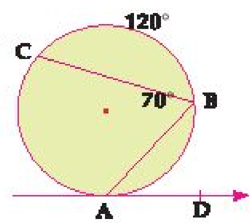
m (∠ADC) = ...°

2



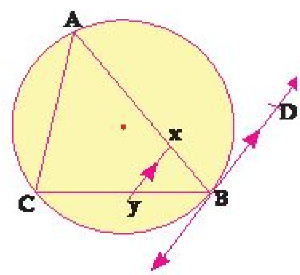
m (∠ BAC) = ...°

3



m (∠ BAD) =..........°

Example 1



ABC is a triangle inscribed in a circle, $\overleftrightarrow{BD}$ is a tangent to the circle at B, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$ Where $\overline{XY}$ // $\overleftrightarrow{BD}$.

Prove that : AXYC is a cyclic quadrilateral.

Proof :

∵ $\overleftrightarrow{BD}$ is tangent to the circle at B, $\overline{AB}$ is a chord of tangency.

∴ m (∠DBA) = m (∠C)

∵ $\overline{XY}$ // $\overleftrightarrow{DB}$, $\overleftrightarrow{AB}$ *intersecting both of them*

∴ m (∠ DBA) = m (∠BXY) ∴ m (∠BXY) = m (∠C)

∵ ∠BXY is exterior from the quadrilateral XYCA.

∴ XYCA is a cyclic quadrilateral. ***(Q.E.D.)***

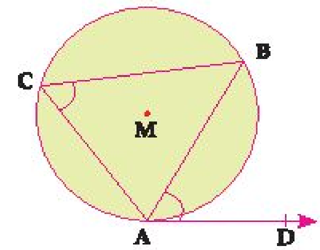
The converse of theorm 5

If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side, then this ray is a tangent to this circle.

i.e. :

If we draw $\overrightarrow{AD}$ from one end of the chord $\overline{AB}$ in circle M and :

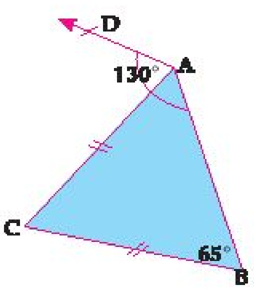
m (∠ DAB) = m (∠ C) **then :** $\overrightarrow{AD}$ is a tangent to circle M.



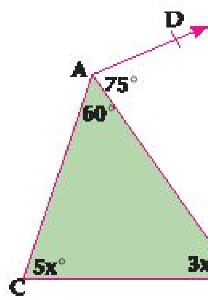
Drill

In each of the following shapes show that AD is a tangent to the circle passing through the vertices of the triangle ABC.

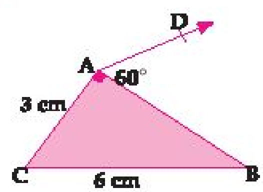
1



2



3

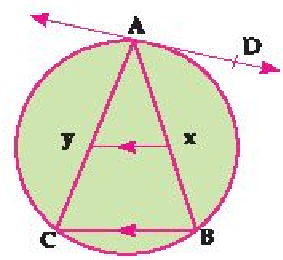


Example 4

ABC is a triangle inscribed in a circle, $\overleftrightarrow{AD}$ is a tangent to the circle at A, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A, X and Y .

Solution



Given: $\overleftrightarrow{AD}$ is a tangent to the circle and, $\overline{XY}$ // $\overline{BC}$

R.T.P.: **Prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y.

Proof: ∵ $\overleftrightarrow{AD}$ is a tangent and, $\overline{AB}$ is the chord of tangeny

∴ m (∠ D A B) = m (∠ C) (1)

∵ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ ***intersector*** ∴ m (∠ AYX) = m (∠ C) (2)

From (1) **and** (2) **we get :** m (∠ DAB) = m (∠ AYX)

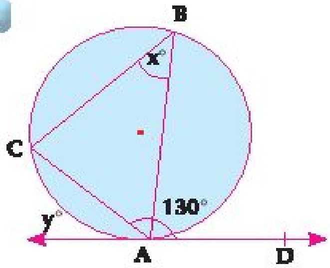
i.e.: m (∠ DAX) = m (∠ AYX)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A, X and Y .

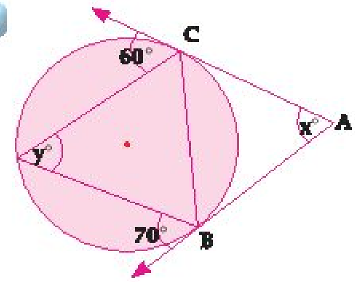
Exercises 5-7

1 **Use the given data on each shape to calculate the used symbol in measure.**

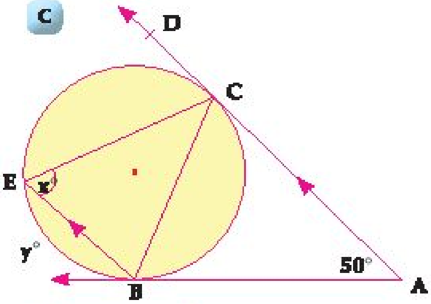
A



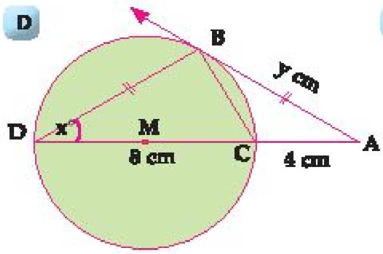
B



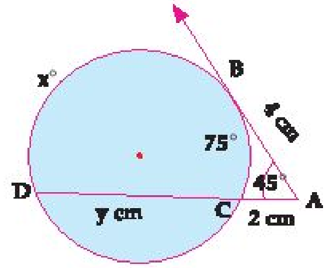
C



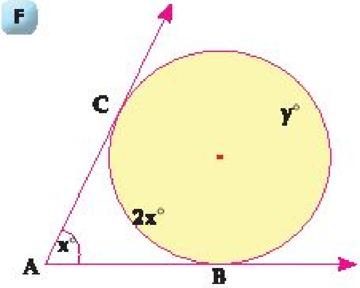
D



F



F



2 ABCD is aquadrilateral inscribed in a circle E is a point outside the circle and, $\overrightarrow{EA}$, and $\overrightarrow{EB}$ are two tangents to the circleat A and B, If m ($\angle$AEB) = 70 ° and m ($\angle$ADC) = 125°

Prove that : First : AB = AC

Second: $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A, B and E

3 ABCD is a quadrilateral inscribed in circle its two diagomals intersect at E, $\overleftrightarrow{XY}$ is drawn to be a tangent to the circle at C Where $\overleftrightarrow{XY}$ // $\overline{BD}$.

Prove that : First : $\overrightarrow{AC}$ bisects $\angle$BAD

Second: $\overleftrightarrow{BC}$ touched the circle passing through the vertices of the triangles ABE

4 ABCD is a parallelogram in which AC = BC.

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC.

General Exercises

1 $\overline{AB}$ is a diameter in circle M, m ($\angle$BAC) = 65°, D ∈ $\overset{\frown}{BC}$

Calculate m ($\angle$ACB), m ($\angle$CDB)

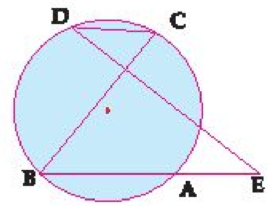
2 $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in circle M, $\overline{AC}$ and $\overline{BD}$ are two perpendicular and intersecting chords at E.

A ***Find*** m ($\angle$CBD)   **B** **Prove that :** $\overleftrightarrow{AD}$ // $\overleftrightarrow{BC}$

3 **In the opposite figure :**

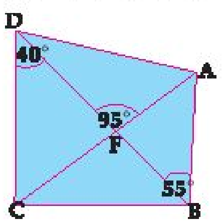
E is a point outside the circle.

Prove that : m ($\angle$E) < m ($\angle$BCD)

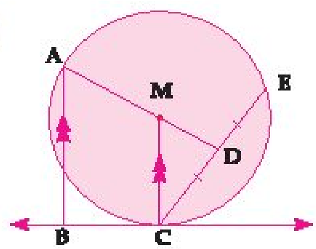


4 **In each of the following shapes, prove that ABCD is a cyclic quadrilateral :**

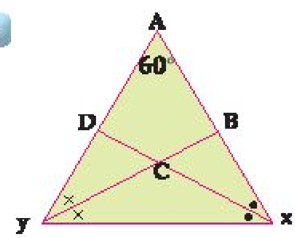
A



B

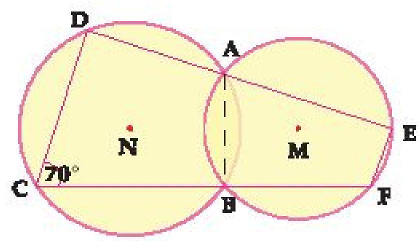


C



5 ABCD is a parallelogram the circle passing through the points, A, B and D intersects $\overline{BC}$ in E. **Prove that :** CD = ED

6 M and N are two intersecting circles at A and B, $\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D. $\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C m ($\angle$C) = 70°.

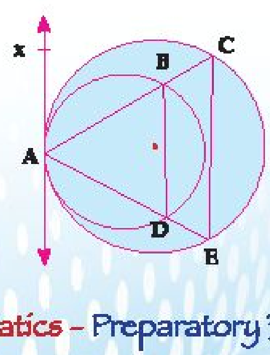


A ***Find*** m ($\angle$F)

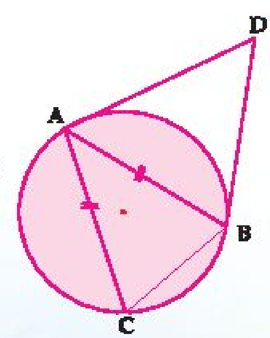
B **Prove that :** $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$.

7 **Use the given data to prove that :**

A $\overline{BD}$ // $\overline{CE}$



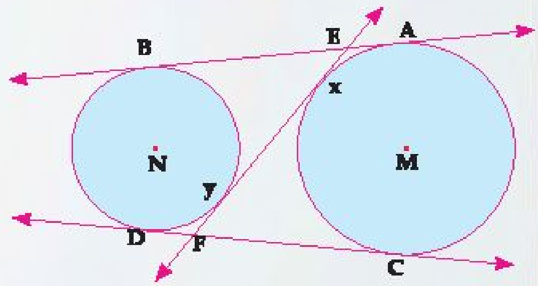
B $\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD



Activity

1 In the opposite figure :

Each point on circle N lies outside circle M. the two points E and F are the two intersetion points of one of internal common tangent $\overleftrightarrow{XY}$ with the two external common tangents $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ respectively:



A What's the relation between the length of $\overline{EF}$, and $\overline{AB}$. Explain your answer.

B **Discuss :** Does the relation change between the length of $\overline{EF}$, and $\overline{AB}$ in the following cases :

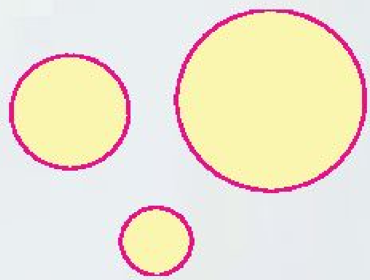
First : If M and N are two congruent circles.

Second: If the surface of circle M $\cap$ the surface of circle N = { Z }

2 The problem of Abulonius :

In the opposite figure: three circles of different radii lengths.

How many circles can be drawn to be tangent to the three circles ?



This problem is known as Abulonius circles.

He was a well known Greek astronomer, engineer and mathematician born in 262 BC and died in 190 BC in Alexandria.

To check your answer, you can use the internet.

Unit test

1 First Complete :

A In the cyclic quadrilateral shape each two opposite angles are ....

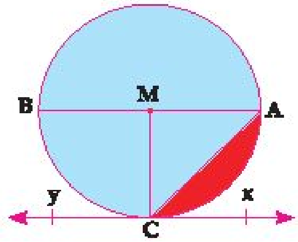
B The center of any circle inscribed in a triangle is the intersection point of ....

Second: In the opposite figure :

M is a circle of a radius length 7 cm,

$\overline{AB}$ is a diameter, $\overleftrightarrow{XY}$ is a tangent to the circle at C

$\overleftrightarrow{XY} // \overleftrightarrow{AB}$.



Choose the correct answer : ($\pi = \frac{22}{7}$)

1 m $(\overset{\frown}{BC})$= ....

A 45 ° B 60 °

C 90° D 180°

2 length $(\overset{\frown}{AC})$ = ...

A 11 cm B 22 cm

C 33 cm D 44 cm

3 The area of the red part = ...

A 154 cm^2 B 77 cm^2

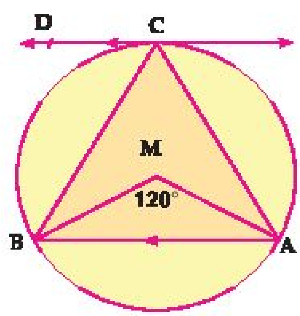
C 38.5 cm^2 D 14 cm^2

2 In the opposite figure :

$\overleftrightarrow{CD}$ is a tangent to the circle at C, $\overleftrightarrow{CD} // \overline{AB}$,

m ($\angle$ AMB) = 120°

Prove that : the triangle CAB is an equilateral triangle.

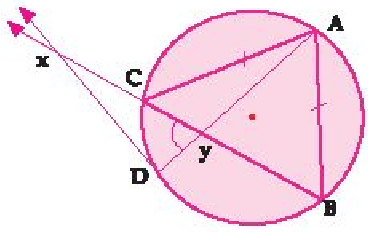


3 ABC is a triangle is inscribed in a circle in which AB = AC

$D \in \overparen{BC}$, $\overrightarrow{DX}$ is drawn to be a tangent to the circle at D

where $\overrightarrow{DX} \cap \overleftrightarrow{BC} = \{X\}$, $\overline{AD} \cap \overline{BC} = \{Y\}$.

Prove that : XY = XD



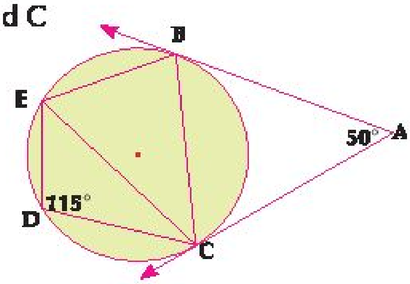
4 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent segment to the circle at B and C

m (∠A) = 50° , m(∠CDE) = 115°

Prove that : First : $\overrightarrow{BC}$ bisects ∠ABE

Second: CB = CE



Model Tests

Algebra - Statistics

Model (1)

(Calculator is allowed) يُسمح بإستخدام الآلة الحاسبة

Answer the following questions :

Question (1) : Choose the correct answer from those given :

1) The domain of the function n (x) = $\frac{x}{x-1}$ is ....

(a) R - {0} (b) R - {1} (c) R - {0 , 1} (d) R - {-1}

2) The number of solutions of the two equations : x + y = 2 and y + x = 3 together is ....

(a) zero (b) 1 (c) 2 (d) 3

3) If x ≠ 0 then $\frac{5x}{x^2+1} \div \frac{x}{x^2+1}$ = ...

(a) - 5 (b) - 1 (c) 1 (d) 5

4) If the ratio between the perimeters of two squares is 1 : 2 then the ratio between their areas = ...

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

5) The equation of the symmetric axis of the curve of the function f where f(x) = x^2 - 4 is ....

(a) x = - 4 (b) x = 0 (c) y = 0 (d) y = - 4

6) If : A ⊂ S of random experiment and P(A`) = 2 P(A), then P(A) = ...

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

Question (2) :

a) By using the general formula : find in R the solution set of the equation :
$2x^2 - 5x + 1 = 0$

b) Find n(x) in the simplest form showing the domain where :

$n(x) = \frac{x-3}{x^2-7x+12} - \frac{4}{x^2-4x}$

Question (3) :

a) Find the solution set of the two equations :

$x - y = 0$ and $x^2 + xy + y^2 = 27$

b) Find n(x) in the simplest form showing the domain where :

$n(x) = \frac{x^2 + 4x + 3}{x^3 - 27} \div \frac{x + 3}{x^2 + 3x + 9}$ then find n(2), n(-3) if possible

Question (4) :

a) A rectangle with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm, Find the area of the rectangle.

b) If : $n(x) = \frac{x^2 - 2x}{x^2 - 3x + 2}$ then find :

(1) $n^{-1}(x)$ in simplest form showing its domain

(2) If $n^{-1}(x) = 3$ then find the value of x

Question (5) :

a) If : $n_1(x) = \frac{x^2}{x^3 - x^2}$ and $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$ then prove that :

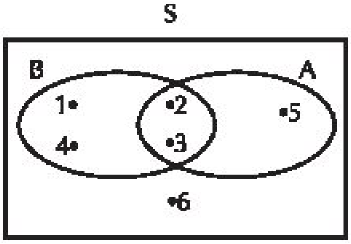
$n_1(x) = n_2(x)$

b) In the opposite figure :

If A and B are two events in a sample space S of a random experiment then, Find :

(1) $P(A \cap B)$ (2) $P(A - B)$

(3) the probability of non-occurrence of the event A.



Model (2)

Choose the correct answer :

1) The solution set of the two equations :
   $x = 3$, $y = 4$ is ....
   [{(3,4)} , {(4,3)} , R , ϕ]

2) The set of zeroes of the function f, where $f(x) = x^2 + 4$ in R is ....
   [{2} , {2 , - 2} , R , ϕ]

3) If A , B are two mutually exclusive events of a random experiment, then
   $P(A \cap B) =$ ....
   [0 , 1 , 0.5 , ϕ]

4) The domain of the multiplicative inverse of the function $f(x) = \frac{x + 2}{x - 3}$ is ....
   [R - {3} , R - {-2,3} , R - {-3} , R]

5) The two straight lines $3x + 5y = 0$, $5x - 3y = 0$ are intersect in ....
   [First quadrant , second quadrant , third quadrant , the origin point]

6) If $P(A) = 0.6$, then $P(A`) =$ ....
   [0.4 , 0.6 , 0.5 , 1]

Question (2) :

a) Find the solution set of the equation :
   $3x^2 - 5x + 1 = 0$ by using the formula

b) Simplify :

   $n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$, showing its domain

Question (3) :

a) Find the solution set of the two equations $x - y = 1$, $x^2 + y^2 = 25$
   $x - y = 0$ and $x^2 + xy + y^2 = 27$

b) If A , B are two events of a random experiment and
   $P(A) = 0.3$, $P(B) = 0.6$, $P(A \cap B) = 0.2$, find $P(A \cup B)$, $P(A - B)$.

Question (4) :

a) Solve in R × R

$2x - y = 3$, $x + 2y = 4$

b) Simplify :

$n(x) = \frac{x^2 + 3x}{x^2 - 9} \div \frac{2x}{x + 3}$, showing its domain.

Question (5) :

a) Simplify :

$n(x) = \frac{x^2 + 2x}{x^2 - 4} + \frac{x - 3}{x^2 - 5x + 6}$, showing its domain.

b) Graph the function f , where $f(x) = x^2 - 1$, $x \in [-3, 3]$, from the graph find the solution set of the equation $x^2 - 1 = 0$

Model Exam

« لطلاب الدمج »

يُسمح بإستخدام الآلة الحاسبة

Answer the following questions in the same paper :

Question (1) : Complete each of the following :

1) The probability of the impossible event = ...

2) The simplest form for the algebraic fraction : $\frac{x - 3}{x^2 - 5x + 6}$ is ....

3) If : $A \subset S$ of random experiment and $P(A) = \frac{1}{3}$, then $P(A`) =$ ....

4) The equation : $3x - x^2 + 1 = 0$ of .... degree.

5) The intersection point of the two straight lines $x = -1$ and $y = 1$ lies on .... quadrant

6) The set of zeroes of the function f where $f(x) = x - 5$ is ....

Question (2) : Choose the correct answer from those given :

1) The solution set of the two equations : $x = 2$ and $xy = 6$ is ....
 a) { (2 , 3) } b) {2 , 3} c) { (3 , 2) } d) {3}

2) The domain of the additive inverse of the fraction $n(x) = \frac{x - 2}{x - 5}$ is ....
 a) R - { 2 } b) R - { 5 } c) R - { 2 , 5 } d) { 2 , 5}

3) The multiplicative inverse of the algebraic fraction $\frac{3}{x^2 + 1}$ is ....
 a) $\frac{-3}{x^2 + 1}$ b) $\frac{x^2 + 1}{-3}$ c) $\frac{x^2 + 1}{3}$ d) $\frac{x^2 - 1}{3}$

4) The domain of the fraction $n(x) = \frac{x + 2}{x - 1}$ is ....
 a) R - { - 2 } b) R - { 1 } c) R - { 1 , - 2 } d) R - { 2 }

5) If $y = 2$ and $x^2 - y^2 = 5$ then $x =$ ....
 a) - 3 b) 3 c) ± 3 d) 9

6) The two straight lines : $x + 2y = 1$ and $2x + 4y = 6$ are ....
 a) parallel b) intersecting c) perpendicular d) coincide

Question (3) : Put (✓) for the Correct Statement and (✗) for the incorrect one:

1) In the equation : $2x^2 - 5x - 4 = 0$
   where $a = 1$, $b = -5$, $c = 4$ ( )

2) The Simplest form of the function n(x) , where
   $n(x) = \frac{x}{x+1} + \frac{1}{x+1} = x + 1$, $x \neq -1$ ( )

3) $\frac{x-1}{5} \times \frac{x+1}{x^2-1} = \frac{1}{5}$, $x \neq \pm 1$ ( )

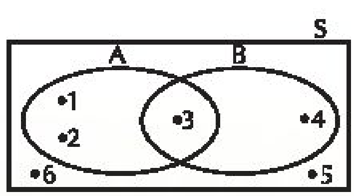
4) If the sum of two numbers is 3 and the sum of their squares is 5 , then
   the two numbers are 1 , 2 ( )

5) If A , B are two mutually exclusive events in the sample space, then
   $P(A \cap B) = 1$ ( )

6) If the probability that of winning a team = 0.7 , then the probability of
   non winning is 0.3 ( )

Question (4) : Join from the Column (A) to the suitable of the Column (B) :

(A)	(B)
1) The solution set of the two equations $x = 2$, $y - 1 = 0$ is	• $\{(2 , 1)\}$
2) The solution set of the equations $ax^2 + bx + c = 0$ is $x =$ .... Where $a \neq 0$, $a,b,c \in R$	• $\frac{x}{x^2+4}$
3) If $n(x) = \frac{x-1}{x+1}$, then the domain of $n^{-1}(x)$ is	• $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
4) If $n_1 = n_2$, $n_1 = \frac{5x}{5x^2 + 20}$, then $n_2 =$	• $R - \{1 , -1\}$
5) The set of zeroes of f(x) , $f(x) = \frac{x-5}{x}$ is	• $\frac{1}{3}$
6) In the opposite figure $P(A - B) =$	• $\{5\}$



Model Tests of Geometry

Model (1)

(Calculator is allowed) يُسمح بإستخدام الآلة الحاسبة

Answer the following questions :

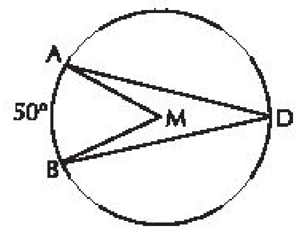
Question (1) : Choose the correct answer from those given :

1) The inscribed angle drawn in a semicircle is ....

(a) an acute (b) obtuse (c) straight (d) right

2) In the opposite figure : circle of center M

If m ($\widehat{AB}$) = 50° then m (∠ADB) = ... °



(a) 25 (b) 50

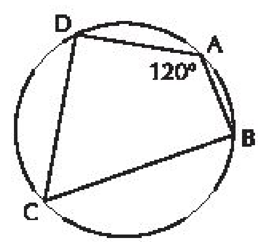
(c) 100 (d) 150

3) The number of symmetric axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number

4) In the opposite figure :

If m(∠A) = 120 °, then m(∠C) = ... °



(a) 60 (b) 90

(c) 120 (d) 180

5) If the straight line L is a tangent to the circle M of diameter 8 cm, then the distance between L and the center of the circle equals .... cm

(a) 3 (b) 4 (c) 6 (d) 8

6) The surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them 3 cm and MN = 8 cm, then the radius length of the other circle = ... cm

(a) 5 (b) 6 (c) 11 (d) 16

Question (2) :

a) Complete and prove that :

In a cyclic quadrilateral, each two opposite angles are ....

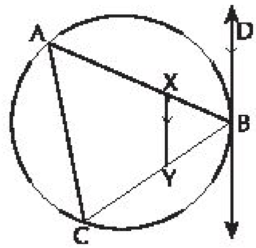
b) In the opposite figure :

ABC is a triangle inscribed in a circle,

$\overleftrightarrow{BD}$ is a tangent to the circle at B

$X \in \overline{AB}$, $Y \in \overline{BC}$ Where $\overline{XY} \,//\, \overleftrightarrow{BD}$.

Prove that : AXYC is a cyclic quadrilateral.



Question (3) :

a) In the opposite figure :

Two circles are touching internally at B.

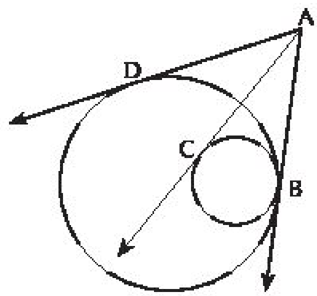
$\overrightarrow{AB}$ is common tangent,

$\overrightarrow{AC}$ is a tangent to the smaller circle at C,

$\overrightarrow{AD}$ is a tangent to the greater circle at D,

AC = 15 cm, AB = (2x - 3) cm and

AD = (y - 2) cm. Find the value of each of x and y.



b) In the opposite figure :

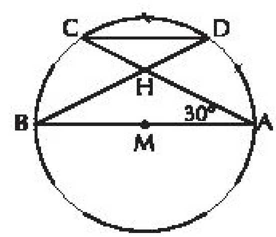
$\overline{AB}$ is a diameter in the circle M, C $\in$ the circle M,

m ($\angle$CAB) = 30°, D is midpoint of $\overset{\frown}{AC}$,

$\overline{DB} \cap \overline{AC}$ = { H } Find :

First : m($\angle$BDC) and m($\overset{\frown}{AD}$)

Second : prove that : $\overline{AB} \,//\, \overline{DC}$



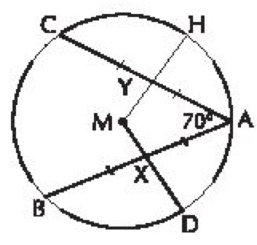
Question (4) :

a) In the opposite figure :

$\overline{AB}$, and $\overline{AC}$ are two chords equal in length in circle M , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$, m($\angle$CAB) = 70°.

(1) Calculate m($\angle$DMH). (2) Prove that : XD = YH.

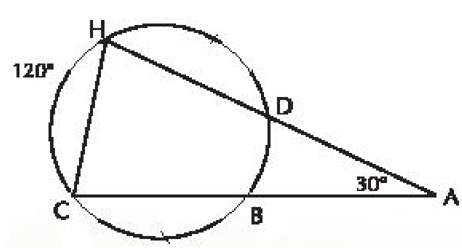


b) In the figure opposite : m($\angle$A) = 30°,

m($\overset{\frown}{HC}$) = 120° , m($\overset{\frown}{BC}$) = m($\overset{\frown}{DH}$)

(1) Find : m($\overset{\frown}{BD}$) «the minor arc»

(2) Prove that : AB = AD

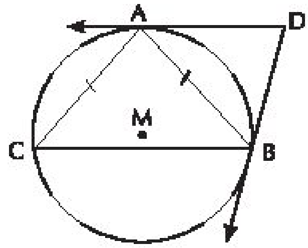


Question (5) :

a) In the opposite figure :

$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents of the circle M and AB = AC

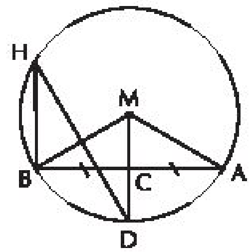
prove that : $\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD



b) In the opposite figure : C is midpoint of $\overline{AB}$

$\overrightarrow{MC} \cap$ the circle M = { D }, m($\angle$MAB) = 20°

Find : m($\angle$BHD) and m($\overset{\frown}{ADB}$)



Model (2)

Question (1) : Choose the correct answer :

1) The measure of the arc which equals half the measure of the circle = ...

(360° , 180° , 120° , 90°)

2) The number of common tangents of two touching circles externally = ...

(0 , 1 , 2 , 3)

3) The measure of the in scribcel angle drawn in a semi-circle = ...

(45° , 90° , 120° , 80°)

4) The angle of tangency is included between ....

(two chords , two tangents , chord and a tangent , a chord and a diametre)

5) ABCD is a cylic quadrilateral, m(∠A) = 60°, then m(∠C) = ...

(60° , 30° , 90° , 120°)

6) If M, N are two touching cicles internally, their radii 5 cm, 9 cm, then MN = ... cm

(14 , 4 , 5 , 9)

Question (2) :

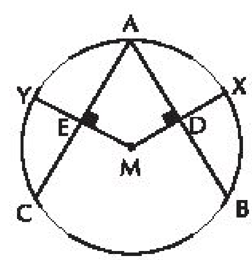
a) In the opposite figure :

AB = AC ,

$\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$, Prove that :

XD = YE



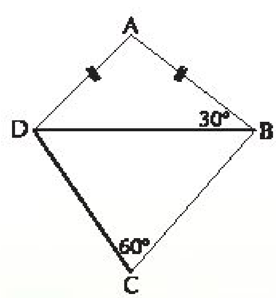
b) In the opposite figure :

ABCD is a quadrilateral in which AB = AD ,

m (∠ABD) = 30°,

m (∠C) = 60°

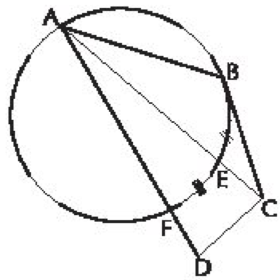
, Prove that : ABCD is a cyclic quadrilateral



Question (3) :

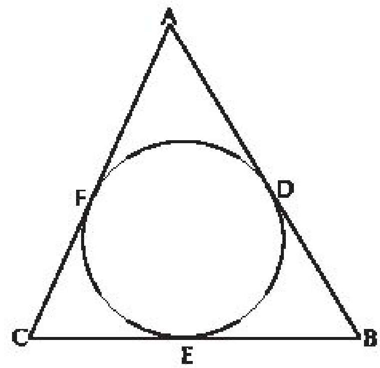
a) State two cases of a cyclic quadrilateral.

b) In the opposite figure :
$\overline{BC}$ is a tangert at B
E is the midpoint of $\overset{\frown}{BF}$,
Prove that : ABCD is
a cyclic quadrilateral.

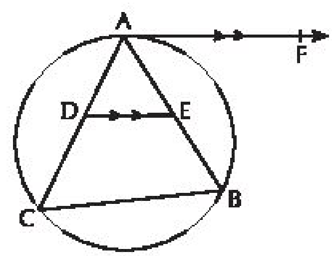


Question (4) :

a) In the opposite figure :
A circle is drawn touches
the sides of a triangle
ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at
D , E , F , AD = 5 cm,
BE = 4 cm , CF = 3 cm
Find the perimeter of Δ ABC.



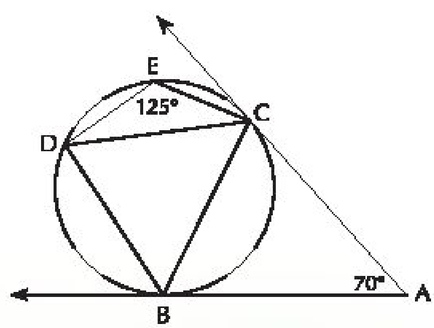
b) In the opposite figure :
$\overrightarrow{AF}$ is a tangent to the
Circle at A, $\overrightarrow{AF}$ // $\overline{DE}$
prove that :
DBCE is a cyclic quad.



Question (5) :

In the opposite figure :
$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents
to the Circle at B , C
, m ($\angle$A) = 70° ,
m ($\angle$CED) = 125° ,
prove that :

- CB = CD
- $\overrightarrow{AC}$ // $\overline{BD}$



Model Exam

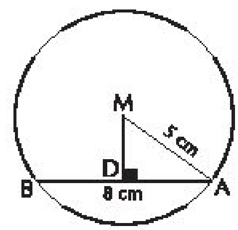
« لطلاب الدمج »

يُسمح بإستخدام الآلة الحاسبة

Answer the following questions in the same paper :

Question (1) : Complete each of the following :

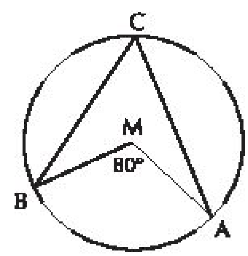
1) The longest chord in the circle is called ....
2) The straight line passing through the center of the circle and the midpoint of any chord is ....
3) The two tangent - segments drawn to a circle from a point outside it are .... in length.
4) **In the opposite figure :**
   The length of $\overline{MD}$ = ... cm



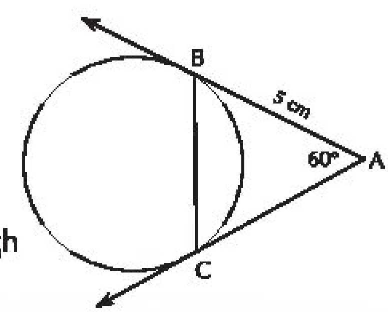
5) The number of symmetric axes of a circle is ....
6) If $\overline{AC}$ is a diameter in a circle M, then m $(\widehat{AC})$ = ... °

Question (2) : Choose the correct answer from those given :

1) If A ∈ the circle M of diameter 6 cm, then MA = ... cm
   (3 , 4 , 5 , 6)
2) **In the opposite figure :**
   m(∠ACB) = ... °
   (40 , 80 , 90 , 180)



3) The number of the common tangents of two distant circles is ....
   (1 , 2 , 3 , 4)
4) **In the opposite figure :**
   The length of $\overline{BC}$ = ... cm
   (3 , 4 , 5 , 6)

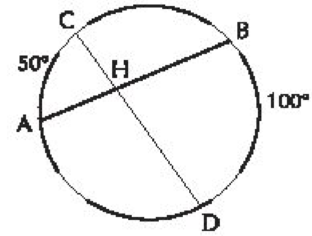


5) The number of circles which can be drawn passes through the end points of a line segment $\overline{AB}$ equals ....
   (1 , 2 , 3 , an infinite)

6) **In the opposite figure :**

m($\angle$AHC) = ... °

(25 , 50 , 75 , 100)



Question (3) :

Put (✓) for the Correct Statement, (✗) for the incorrect Statement :

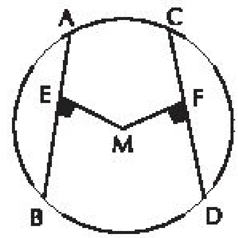
1) If M , N are two touching externally Circles with radii length r_1 = 5 cm, r_2 = 3 cm, then MN = 15 cm. (   )

2) **In the opposite figure :**

If AB = CD ,

ME = 3 cm , then

MF = 3 cm (   )

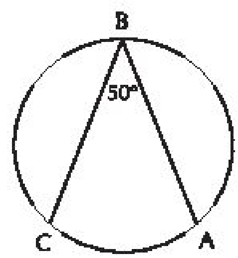


3) The quadrilateral ABCD is a cyclic quadrilateral if

m($\angle$A) + m($\angle$C) = 90° (   )

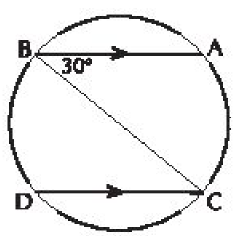
4) **In the opposite figure :**

m ($\overset{\frown}{AC}$) = 100° (   )



5) **In the opposite figure :**

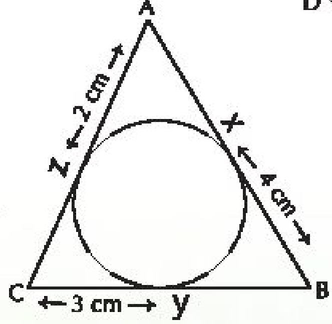
m($\overset{\frown}{AB}$) + m($\overset{\frown}{CD}$) = 300° (   )



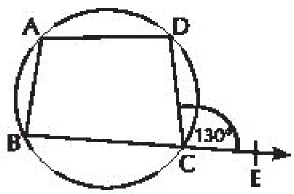
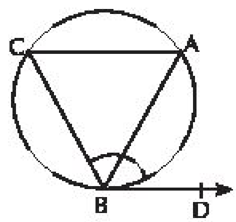
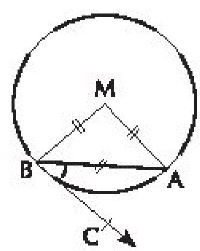
6) **In the opposite figure :**

The Perimeter of

Δ ABC = 9 cm (   )



Question (4) : Join from the Column (A) to the suitable one of the Column (B):

(A)	(B)
1) The measure of the inscribed angle which drawn in a semicircle equals	• 130°
2) **In the opposite figure :** m (∠A) = ... 	• 40°
3) **In the opposite figure :** $\overrightarrow{BD}$ is a tangent at B, m (∠DBC) = 140° , then m (∠A) = ... 	• 90°
4) The radius of the circumcircle of the vertices of right-angled triangle of hypotenuse length 10 cm equals .... cm	• 30°
5) **In the opposite figure :** Δ MAB is an equilateral triangle, $\overrightarrow{BC}$ is a tangent at B, then m (∠ ABC) = ... 	• 2 : 1
6) The ratio between the measure of the central angle, in scribed angle subtended by the same arc is	• 5°

Book size	:	20 x 28
Number of pages	:	156
Weight of paper sheet	:	80 gm
weight of cover sheet	:	200 gm
print colour	:	4 colour
Book No. :	:	1536 / 10 / 15 / 22 / 3 / 25

http://elearning.moe.gov.eg