Scientific

Student book

Mathematics Applications

second secondary grade

SECOND TERM

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2025/2026

Introduction

بسم الله الرحمن الرحيم

Today's world live an age of continuous scientific progress. Tomorrow›s generation needs to be well prepared with the materials of the future in order to be able to match with the massive progress of different science. According to this principle, the Ministry Of Education does its best to develop the curricula via placing the learners in the position of being explorer to the scientific truth besides, training the students on the scientific researches as a way of thinking to make the minds the real materials to the scientific thinking and not to be stores for the scientific facts.

We introduce this book Mathematics Applications» for second secondary grade to be assisting tool to lighten the scientific thoughts of our students and motivate them to search and explore.

In light of what was previously mentioned, the following details have been considered:

★ This book contains mechanics. The book has been divided into related and integrated units. Each unit has an introduction illustrating the learning outcomes, the unit planning guide, and the related key terms. In addition, the unit is divided into lessons where each lesson shows the objective of learning it through the title You will leam. Each lesson starts with the main idea of the lesson content. It is taken into consideration to introduce the content gradually from easy to hard. The lesson includes some activities, which relate Math to other school subjects and the practical life. These activities suit the students different abilities, consider the individual differences throughout Discover the error to correct some common mistakes of the students, confirm the principle of working together and integrate with the topic. Furthermore, this book contains some issues related to the surrounding environment and how to deal with.

★ Each lesson contains examples starting gradually from easy to hard and containing various levels of thoughts accompanied with some exercises titled Try to solve. Each lesson ends in Exercises that contain various problems related to the concepts and skills that the students learned through the lesson.

Last but not least. We wish we had done our best to accomplish this work for the benefits of our dear youngsters and our dearest Egypt.

Contents

Mechanics

Introduction to the development of the science of mechanics

Mechanics, as a general concept, is the science that studies the motion or the balance of bodies through using its own laws; for example there are laws which are applied to the Earth's rotation around the sun and the firing of rockets, a cannon ball or otherwise. It is intended to the change that happens over time to the position of bodies in space. The mutual mechanical effect between bodies is the one by which these bodies change their motions according to the effect of different forces on them. So, the main issue in mechanics is the study of the general laws of the motion and balance of bodies subjected to the action of forces. Mechanics is divided into two branches:

Statics

(**The science of the equilibrium of bodies**) It is concerned with the forces that produce a state of rest in a system of bodies. These forces are known as equilibrium if they don't change the state of the body which is said to be equilibrium under the effect of these forces.

Dynamics

(**The science of the motion of bodies**) It is concerned with the study of forces and their effect on motion of bodies. Dynamics is divided into **Kinematics** which studies motion geometrically (describing motion without reference to the forces causing it), and **Kinetics** which studies the relationship between the motion of bodies and its causes, namely forces.

There are:

Mechanics of Particles (You can ignore the dimensions of the body on studying its motion or equilibrium.)

Mechanics of Rigid Bodies (the body which is consisted of a very large number of connected particles, so close to each other and the distance between any two particles of them is fixed and not affected by any external effect).

Mechanics of Bodies of Variable Mass (Some systems and bodies have varies in which the mass varies as time due to separating out or joining up of particles which decrease or increase during motion. As examples for these bodies, there are the jet rockets and the mining cars; their masses vary as a result of the consumption of fuel and other different systems).

Mechanics of Elastic Bodies (**Elasticity**) It is the property of bodies that are able to return to their original shape and dimensions after being formed; whereas in **plasticity**, if bodies are exposed to external effects, they don’t return to their normal shape on dismissing these effects.

The revelation of mechanics

Classical machanics

It is the oldest branch of mechanics which concerns with the study of forces that act on bodies, It also concerns with the motion of the planets. It helps in many modern technics (constitutive engineering, civil engineering and space's remarks..)

Quantum mechanics

It is a set of physical theories that emerged in the twentieth century, to explain the phenomena at the level of the atom and the particles. It combines between the particle property and the wave length property to show the term of dual wave-particle. Thus, quantum mechanics is responsible for the physical interpretation at the atomic level. Moreover, it is applied on classical mechanics, but its effect doesn’t appear at this level. So, quantum mechanics is the generalization of classical physics to be applied on both atomic and normal levels. It is called quantum mechanics due to the importance of quantity in its structure (it is a physical term used to describe the smallest quantity of energy which could be exchanged between particles, and used also to refer to the finite quantity of energy which emits in a discontinuous state.)

Fluid Mechanics

It is a branch of quantum mechanics and it studies mainly fluid (liquids, gases). This branch studies the physical behavior for the fluids and is divided into fluid statics (studying its rest state), and fluid dynamics(studying its motion state).

Biomechanics:

It is the application of the mechanical principles on the living organisms; this includes the study and analysis of the mechanism of living organisms physically, mechanically , physiologically systems. Some simple examples of biomechanics researches include the study of the forces that act on limbs (organs) in its rest or movement status. Some simple examples of that "the movement of the intestine, the blood flowed, the movement of the nucleus in fallopian tube, the transfer of the liquids in the ureter to the kidney and the digested operation of the food and its movement. The Applied Mechanics plays key roles in the study of biomechanics by which we can discover new cases, suitable to improve the applied state.

General relativity theory

The theory of relativity by Einstein changed a lot of concepts with respect to the basic terms in physics, time, place, mass, and energy which brought about a quantum leap in theoretical physics and space physics in the twentieth century. When first published, it modified Newton’s mechanical theory that existed for two hundred years. The theory of relativity converted Newton’s concept of motion; it stipulates that every motion is relative. The concept of time has been changed from being fixed and definite to yet another non spatial. Time and place has become one thing after being dealt with as two different things. The concept of time is made to depend on the speed of bodies. The dilation and contraction of time has become a key concept for understanding the universe; and so all the Newtonian classical physics have been changed.

Activity

1 - The international web for information (internet) is used to search for the role of mathematicians in improving the science of mechanics. There are some of the searching results:

Thanks to the English scientist **Isaac Newton**, the route of classic mechanics has been prefaced through the laws of motion which illustrated the most of a strological and natural phenomena. The German scientist **Johannes Kepler** and the Italian **Galileo Galilei** have had a great role in putting laws which describe the planets movements.

Kepler's laws show that, there is an attraction power among each of them, and also it shows the movement of planets around the sun according to the new perspective which depends on Heliocentric in a form that calculation in it is matching the astronomical observations substantially. All these rules have been used since the seventeenth century and led to the appearance of the theory of relativity composed by **Einstein** through the years 1905-1916 and the quantum mechanics that composed by the help of **Max plank** , **Heisenberg** , **Schrodinger** and **Dirac** at the beginning of the twentieth century.

Dr. Ahmed Zewail invented a very fast photographic system using laser. It has the ability to determine the motion of the particles when they are formed and when they are connected to each other.

Ahmed Zewail is recorded in the honor list in the United States of America which included **Albert Einstein** and **Alexander Graham Bel**.

For more information search in the Wikipedia using the site ***http: //ar.wikipedia.org***

Measuring Units

When students apply to join the military faculties, some medical tests must be done as height measurment, weight, blood pressure, and average of the beats of the hearts, ...etc.

Measurement operation compares a quantity to another quantity from the same type, to know the number of times the first quantity included into the second quantity.

The system used in most of the parts of the world is called 'International system of units (SI)".

The 'International system of units (SI) include seven basic units. The units of these basic quantities are determined by the direct measurements that depends on the standard units for each of the length, the time, and the weight that was saved in the department of weights and measurements in France. The other units are derivative from the basic units, We are concerned with the following quantities in our study:

First: Fundamental quantities and its measurements units in (SI)

Basic quantities	basic unites	symbols
length	metre	(m)
mass	kilogram	(kg)
time	second	(s)

One of the benefits of using the international system is that: it's too easy to transfer among the units.

Add to your information:

1- Femtosecond:

is a part of a million billion of a second, i.e. (ten raised to the power of -15) and the ratio between the second and femtosecond is as that between the second and thirty two million years.In 1990, the Egyptian scientist, Ahmed Zweil, was able to install his invention which is known as femto-chemistry, after painstaking effort with his research at California Institute of Technology since 1979. His invention can be summed up in inventing of the unit time surpassed the normal time and access to the femtosecond unit of time.He achieved his scientific discovery using ultra short laser flashes and a molecular beam inside a vacuum chamber, a digital camera with unique specifications so as to photo the motion of the particle since birth and before joining the rest of the other particles.It was then possible to intervene rapidly and surprise the chemical reactions as they occur using the laser flashes as a telescope to watch and follow the destruction and construction processes in the cell. This giant Arab scientist has left the door open to the use of this scientific discovery in the field of medicine, physics, spaces researches and others fields; a new scientific school has been recorded by his name and known as femto-chemistry.

2- The multiples of units:

Unit	symbol	measure
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	K	10^{3}

Fractions of units:

units	symbol	measure
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	u	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

According to that we can transfer each of the following units to their corresponding units:

1. 2.75 Km into m.
2. 635 mm. into dm
3. 750 k.Hertz into M.Hertz.
4. 1970 gm into K.gm.

As follow:

1. 2.75 Km =2.75 × 1000 = 2750 m
2. 635 mm. $=635\times10^{-2}$ = 6.35 dm.
3. 750 k.Hertz= 750×10^{-3} =0.75 M.Hertz
4. 1970 gm = 1970×10^{-3} =1.97 kgm

Remember that

Km =1000m

M =10 dm

dm = 10 cm

cm = 10 mm

Second: Derived quantities:

1 Unit of measuring the velocity

Velocity is known as the rate of changing of displacement according to time.

Unit of measuring the velocity = unit of measuring the distance÷unit of measuring the time.

So that, velocity is measured by the unit: m/sec. (m/s)

Do you Know

Normative second : is the time interval in which the cesium atom use to ascillate by one complete cycle

2 Acceleration

Acceleration is known as the rate of changing of velocity according to time. So that, acceleration is measured by the unit: m/sec. square (m/s^2)

Note that

The units used for the vector quantities (velocity - Acceleration - force) concerned on their magnitudes regardless of its direction.

According to that we can transfer each of the following units to their corresponding units:

1. 1 km/h into m/sec.
2. 1 Km/h into cm/sec.
3. 1 km/h/sec into m/sec^2
4. 1 km/h/sec into cm/sec^2

Remember that

The average sun day= 24 hour

Hour = 60 min.

Min. = 60 sec.

As follow:

1. $1Km/h = \frac{1\times1000\ sec}{60\times60\ sec} = \frac{5}{18}$ m/sec

② $1\ \text{Km/h} = \dfrac{1 \times 1000 \times 100^{\text{m}}}{60 \times 60\ \text{sec}} = \dfrac{250}{9}\ \text{cm/sec}$

③ $\text{km/h/sec} = \dfrac{1000\ \text{m}}{60{\times}60\ \text{sec} \times \text{sec}} = \dfrac{5}{18}\ \text{m/sec}^2$

④ $\text{Km/h/sec} = \dfrac{1000 \times 100\ \text{m}}{60 \times 60\ \text{sec} \times \text{sec}} = \dfrac{250}{9}\ \text{cm/sec}^2$

Activity

1. Transfer each of the following units into their corresponding units:
   a. 72 km/h into m/sec
   b. 1000 cm/sec into km/h
   c. 36 km/h/sec into cm/sec^2

3 Force

Force is defined as the product of the mass(m) with the acceleration (a)
If we denoted by (F) to the force ,then $F = m \times a$

Units of measuring the magnitude of the force

Absolute units:

As: Dyne and Newton, where: 1 Newton = 10^5 Dyne .and they will define as follow:

Newton: is the magnitude of the force that if it is acts on a mass equals 1 kilogram it gains an acceleration of magnitude 1 m /sec^2

Dyne: is the magnitude of the force that if it is acts on a mass equals 1 gram it gains an acceleration of magnitude 1 cm /sec^2

Gravitational units:

As: Gram weight (gm.wt) and kilogram weight (Kg.wt), where: 1 Kg.wt = 10^3 gm.wt.

and they will define as follow:

Kilogram weight: is the magnitude of the force that if it is acts on a mass equals 1 Kilogram it gains an acceleration of magnitude 9.8 m /sec^2

Gram weight: is the magnitude of the force that if it is acts on a mass equals 1 gram it gains an acceleration of magnitude 980 cm /sec^2

The Gravitational units joined with **the Absolute units** by the relations: 1 Kg.wt = 9.8 Newton and 1 gm.wt = 980 Dyne

Add to your knowlage

All bodies fell to the ground with uniform acceleration between 9.78. 9.82 m/sec^2 ragardless of their masses, Counting on latitude we will consider the acceleration equals 9.8 m/sec^2 for case of use if there is no other values of it are set.

According to that we can transfer each of the following units into their corresponding units:

1. 3.14 Newton into Dyne
2. 6.75×10^7 Dyne into Newton

As follow:

1. 3.14 Newton $= 3.14 \times 105 = 314000$ Dyne
2. 6.75×10^7 Dyne $= 6.75 \times 10^7 \times 105 = 675$ Newton

Activity

2. Transfer each of the following units into their corresponding units:
   a. $\frac{1}{7}$ gm.wt into Dyne
   b. 5.36×1250 Dyne into Newton
   c. 2.50 Newton into Dyne

You can put the derived quantities in the following table as follow:

Derived quantity	Relation with other quantities	measurement unit
Velocity (V)	Displacement ÷ time	m/s
Acceleration (a)	velocity ÷ time	m/s^2
Force (F)	mass × acceleration	N

Check your understanding

Choose the correct answer from those given:

1. Mass is measured by:
   - a Dyne
   - b Newton
   - c kilogram
   - d kilogram weight

2. From the basic quantities in the international system:
   - a Mass
   - b Velocity
   - c Acceleration
   - d Force

3. Millimeter unit is equivalent to:
   - a 10^{-3} meter
   - b 10^{-3} meter cube
   - c 10^{-2} centimeter
   - d 10^{-4} decimeter

Answer the following questions

4. Mention the name of the following values
   - a 10^{-2} meter
   - b 10^{-3} meter
   - c 1000 meter

5. Transfer each of the following into meter:
   - a 63.4 centimeter
   - b 512.6 millimeter
   - c 0.534 decimeter

6. **Critical thinking:** Calculate in kilogram unit the mass of water that must fill in a container in a form of cuboid with length 1.6 m ,width 0.650 m and height 36 cm ,known that the density of the water equals 1 gm/cm^3 approximating the result to the nearest integer number.

[Hint: mass = volume × density]

Unit one

Dynamics

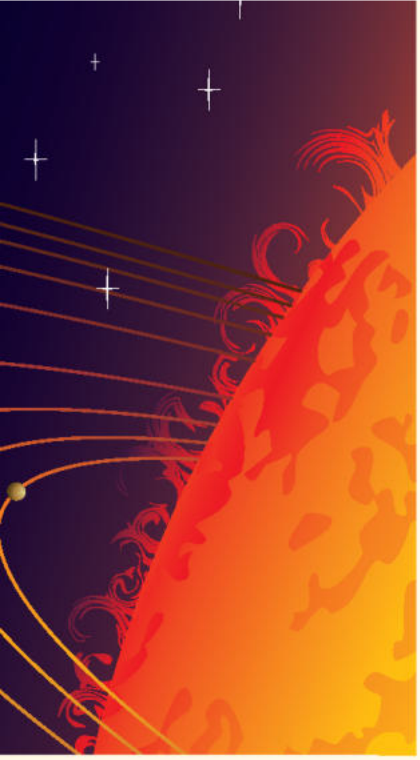
introduction

Dynamics concerns with the study of the object movements and forces causing that movement. Dynamics is divided into kinematics and kinetics in this unit, we are going to study kinematics. Kinematics describes the object movements disregarding the forces cting on . kinematics has its own applied importance in the practical life such as accounting solar and lunar aclipses before they occur. Furthermore, it helps to direct the missles to their targets very accurately and to identify the pathway of spaceships or sattclites and their landing ground points. Kinematics is also used to design mechanical machinaries. As a resut, we are going to study the object movement and the phenomena associated with this movement and its causes.

Unit objectives

By the end of the unit the students should be able to:

- Recognize the concept of the partice as a geometrical point.
- Understand the meaning of the translational motion of a particle from a position to another.
- Realize that the trnaslational motion is taken place if all points of the body move in parallel during the motion .
- Distinguish between distance and displacement .
- Recognize the concept of the uniform velocity (velocity vector - unifotm motion - Average velocity vector - instantaneous velocity vector - measuring units of velocity.
- Distinguish between the concepts of the average velocity vector and the magnitude of the average velocity in the rectilinear motion in a fixed direction.
- Apply the concepts of the velocity, average velocity, acceleration in modelling physical and life application including the movements of (planes, nechets, satellites) as activities .
- Recognize the concept of the relative velocity.
- Deduce the laws of motion with uniform acceleration if a body moves with uniform acceleration and : $v = u + at$, $s = ut + \frac{1}{2} at^2$, $v^2 = u^2 + 2as$
- Recognize the vertical motion under gravity.
- Recognize the applications on the laws of the straight motion with uniform acceleration
- Recognize the laws of the vertical motion"up and down under gravity.
- Recognize the graphical representation of the (displacement - time curve), (velocity - time curve).
- Use graphical calculator to represent the relation between (displacement - time) and (velocity - time) as an activity



Key Terms

- Rectilinear Motion
- Distance
- Vector Velocity
- Average Velocity
- Average Speed
- Relative Velocity
- Vertical Motion
- Displacement
- Uniform Velocity
- Instantaneous Velocity
- Position Vector
- Uniform Acceleration
- Free fall
- Gravity

Materials

- Scientific calculator
- Graphical calculator
- Graphical programs

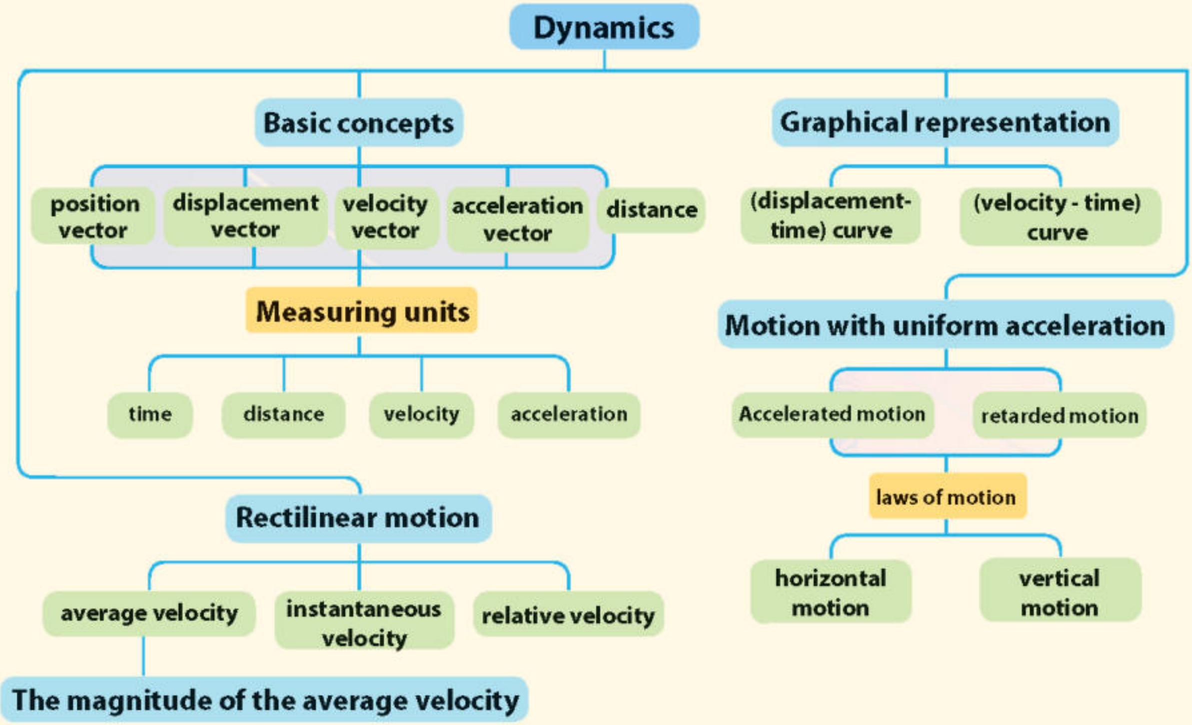
Lessons of the unit

Lesson (1 - 1): Rectilinear motion.

Lesson (1 - 2): Uniformly accelerated rectilinear motion.

Lesson (1 - 3): Vertical motion under the effect of gravity (Free fall).

Chart of the unit



Rectilinear motion

You will learn

- The relation between position vector and displacement vector
- Average velocity
- Average speed
- Instantaneous velocity
- The relative velocity

Key terms

- Rectilinear Motion
- IS
- Displacement Vector
- Position Vector
- Velocity Vector
- Uniform mation
- Average Velocity
- Average speed
- Instantaneous Velocity
- Relative Velocity

Tools

- Squared paper
- Scientific calculator
- Drawing programs

Introduction:

You have learned some measurement systems that began in people's lives. Until the decimal system has been adopted which was invented by the French in 1970 and continued until the consolidated international system IS. It is derived from the word international system of units. This system is formed of basic units in mechanics (mass , length , time) as well as from the derived units which come from products of powers of basic units according to some algebraic relations as (velocity , acceleration , force).

Motion

Rest and Motion:

If a body changes its position with respect to another body related to time, then it said that the first body is in a movement state with respect to the other body. If the relative positions of the two bodies do not change over time, then they will be at rest to each other.

Rest and Motion are two relative concepts. We know that trees and houses are at rest but they seem to be in motion with respect to a moving train.

Motion and its Types

There are many types of motion as the translational motion, rotational motion and harmonic motion .For example, the projected football translates from a position to another position. It may rotate around itself, so it moves in a translational motion and a rotational motion at the same time. The falling water drops move in a translational motion and a harmonic motion at the same time.

We will study only the translational motion assuming the motion of a very small body called particle ,the particle is considered a geometrical point without any dimensions to avoid the theoretical complexes resulted in the rotational and harmonic motions which will be listed in this study.

Translational Motion

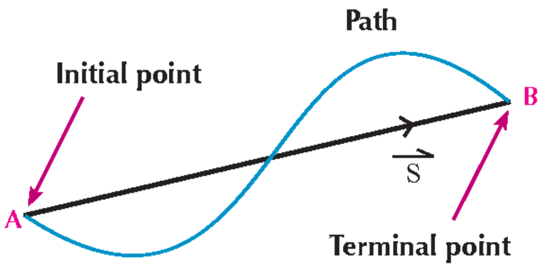
In the translational motion the body moves between two points, the first point is called the initial point and the second point is called the terminal point. The motion of the body in a straight line is an example.

Distance

If a train moves from Cairo to Mansoura, then it covers a distance of 126 Km. The distance is a scalar quantity. It is sufficient to determine the magnitude of the distance. If the magnitude of the distance between the two cities is 126 km, then the number 126 represents a numerical value and (km) is the measuring unit of distance.

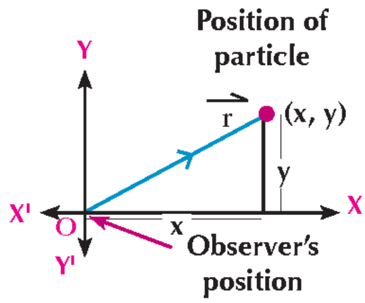
Displacement vector

The displacement vector is the vector represented by the directed line segment $\overrightarrow{AB}$ where its starting point A coincides with the initial position of the body and its terminal point B on its end. The displacement vector $\overrightarrow{AB}$ is denoted by $\overrightarrow{S}$, the norm of the displacement vector $\overrightarrow{AB}$ is denoted by $\|\overrightarrow{AB}\|$ and that may not be equal to the distance covered by the body during its motion.



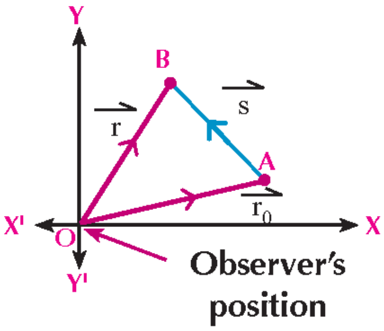
Position vector

The position vector of a praticle is the vector whose initial point coincides with the position of the observer (o) and its end point coincides with the position of the particle. The position vector of the particle is denoted by $\overrightarrow{r}$ where $\overrightarrow{r} = x\overrightarrow{i} + y\overrightarrow{j}$ $\overrightarrow{i}$, $\overrightarrow{j}$ are two perpendicular fundamental unit vectors.



Relation between position vector and displacement vector:

If (O) is the position of the observer and $A(x_1 . y_1)$, $B(x_2 . y_2)$ are the two positions of the particle at two successive moments, then $\overrightarrow{AB}$ is the displacement vector of the particle let it $\overrightarrow{S}$. If we denote the position vector at the moment t with the symbol $\overrightarrow{r_0}$, and the position vector the moment (t + E) with the symbol $\overrightarrow{r}$, then $\boxed{\overrightarrow{S} = \overrightarrow{r} - \overleftarrow{r_0}}$



$\overrightarrow{S} = (x_2\overrightarrow{i} + y_2\overrightarrow{j}) - (x_1\overrightarrow{i} + y_1\overrightarrow{j})$

$= (x_2 - x_1)\overrightarrow{i} + (y_2 - y_1)\overrightarrow{j}$, $\|\overrightarrow{S}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ $\overrightarrow{S} = \|\overrightarrow{S}\|\overrightarrow{n}$, $\overrightarrow{n}$ is the unit vector in direction of $\overrightarrow{S}$ (direction of motion)

Example

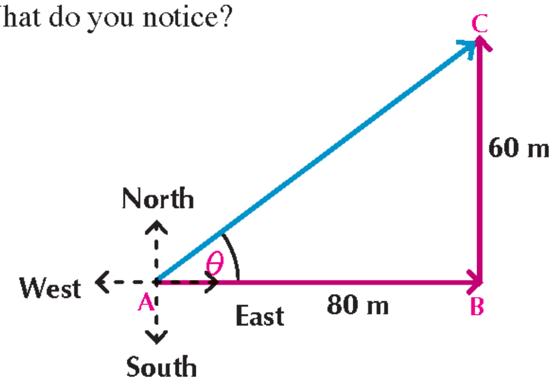
1. A runner moves 80 m. due to East, then he moves 60 m. due to North. Find the distance and the displacement covered by the runner. What do you notice?

Solution

The total distance covered by the runner is the sum of the two distances from A to B then from B to C.

The distance = A B + B C = 80 + 60 = 140 m

The displacement is represented by the directed line segment $\overrightarrow{AC}$



From Pythagoras :

$$AC = \sqrt{(80)^2 + (60)^2} = \sqrt{10000} = 100, \quad \tan\theta = \frac{60}{80}, \quad \text{then } \theta = 36^\circ\ 52'\ 12''$$

So that: the magnitude of the displacement = 100 m and it works in the direction of $36^\circ\ 52'\ 12''$ north of the east.

We notice that:

- The covered distance is a scalar quantity (identified in terms of its value), while the displacement is a vector quantity (identified in terms of its magnitude (value) and its direction).
- The magnitude of the displacement vector $\leqslant$ the covered distance.

Try to solve

1. A bicyclist moves 6 Km. due to West, then he moves 8 Km. with angle 60°south of the west. Find the distance and the displacement covered by the bicyclist.

2. **Critical thinking:** An ant ascends vertically a wall with height 3m, then it return back to its start point. Find the distance and the displacement covered by the ant.

Example

2. A particle moves so that its position vector $\overrightarrow{r}$ is given as a function in time in terms of the fundamental unit vectors $\overrightarrow{i}$, $\overrightarrow{j}$ with the relation: $\overrightarrow{r}(t) = (3t + 2)\overrightarrow{i} + (4t - 1)\overrightarrow{j}$. Find the magnitude of the displacement vector till the moment t = 4

Solution

$$\overrightarrow{r}(0) = 2\overrightarrow{i} - \overrightarrow{j}, \quad \overrightarrow{r}(4) = (3 \times 4 + 2)\overrightarrow{i} + (4 \times 4 - 1)\overrightarrow{j} = 14\overrightarrow{i} + 15\overrightarrow{j}$$

$$\therefore \overrightarrow{S} = \overrightarrow{r}(4) - \overrightarrow{r}(0)$$

$$= (14 - 2)\overleftarrow{i} + (15 + 1)\overrightarrow{j} = 12\overrightarrow{i} + 16\overrightarrow{j}$$

$$, \|\overrightarrow{S}\| = \sqrt{144 + 256} \qquad , S = 20 \text{ length units}$$

Try to solve

(3) In the previous example : Find the magnitude of the displacement vector from $t = 1$ to $t = 3$.

Activity

The curve of (Distance – Time)

The following table shows the relation between the time in seconds and the distance in meter for a runner.

Time in seconds	0	2	4	6	8	10
Distance in meters	0	10	20	30	40	50

1 In the graph paper, determine the time on x- axis and the distance on y-axis.

2 Represent graphically the coordinates of the points in the table.

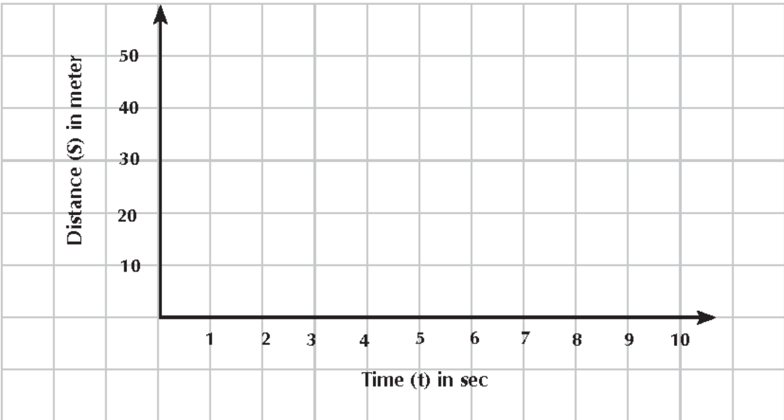
3 Use the ruler to draw the best straight line that passes through the majority of the points in the graph.

4 Use the line that represents the relation between the distance and the time in the times shown in the table. Can you determine the following:

  a The distance covered by the runner after 3 sec ?

  b The time taken by the runner to cover 45 meter ?

5 Can you determine the slope of the straight line that represents the type of the motion of the runner? Explain that.



Velocity

If two runners compete in a certain time interval, then the runner who covers a longer distance is faster than the runner who covers a shorter distance. Velocity can be measured by the covered distance during a certain time interval without determining its direction.The Speedometer in front of the driver determines the magnitude of the velocity without determining the direction of the car.

مقدار السرعة

Remember that

$1\text{km /hr} = \frac{5}{18}\text{ m / sec}$

$1\text{m /sec} = \frac{18}{5}\text{ km / hr}$

Try to solve

4 a Convert 90 km/hr into m / sec   b Convert 15m/sec into km/hr

5 Complete the following table:

$\times\frac{5}{18}$							$\times\frac{18}{5}$
	18km/hr	54km/hr	....km/hr	90km/hr	...km/hr	180km/hr	
	5 m/sec	...m/sec	20m/sec	...m/sec	30m/sec	...m/sec	

Velocity vector

The velocity vector of a particle is the vector whose magnitude equals the value of the velocity, and its direction is the same as the direction of motion.

Verbal Expression:

1- Compare between the velocity and the velocity vector in:

a Definition.   b The type of the quantity (scalar – vector)

Uniform motion and variable motion

Uniform motion: is the case in which the magnintude and the direction of the velocity vector is constant.

So, we will list two important remarks about the uniform motion.

1 - The fixity of the direction of velocity vector: means that the body moves in a fixed direction.

2 - The fixity of the magnitude of the velocity vector: means that the body covered equal distances in equal time intervals in the direction of the motion.

Variable motion: If the motion is not uniform, then the motion is called a variable motion.

In the variable motion the velocity vector of the body changes its magnitude or its direction or both of them from an instant to another.

average velocity

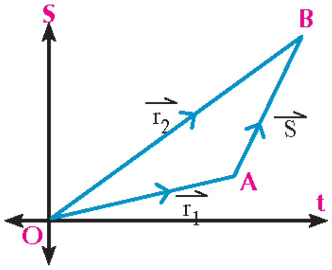
If a car made a trip from Cairo to Hurgada, then the distance between the two towns according to the path of the car reaches 510 km. If the car moves with variable velocities between the two towns and the total time for this trip is 6 hours , So to calculate the average velocity of the car during this trip we will find that:

$$\text{The average velocity } V_A = \frac{\text{Total distance}}{\text{Total time}} = \frac{510}{6} = 85 \text{ Km / hr}$$

So that :

The average velocity is determined by the total covered distance during the whole trip divided by the total time taken in this trip.

Vector of the average velocity

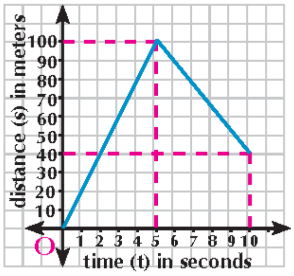


If a particle takes two positions A and B between two successive moments t_1 and t_2 respectively and let $\overrightarrow{S}$ be the displacement vector during the time interval $(t_2 - t_1)$, $\overrightarrow{V_A}$ is known as the vector of the average velocity for that particle through this time interval and it is:

$$\overrightarrow{V_A} = \frac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1} = \frac{\overrightarrow{s}}{t_2 - t_1}$$

Example

الإزاحة



(3) The opposite figure shows the relation between time and distance for the motion of a cyclist the motion starts from the point (0) in a straight line , find:

a) The vector of the average velocity.

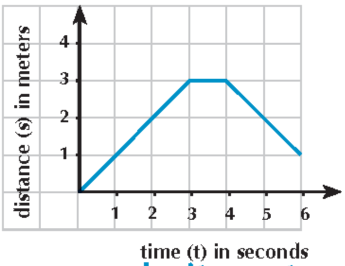
b) The average velocity.

Solution

a) $\overrightarrow{V_A} = \frac{40\ \overrightarrow{n}}{10} = 4\ \overrightarrow{n}$ and its magnitude is 4 m/sec.

b) $V_A = \frac{100 + 60}{10} = 16$ m/sec.

Try to solve

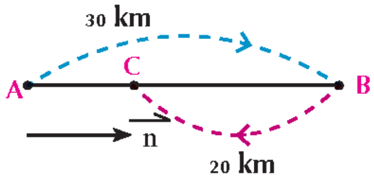


(6) The opposite figure shows a chart for the curve (distance - time) of a rat escapes from a cat. Redraw this figure if the rat escapes from the cat with double its speed.

Example Calculating the average velocity and the average velocity vector

(4) A cyclist covered 30 km on a straight road with velocity 18 km/hr., and then he returned on the same road and covered 20 km in the opposite direction with velocity 15 km/hr. Find the average velocity and the average velocity vector during the whole journey.

Solution



If the cyclist starts his motion from position A towards position B in the first stage, then he returned from B to C in the second stage, let $\overrightarrow{n}$ be the unit vector in the direction of $\overrightarrow{AB}$.

Time of the first stage $t_1 = \frac{d}{v_1}$ then : $t_1 = \frac{30}{18} = \frac{5}{3}$ hours.

Time of the second stage $t_2 = \frac{20}{15} = \frac{4}{3}$ hours.

The total time for the whole journey $= \frac{5}{3} + \frac{4}{3} = \frac{9}{3} = 3$ hours

The displacement $\overrightarrow{S} = 30\overrightarrow{n} - 20\overrightarrow{n} = 10\overrightarrow{n}$

$\because \overrightarrow{V_A} = \frac{\overrightarrow{S}}{t}$ $\therefore \overrightarrow{V_A} = \frac{10\overrightarrow{n}}{3} = 3\frac{1}{3}\overrightarrow{n}$

$\therefore$ The average velocity vector has the same direction as $\overrightarrow{n}$ as in the same direction of $\overrightarrow{AB}$ and it is magnitude equals $3\frac{1}{3}$ km / hr.

Average velocity $= \frac{\text{total distance}}{\text{total time}} = \frac{30 + 20}{3} = \frac{50}{3}$ km/h

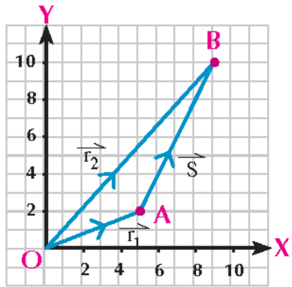
Try to solve

7. A bicyclist covered a distance 25 km on a straight road with velocity 15 km/hr, then he covered a distance of 7 km in the same direction with velocity 7 km / hr. Find the average velocity vector and the average velocity during the whole journey.

Example

5. If a particle takes two positions A(5,2) and B(9, 10) between two successive moments 3sec. and 7sec. respectively. Find the direction of the average velocity of the particle during this time interval, then find the magnitude and the direction of this average velocity.

Solution



The opposite figure shows:

Primary position vector $\overrightarrow{OA}$ ($\overrightarrow{r_1}$),

Final position vector $\overrightarrow{OB}$ ($\overrightarrow{r_2}$),

Displacement vector $\overrightarrow{AB}$ ($\overrightarrow{S}$)

where: $\overrightarrow{S} = \overrightarrow{r_2} - \overrightarrow{r_1}$

$\overrightarrow{S} = (9\,,\ 10) - (5\,,\ 2)$

$\overrightarrow{S} = (4\,,\ 8)$

$\because \overrightarrow{V_A} = \frac{\overrightarrow{S}}{t_2 - t_1}$

$\therefore \overrightarrow{V_A} = \frac{1}{(7 - 3)}(4\overrightarrow{i} + 8\overrightarrow{j})$

$\overrightarrow{V_A} = \overrightarrow{i} + 2\overrightarrow{j}$ (The vector form of the average velocity)

$\|\overrightarrow{V_A}\| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$ unit of velocity

and it makes a polar angle with $\overrightarrow{Ox}$ whose tangent is 2,

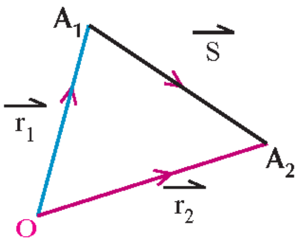
the measure of the angle equals : $63^\circ\ 26'\ 6''$.

Try to solve

8. If a particle takes two positions A(7,2) and B(4, 6) between two successive moments 3sec. and 8sec. respectively. Find the direction of the average velocity of the particle during this time interval, and then find the magnitude and the direction of this average velocity.

Instantaneous Velocity

In the opposite figure: $\overrightarrow{V_A} = \dfrac{\overrightarrow{S}}{t_2 - t_1} = \dfrac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1}$.



If the time interval ($t_2 - t_1$) was very small and the instance (t) is its middle, then the velocity vector in this case is named the instantaneous velocity vector at this instance (t) and is denoted by $\overrightarrow{V}$

Think and discuss

Relative velocity

What do you notice?

- If you sit in a moving train and watch the lamp posts and trees on the side of the road from the window.
- If you sit in a moving car that moves in a certain direction and velocity while you observe other cars moving in the same direction of your car.
- If the other cars are moving in the opposite direction to your car.

We notice from the previous that the movement is a relative concept that differs from an observer to another in another location. In all cases the viewer observes the movements of the other objects conceder himself at rest, even if he is not. The viewer will see the objects moving with velocities that are not the actual velocities, but they are the relative velocities.

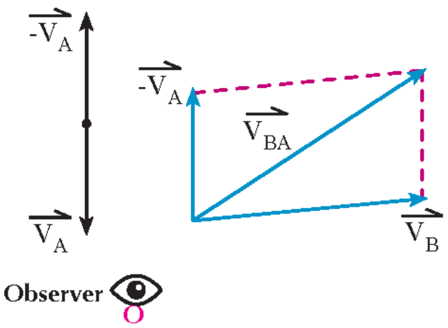
The concept of relative velocity:

The relative velocity for a particle (B) with respect to another particle (A) is the velocity that the particle (B) appears to move with if we assume that the particle (A) is at rest.

Relative velocity vector:

Consider $\overrightarrow{V_A}$, $\overrightarrow{V_B}$ are the two velocity vectors of two bodies A and B with respect to the observer (o) and $\overrightarrow{V_{BA}}$ is the relative velocity vector of B with respect to A.

By adding (- $\overrightarrow{V_A}$) vector to both of the vectors $\overrightarrow{V_A}$, $\overrightarrow{V_B}$ of the bodies A and B, then A become at rest, and the velocity vector of B with respect to A becomes.



($\overrightarrow{V_B}$ - $\overrightarrow{V_A}$) **i.e.:** $\boxed{\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}}$

Critical thinking: If $\overrightarrow{V_{BA}}$ is the velocity vector of B with respect to A , $\overrightarrow{V_{AB}}$ is the velocity vector of A with respect to B, then write the relation between $\overrightarrow{V_{BA}}$, $\overrightarrow{V_{AB}}$

Example

6 A car moves on a straight road with velocity 80 km/hr. A motorcycle moves on the same road with velocity 50 km/hr. Find the velocity of the motorcycle relative to the car If:

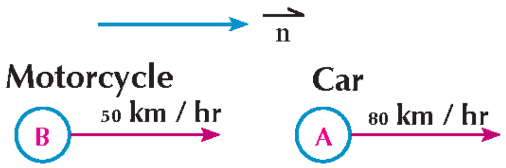
a The motorcycle moves in the same direction of the car.

b The motorcycle moves in opposite direction to the car.

Solution

Let the car be A and the motorcycle be B, and $\vec{n}$ be the unit vector in the direction of the car.

a The motorcycle (B) moves in the same direction of the car (A):



$\vec{V}_B = 50\,\vec{n}$, $\vec{V}_A = 80\,\vec{n}$, The velocity of the motorcycle with respect to the car $\vec{V}_{BA} = ?$

$\because \vec{V}_{BA} = \vec{V}_B - \vec{V}_A$ $\qquad \therefore \vec{V}_{BA} = 50\,\vec{n} - 80\,\vec{n} = -30\,\vec{n}$

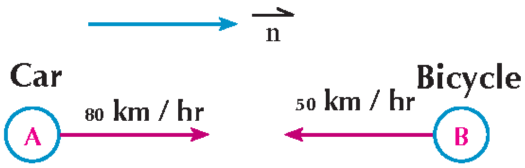
i.e. The motorcycle seems to the driver of the car as it moves away of the car with velocity of magnitude 30 km/hr in the opposite direction of $\vec{n}$.

b The motorcycle (B) moves in the opposite direction of the car (A):

$\vec{V}_B = -50\,\vec{n}$, $\vec{V}_A = 80\,\vec{n}$,

$\because \vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$\therefore \vec{V}_{BA} = -50\,\vec{n} - 80\,\vec{n} = -130\,\vec{n}$



i.e. The motorcycle seems to the driver of the car as it moves towards him with velocity of magnitude 130 km/hr.

Try to solve

9 A car moves on a straight road with velocity 72 km/hr. A motorcycle moves on the same road with velocity 28 km/hr. Find the velocity of the motorcycle relative to the car If:

a The motorcycle moves in the same direction of the car.

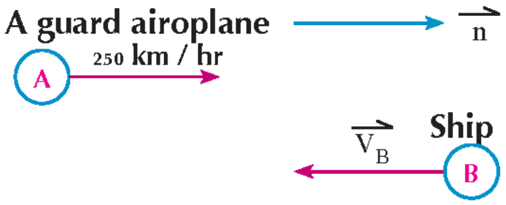
b The motorcycle moves in opposite direction of the car.

Example

7 A ship is moving in a straight way towards a port at 100 km. Far from it, a guard airoplane passes over the ship in the opposite direction with velocity 250 km/hr. The guard airoplane observes the ship, which seems to be moving with velocity 300 km/hr. Find the time from the moment of observation till it reaches the port.

Solution

Let the ship be B and the guard airoplane be A and let $\overrightarrow{n}$ be the unit vector in the direction of the velocity vector of the guard airoplane (A). And the actual velocity of the ship $\overrightarrow{V_B}$ (in opposite direction of the motion of the guard airoplane).



$\therefore\ \overrightarrow{V_A} = 250\ \overrightarrow{n}\ ,\ \overrightarrow{V_{BA}} = -300\ \overrightarrow{n}$

$\because\ \overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}\qquad \therefore -300\ \overrightarrow{n} = \overrightarrow{V_B} - 250\ \overrightarrow{n}$

i.e.: $\overrightarrow{V_B} = -50\ \overrightarrow{n}$

i.e. The magnitude of the actual velocity of the ship equals 50 km/hr, which act in the opposite direction of the motion of the guard airoplane.

$\because\ \overrightarrow{S} = \overrightarrow{V}t \qquad \therefore\ 100 = 50t$

i.e. t = 2 hours

Try to solve

(10) A moving radar car to monitor the velocity on the desert road moves with velocity 40 km/hr. This car observes the movement of a truck coming in the opposite direction. It seems like it is moving with velocity 120 km/hr. What is the actual velocity for the truck?

Exercise (1 - 1)

Complete the following:

(1) 20m/sec = ... km/hr

(2) 90 km/hr = ... m/sec

(3) A car moves with a uniform velocity of magnitude 72 km/hr for a quarter of an hour, then the covered distance = ... km.

(4) If $\overrightarrow{V_A} = 15\ \overrightarrow{i}\ ,\ \overrightarrow{V_B} = 22\ \overrightarrow{i}$ $\qquad \therefore\ \overrightarrow{V_{BA}} =$ ....

(5) If $\overrightarrow{V_{AB}} = 65\ \overrightarrow{n}\ ,\ \overrightarrow{V_A} = 50\ \overrightarrow{n}$ $\qquad \therefore\ \overrightarrow{V_B} =$ ....

(6) A cyclist (A) moves on a straight road with a velocity of 15 km/hr. If he met another cyclist (B) moves with a velocity of 12 km/hr, then the velocity of B with respect to A equals .... km/hr.

Choose the correct answer:

(7) If a car moves with uniform velocity 75 km/hr for 20 minutes, then the covered distance equals .... km

a) 15   b) 20   c) 25   d) 30

8. A car covered a distance of 180 km. With velocity 20 m/sec on a straight road, then the time taken to cover this distance = ... hours

a) $1\frac{1}{2}$ b) - 2 c) $2\frac{1}{2}$ d) 3

9. If $\overrightarrow{V_{AB}} = 15 \overrightarrow{i}$, $\overrightarrow{V_A} = 35 \overrightarrow{i}$ $\therefore \overrightarrow{V_B}$ equals:

a) $-50 \overrightarrow{i}$ b) $-20 \overrightarrow{i}$ c) $20 \overrightarrow{i}$ d) $50 \overrightarrow{i}$

10. If the position vector of a particle moves in a straight line from a point and gives a function in time t by the relation: $\overrightarrow{r} = (2t^2 + 3) \overrightarrow{n}$ then the magnitude of position vector $\overrightarrow{S}$ its norm is measured by meter after 2 seconds equal:

a) 4m b) 6m c) 8m d) 11m

11. **Join with space:** If the sun light reaches the earth in 8.3 min. and the distance between the earth and the sun equals 1.494×10^{11} meter , find the velocity of the light.

12. Two cars moves at the same time from Banha towards Cairo with a constant velocity for each of them. If the velocity of the first car equals 70 km\hr and the velocity of the second car equals 84 km\hr .Find the taken time by the driver of the second car to reach the first car at the end of the trip whose length 49 km?

13. A train of length 150 meter entered a straight tunnel of length S meter. It took the entire crossing of the tunnel in a time of 15 seconds. Find the length of the tunnel if the train moves with uniform velocity equals 90 km/hr.

14. A cyclist covered 30 km on a straight road with a velocity of 15 km/hr, then he returned back and covered 10 km in the opposite direction with a velocity of 10 km/hr. Find the average velocity during his whole trip.

15. A traveller moved on a straight road, he covered 800 meters with velocity 9 km/hr, and then he returned back and covered the same distance in the same direction with velocity 4.5 km/hr., Find the magnitude of the average velocity of the traveller during the whole trip.

16. The distance between two cities A and B is 120 km. A car moved from the city A towards the city B with a velocity of 88 km/hr. At the same moment , another car moved from the city B towards the city A with a uniform velocity of 72 km/hr. Find when and where do the two cars meet.

17. A car (A) moves on a straight road with a uniform velocity 60 km/hr., If another car (B) moves with a uniform velocity of 90 km/hr. on the same road. Find the velocity of car (A) relative to the car (B) if:

a) The two cars are moving in opposite direction.

b) The two cars are moving in the same directions.

18. A police car moves in a straight line with a uniform velocity, if it recorded the relative velocity of a truck moves in its direction in front of it which equals 60 km/hr. ,If the police car doubles its velocity, and recorded the relative velocity of a truck again which seems to be at rest. Find the real velocity of each of the police car and the truck.

Rectilinear motion with Uniformly accelerated

You will learn

- Acceleration
- Time - velocity curve
- Uniformly variable motion
- Relation between velocity - time
- Relation between distance - time
- Relation between velocity - distance

Key terms

- Acceleration
- Uniform variable motion
- Uniform acceleration
- Uniform deceleration

Tools

- Squared paper.
- Scientific calculator.
- Drawing computer programs.

Introduction:

We have studied the regular motion in a straight line with regular speed. It is observed that a small number of objects move this way for a long time, and that each car has three tools to control its speed. They are the gas pedal, brake pedal and the steering wheel which controls the direction of movement of the car. We also observe the change in objects, velocities during their falling and during their projecting upwards.

Learn

Def. 1

Rectilinear Uniformly accelerated motion: it is the motion occurred by the change of the magnitude of the velocity regularly by time and it is called acceleration. Such that :

$$\text{Acceleration (a)} = \frac{\text{final velocity - initial velocity}}{\text{time}}$$

Measuring units: m/sec^2 **or** cm/sec^2 **or** km / hr^2

Notice that:

The change in the velocity at a specific instance of time is called the instantaneous acceleration.

Velocity-Time curve

The concept of acceleration associated with the change in velocity, so if the value of velocity increases with time we say: the motion is accelerated, and the acceleration is positive (as velocity is positive) as in figure (1).

And if the value of the velocity decreases with time we say: The acceleration is negative as in figure (2).

And if the velocity is constant with time we say: the motion is uniform (regular).

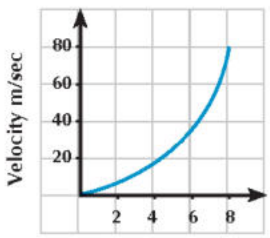


Fig (1)

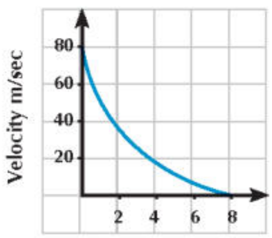


Fig (2)

Uniformly accelerated motion

It was said that the particle moves in a uniformly accelerated motion or with uniformly acceleration if the acceleration vector is constant in magnitude and direction at all times.

Verbal expression: What do the following statements mean?

a. The magnitude of the velocity of a particle increases during its motion a uniformly increasing by the rate 4 m/sec^2.

b. The magnitude of the velocity of a particle decreases during its motion a uniformly decreasing by the rate 24 km/hr^2.

Example

1. If the velocity of a moving car in a straight line changes from 50 km/hr to 68 km/hr during 10 seconds. If a truck has been moved from rest, till its velocity becomes 18 km/hr during this period. Which of them move with higher acceleration? Explain your answer.

Solution

From the data, it is shown that both of them (the car, the truck) have an increase in velocity with magnitude 18 km/hr (i.e. 5 m/sec) during a period of time equals 10 sec. which means that the acceleration is equal for each of them.

then: The acceleration that each of them move with is:

$$\because \ a = \frac{\text{change in velocity}}{\text{time interval}} = \frac{5}{10} = \frac{1}{2} \ m/sec^2$$

Try to solve

1. If the velocity of a moving car(A) in a straight line changes from 24 km/hr into 36 km/hr during 5 seconds, and the velocity of another car (B) moves in the same straight line changes from 12 km/hr into 30 km/hr during the same period of time. Which of them moves with higher acceleration? Explain your answer.

Equations of the uniform variable motion in a straight line

There are three basic equations link between the algebraic magnitude for the vectors of displacement, velocity, acceleration and time in the case of moving with uniform acceleration and they are:

First: the relation between the velocity and time:

If a particle moves in a straight line by an initial velocity vector $\vec{V_0}$, fixed acceleration vector $\vec{a}$ and its velocity vector become $\vec{V}$ after an interval of time (t) then :

$$\vec{a} = \frac{\vec{V} - \vec{V_0}}{t} \quad \text{i.e.:} \quad \boxed{\vec{V} = \vec{V_0} + \vec{a}\, t}$$

by taking the algebraic measurement it will be $\boxed{V = v_0 + a\,t}$

Note that:

- From the relations between four unknowns, we can find one of them by knowing the other three.
- If the body started its motion from rest , So $v_0 = 0$, then $V = at$
- If $a = 0$ then $V = V_0$ i.e. the body moves with uniform velocity.

Example

(2) A particle starts its motion in a constant direction with velocity 9 cm/sec and with a uniform acceleration of magnitude $3 cm/sec^2$ acts in the same direction of the initial velocity. Find:

(a) The velocity of the particle after 5 seconds from the starting of the motion.

(b) The time it takes from the starting of the motion until its velocity become 54 cm/sec.

Solution

(a) We assume that the positive direction is the direction of the motion of the particle.
From the givens of the problem: $v_0 = 9cm/sec$, $a = 3cm/sec^2$, $t = 5$ seconds.
$\because\ V = v_0 + a\,t \qquad \therefore V = 9 + 3 \times 5 \qquad \therefore\ V = 24$ cm/sec.

(b) $\because\ V = v_0 + a\,t \qquad \therefore\ 54 = 9 + 3\,t \qquad \therefore\ t = 15$ seconds.

Try to solve

(2) A particle starts it motion in a constant direction with velocity 20 cm/sec and with a uniform acceleration $5 cm/sec^2$ acts in the same direction of vector of the initial velocity. Find:

(a) Its velocity at the end of one minute from the starting of the motion.

(b) The taken time from the starting of the motion until its velocity becomes 18 km/hr.

Example

(3) A particle moves in a straight line so, its velocity changes from 54 km/hr into 3 m/sec in half a minute. Find the magnitude of the acceleration of the motion. Can this particle be at rest instantly? Explain your answer.

Solution

To transfer the velocity from km/hr into m/sec

$54\ km / hr = 54 \times \frac{5}{18} = 15\ m / sec$

"From the given of the problem" $v_0 = 15\ m / sec$, $V = 3m / sec$, $t = 30$ seconds .

$\because\ V = v_0 + a\,t \qquad \therefore 3 = 15 + 30\,a$

i.e.: $30\,a = -12 \qquad \therefore a = -0.4\ m/sec^2$

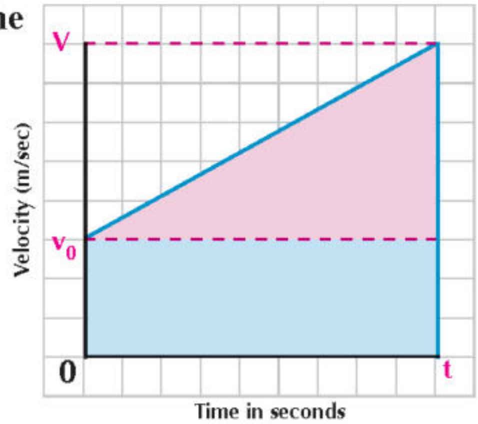
$\because a < 0$ The particle can rest instantly because it moves with a deceleration motion.

Try to solve

(3) A car moves in a straight line so its velocity decreases from 63 km/hr into 36 km/hr in a time of magnitude half minute. Find the acceleration that the car moves with and the time that it takes after that until it comes at rest instiniously.

Second: The relation between displacement and time

The area under the curve (The velocity - the time) equals the displacement of the body. In the opposite figure: The body moves with uniform acceleration starting with initial velocity V_0 and after time t second , its velocity becomes V . The area under the curve can be calculated by dividing it into a rectangle and a triangle.



Area (S) = area of rectangle + area of triangle

$$= v_0 t + \frac{1}{2} t (v - v_0)$$

$$S = v_0 t + \frac{1}{2} t (v_0 + a t - v_0)$$ (and that by substituting from the first law: $v = v_0 + a t$)

$$S = v_0 t + \frac{1}{2} a t^2$$

where: S, V_0, a are the algebraic measurment to the displacement, velocity, acceleration vectors.

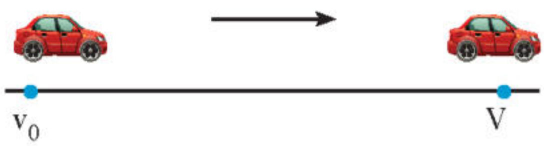
Verbal Expression:

1- Write the formula of the law (The displacement - The time) when the body starts its motion from rest.

2- Write the formula of the previous law when $a = 0$, and mention the type of the motion in this case?

Example

(4) A car moves with velocity 90 km/hr, the driver pressed on the brake, where the velocity decreases with constant rate until the car stopped after 5 seconds. Calculate:



a) Acceleration of the car during the decreasing of the velocity.

b) The distance that the car covered until its motion stopped completely.

Notic

When the body stopped, then V = 0

Solution

a) To convert the velocity from km/hr into m/sec : $90 km/hr = 90 \times \frac{5}{18} = 25$ m/sec

by substitution in the law: $V = v_0 + a t$ where $v_0 = 25$ m/sec , $V = 0$, $t = 5$ seconds

$\therefore 0 = 25 + 5 a$ **i.e.** $a = - 5$ m/sec^2

i.e. The car moves with uniform deceleration of magnitude 5 m/sec^2.

b $\because S = v_0 t + \frac{1}{2} a t^2$ by substitution : $v_0 = 25$m/sec, $t = 5$sec , $a = -5$m/sec^2

$\therefore S = 25 \times 5 + \frac{1}{2}(-5) \times 25 = 62.5$ meters.

Try to solve

4. A small ball is projected horizontally with velocity 20 m\sec. then it moves in a straight line with a retardation motion by a uniform acceleration $\frac{1}{2}$ m\sec^2. Determine the position of the point and its velocity after 2 sec from the starting of the motion.

Third: The relation between the displacement and velocity

We know that: $V = v_0 + a t$ **(1)** $S = v_0 t + \frac{1}{2} a t^2$ **(2)**

By squaring the first equation: $v^2 = V_0^2 + 2 v_0 a t + a^2 t^2$.

$\therefore v^2 = V_0^2 + 2a(v_0 t + \frac{1}{2} a t^2)$ By substitution from the equation **(2)** by the value of S

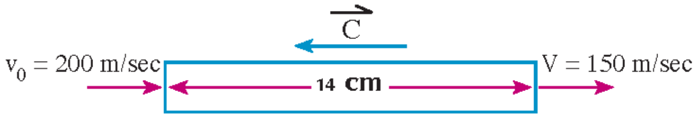
$$v^2 = V_0^2 + 2as$$

Example

5. A bullet was fired with velocity 200 m/sec in perpendicular direction on a vertical wall with a thickness of 14 cm. So, it went out with velocity 150 m/sec. Find the magnitude of the deceleration, and if the bullet was fired with the same velocity on another identical vertical wall with the same resistance. So, Find the distance that it embedded until it rests, If it is known that the acceleration that the bullet moves with it is the same in both cases.

Solution

We assume the positive direction is the direction of motion of the bullet.



The first case: $v_0 = 200$m/sec, $V = 150$ m/sec , $S = 0.14$ m

$\because v^2 = V_0^2 + 2as$ $\therefore (150)^2 = (200)^2 + 2 \times C \times 0.14$

and by simplification: $a = -62500$ m/sec^2

The second case:

$v_0 = 200$m/sec , $V = 0$ $C = -62500$ m/sec^2

$\because v^2 = V_0^2 + 2as$ $\therefore 0 = (200)^2 - 2 \times 62500\, s$

$\therefore s = 0.32$ meters **i.e.** the bullet embedded in the wall a distance 32 cm until it comes to rest.

Try to solve

5. The velocity of the car decreases uniformly from 45 km/hr into 18 km/hr after it covered a distance 625 meters. Find the distance that it covers after that until it comes to rest.
6. A bullet was fired horizontally on a wooden block with velocity 100 m/sec. So, it embedded in it a distance 50 cm. Find the acceleration that the bullet moves with it if it is an uniform acceleration and if it is fired on an identical wooden block for the first time its thickness 18 cm . What is the velocity that the bullet went out with it from the wooden block?

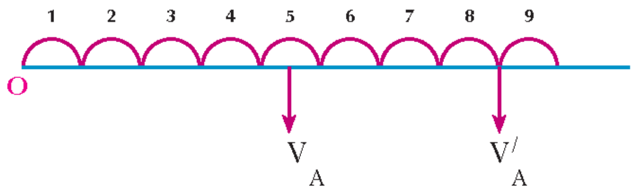
Example The average velocity within n^{th} second:

6 A particle moves with an initial velocity 10 cm\sec in a constant direction and with uniform acceleration 4cm\sec^2 .Find:

First: The covered distance in the 5^{th} second of the motion.

Second: The covered distance in both of the 8^{th} and the 9^{th} seconds together.

Solution



The positive directions consider is the direction of the initial velocity so

$\therefore v_0 = 10$ cm/sec , $a = 4$cm/sec^2

First: The average velocity V_A through the 5^{th} second = the velocity in the middle of the time interval = the velocity after $4\frac{1}{2}$ second.

$\because V_A = V_0 + a\,t \qquad \therefore V_A = 10 + 4 \times 4\frac{1}{2} = 28$cm/sec.

The covered distance in the 5^{th} second = average velocity × time = 28 × 1 = 28 cm.

Second: The average velocity V_A through the 8^{th} and 9^{th} second = the velocity in the middle of the time interval = the velocity after 8 second from the start of the motion.

$\because V_A = V_0 + a\,t \qquad \therefore V_A = 10 + 4 \times 8 = 42$ cm/sec

The covered distance in the 8^{th} and 9^{th} second = average velocity × time = 42 × 2 = 84 cm

Think:

Try to solve the previous example by another method.

Try to solve

7 A particle started its motion in a constant direction with velocity 30 cm/sec and uniform acceleration 6 cm/sec^2 in the same direction of its velocity. Calculate:

a The distance covered after 5 seconds from the starting of the motion.

b The distance covered in the 5^{th} second only.

8 A particle moves with an initial velocity in a constant direction and with uniform acceleration, if it covered in the 3^{rd} second from its motion a distance of 20 meters, then it covered in the two seconds, both the 5^{th} and 6^{th} a distance of 60 meters. Calculate the acceleration that the particle moves with it and its initial velocity.

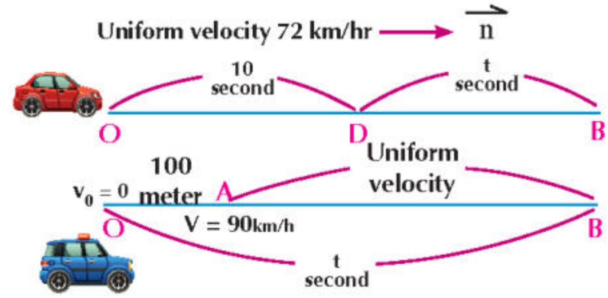
9 A metro moves in a straight line between two stations A, B the distance between them is 700 meters, where it starts from the station A from rest with a uniform acceleration 2 m/sec^2 for 10 seconds ,then it moves after that with a uniform velocity in an interval of time, then it covered a distance of the last 60 meters from its motion with a uniform deceleration until it stops in the station B. Find the time taken in covering the distance between the two stations.

Example Applications on the laws of motion with uniform acceleration

(7) A car moves with a uniform velocity 72 km/hr passed by a police car, so the police car started following it after 10 seconds from its passing moving with uniform acceleration a distance of 100 meters until its velocity reached 90 km/hr , then it moves with that velocity until it reached the first car. Find the time taken through the chasing process starting from the of motion of the police car and the distance covered by the first car.

Solution

We consider the positive direction is the direction of the uniform velocity and the police car was at rest at point (0), then it covered a distance 100 meters until it reached point (A) where its velocity became 90 km/hr ,then it moved uniformly until it reached the first car at (B).



$72 \text{ km/hr} = 72 \times \frac{5}{18} = 20 \text{ m/sec}$, $90 \text{km/hr} = 90 \times \frac{5}{18} = 25 \text{ m/sec}$

With respect to the police car in the interval from O ⟶ A

$v_0 = 0$, $V = 25 \text{ m/sec}$, $S = 100 \text{ m}$ $\quad v^2 = v_0^2 + 2as$

$25 \times 25 = 2 \times a \times 100 \quad \therefore a = \frac{25}{8} \text{ m/sec}^2$

$V = v_0 + at \quad \therefore 25 = \frac{25}{8} t \quad \therefore t = 8$ seconds

$\therefore$ The distance that the police car moved on with uniform velocity $= 25\,(t - 8)$ meters

The chased car covered the distance and reached point B in time of magnitude $= (t + 10)$ seconds

The police car covered the same distance and reached point B in time of magnitude = t second

$\therefore 20\,(t + 10) = 100 + 25\,(t - 8)$, then $t = 60$ second

The covered distance $= 20 \times 70 = 1400$ meters

Try to solve

(10) A car moves with a uniform velocity 54 km/hr passed by a police car, so the police car started following it after 30 seconds from its passing moving with uniform acceleration a distance of 200 meters until its velocity reached 72 km/hr , then it moves with that velocity until it reached the first car. Find the time taken through the chasing process starting from the motion of the police car and the distance covered by the first car.

Exercises (1 - 2)

1 Complete the following:

a A particle moves in a straight line from rest with uniform acceleration of magnitude $4 m/sec^2$ so, its velocity after 6 seconds from starting the motion = ... m/sec.

b The distance that the particle covered in a constant direction from rest with acceleration. of magnitude 5 cm/sec^2 during a time of magnitude 4 seconds = ... cm.

c The average velocity for a particle moving with initial velocity v_0 and a uniform acceleration (a) through the sixth second from its motion = ...

d The average velocity for a particle moving with initial velocity V_0 and a uniform acceleration (a) through the seconds 7^{th}, 8^{th} and 9^{th} from the starting of the motion = ...

e A particle moves from the rest in a straight line with uniform acceleration. So, it covered 24 meters in the first four seconds from its motion , then the magnitude of its acceleration = ...

f A particle started its motion from rest in a straight line with uniform acceleration of magnitude 2 cm/sec^2 so, it covered a distance 25 cm ,then its velocity at the end of this distance = ... cm/sec.

2 A car moves from rest with acceleration of magnitude 4 m/sec^2. What is the distance that the car covered when its velocity became 24 m/sec?

3 A racing car moves in the track with velocity 44 m/sec then its velocity decreases with a constant rate until it becomes 22 m/sec through 11 seconds. Find the distance that the car covered through that time .

4 A particle moves in a straight line with a uniform acceleration so its velocity increased from 15 m/sec into 25 m/sec. after covering 125 meter .Find the time takes for that .

5 A cyclist moves with a uniform acceleration until its velocity became 7.5 m/sec through 4.5 seconds. If the displacement of the bicycle through the accelerating interval equals 19 meters, find the initial velocity for the bicycle.

6 Karim practices on riding the bicycle. His father pushes him to gain a constant acceleration of magnitude $\frac{1}{2} m / sec^2$ for 6 seconds and after that Karim rides the bicycle alone with the velocity gained for another 6 seconds before he falls on the ground. Find the distance that Karim will cover.

7 A cyclist descends from the top of a hill with a constant acceleration of magnitude 2 m/sec^2. When he reaches the base of the hill, his velocity reaches 18 m/sec. then he uses the brakes to preserve this velocity for one minute. Find the total distance that the cyclist covered.

8 A car driver moves with a constant velocity of magnitude 24 m/sec. He suddenly saw a child passing the road. If the required time for the brakes to respond is $\frac{1}{2}$ sec then it moves with a uniform deceleration of magnitude 9.6 m/sec^2 until it stopped. Find the total distance covered by the car before it stops.

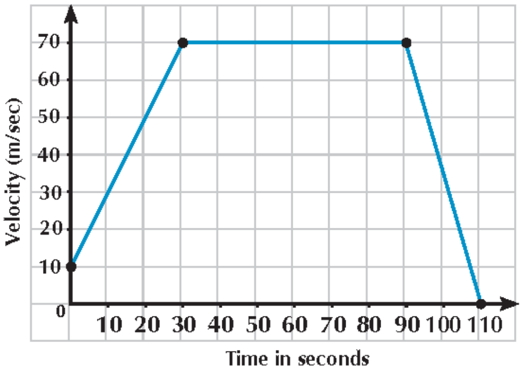
9. A body started its motion from rest in a horizontal straight line with uniform acceleration of magnitude 4 cm/sec^2 for 30 seconds ,then it moves with the velocity it gained for another 40 seconds. Find the magnitude of its average velocity.

10. A body moves in a straight line with uniform acceleration on a smooth horizontal plane till it covered 26 meters through the 4th second from starting the motion and 56 meters through the 9th second only , Find its initial velocity and the magnitude of its acceleration.

11. x, y are two points on a horizontal straight road. The car (A) started the motion from x towards y starting from rest and with uniform acceleration 10 m/sec^2 and at the same moment another car (B) moves from y towards x with uniform velocity of magnitude 54 km/hr , if the relative velocity for the car (A) with respect to the car (B) at the moment of their meeting equals 162 km/hr ,find the time taken by each one of the two cars from the moment of their motion together until the moment of their meeting.

Activity

12. The opposite figure represents the curve (the velocity - the time) for a body started the motion with an initial velocity of magnitude 10 m/sec and until it came to rest after a time of magnitude 110 second. Find:
   - a The acceleration.
   - b The magnitude of the uniform deceleration for the body until it rests.
   - c The total distance that the body moves.

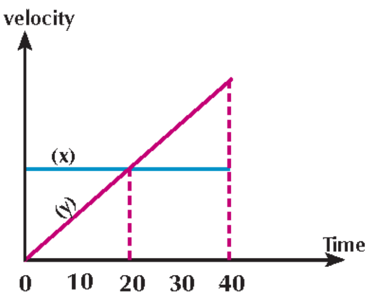


Creative thinking:

13. A lift is at rest at the bottom of a mine. The lift rises a distance 540 cm with acceleration of magnitude 120 cm/ sec^2, then it moves with uniform velocity for a distance 360 cm then with a uniform deceleration a distance 720 cm until it rests at the nozzle of the mine. Calculate the time that the lift takes in ascending from the bottom of the mine to its nozzle.

Creative thinking:

14. The opposite figure represents the curve (velocity – distance) for two cars x and y find the time taken by the two cars till they met.(Explain your answer)



Vertical motion under the effect of gravity (Free Fall)

Introduction:

What happens when an orange falls from a tree?

- The orange moves from rest, then it gains velocity during the free falling due to the effect of the Earth's gravity on it. After one second its velocity will be 9.8m/sec downwards and after another second its velocity will be 19.6m/sec downwards and so on.

Note that: The velocity of the orange is directly proportional with time. The acceleration that the bodies fall with in free falling (neglecting resistance of the air) equals 9.8 m/sec^2 approximately, and it denoted by (g) it varies by the variation of the latitude line so it decreases at the equatorial and increases slightly as we get closer to the poles. The acceleration will be positive or negative according to the coordinates system used, if the body fell or projected towards the surface of the earth then (g) , will be positive but if it projected upwards, then (g) will be negative.

Laws of the vertical motion of the bodies:

The vertical motion applies the same laws of the uniform motion with the uniform acceleration but with the usage of the symbol (g) instead of the symbol (a) for the acceleration when the bodies fall freely. Hence the laws will take the following forms:

$$V = v_0 + g\,t \;,\quad S = v_0\,t + \frac{1}{2}\,g\,t^2 \;,\quad v^2 = v_0^2 + 2gS$$

where V, g, S are the algebraic magnitude of the vectors: velocity, acceleration and displacement.

So, when we apply the rules with the preceding form, we should notice V , v_0 , g , S according to the follows:

First: If the body is falling or projected downwards

We consider the vertical direction downwards is the positive direction. In this case, each of v_0 , V , g , S are positive.

Example

1. A building worker dropped a piece of concrete from a high scaffold (platform).

   a What is the velocity of the piece of concrete after half a second?

You will learn

- Laws of vertical motion
- The study of the movement of the falling bodies or the projected downwards
- The study of the bodies projected upwards

Key terms

- Free fall
- Acceleration of gravity

Tools

- Scientific calculator

b What is the distance covered by the mass building during this time?

Solution

a **Formula of the law**: $V = v_0 + g\,t$

by substitution $v_0 = 0$, $g = 9.8$ m/sec^2 , $t = \frac{1}{2}$ sec.

$$V = 0 + 9.8 \times \frac{1}{2} = 4.9 \text{ m/sec}$$

b Formula of the law: $S = v_0\,t + \frac{1}{2}\,g\,t^2$

by substitution $v_0 = 0$, $g = 9.8$ m/sec^2 , $t = \frac{1}{2}$ sec.

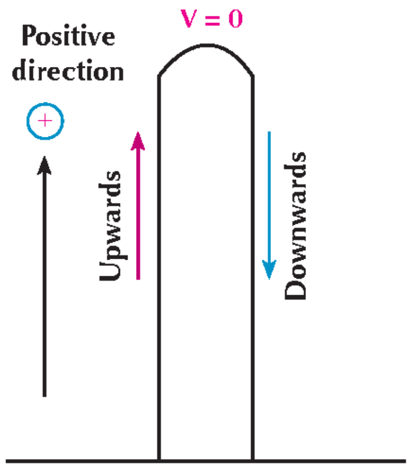
$$S = 0 + \frac{1}{2} \times 9.8 \times \frac{1}{4} = 1\frac{9}{40} \text{ meter.}$$

Try to solve

1. An apple fell from a tree and after one second it reached the ground.
   - **a** Calculate the velocity of the apple at the moment of reaching the ground surface, and then calculate the average velocity at the time taken to reach the ground surface.
   - **b** What is the distance of the apple from the ground surface at the moment of the beginning of its falling?

Second: If the body is projected vertically upwards

Activity



A ball is projected vertically upwards with initial velocity of magnitude 19.6 m/sec^2. By considering that the vertical direction upwards is the positive direction thus the initial velocity becomes positive but the acceleration becomes negative - **Why?**

➢ Use (geogebra) program in drawing the relation between (the velocity – the time) i.e. **$V = 19.6 - 9.8t$** when $t \in [0 , 4]$

What do you notice?

➢ Use the same program in drawing the relation between **(the distance - the time):**

i.e. $S = 19.6\,t - 4.9\,t^2$, **What do you notice?**

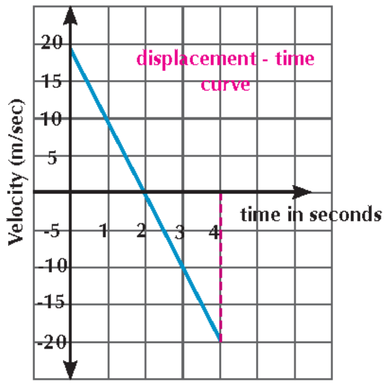


Fig (1)

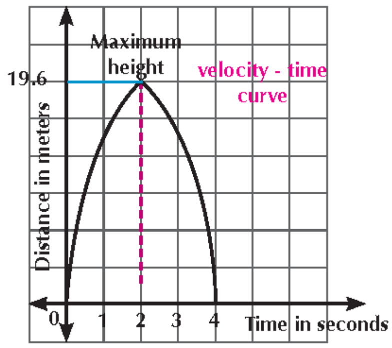


Fig (2)

We notice from the graph that:

- Velocity of the body during ascending is positive and during descending is negative.

  For example: when $t \in [0, 2[$ **we notice that velocity** $V > 0$, when $t \in]2, 4]$ So $V < 0$

- Velocity of the body at the point of maximum height equals zero.
- Time for ascending of the body equals time of descending.
- The magnitude of velocity of body when it returns to the projection point equals the magnitude of velocity of projection but with opposite sign.
- The displacement of the body during a time interval is not necessary to be equal to the distance covered by the body during this interval.

notice

The time of the maximum height = $\frac{v_0}{g}$

The maximum height = $\frac{V_0^2}{2g}$

Critical thinking:

1- If a body projected vertically upwards with initial velocity (V_0) and its final velocity reached (V) in time of magnitude (t) find.

a The time taken by the body to reach the maximum height.

b The maximum height that the body reaches.

Example

(2) If a body is projected vertically upwards with velocity 49 m/sec Find the time needed to reach the maximum height and the distance that it reaches.

Solution

Assume that the positive direction is vertically up ward :

$v_0 = 49$m/sec , $g = -9.8$ m/sec^2 , $V = 0$ (at the maximum height)

a **To find the time needed to reach the maximum height:**

$\because V = v_0 + g t$ $\quad \therefore 0 = 49 - 9.8 t.$ $\quad \therefore t = 5$seconds.

b **To find the distance of the maximum height:**

$\because v^2 = V_0^2 + 2 gS$ $\quad \therefore 0 = (49)^2 - 2 \times 9.8 \times S$ $\quad \therefore S = 122.5$ meter

Think:

1- Can you use other laws to find the distance of the maximum height? (Explain that)

Try to solve

(2) A body projected vertically up wards with velocity 39.2 m/sec. Find the time for the maximum height and the maximum height the body can reach.

Example

(3) A body projected vertically upwards with velocity 16 m/sec. Find the time taken by the body to reach 330 meters under the projection point.

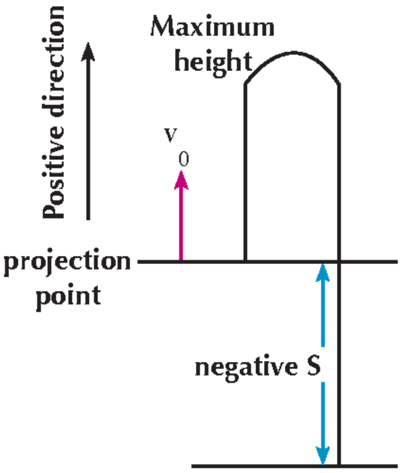
Solution

We consider the vertical direction upwards is the positive direction

$v_0 = 16$m/sec because it has the same direction of projection.

$g = -9.8$ **because it is opposite to the direction of the Earth's gravity.**

$S = -330$ **because it is under (down) the point of the projection.**



$S = v_0 t + \frac{1}{2} g t^2$

$-330 = 16t - \frac{1}{2} \times 9.8t^2$ **by simplification** $49t^2 - 16t - 330 = 0$

By factorization: $(t - 10)\ (49t + 330) = 0$

$t = 10$, $t = -\frac{330}{49}$ **(Refused)**

Think:

1- Do you have other solutions? Explain that.

Try to solve

3) A small ball is projected vertically upwards from a window of a house and it was seen during its falling in front of the window after 3 seconds from its projection. It reached the ground surface after 4 seconds from the moment of projection. Find the height of this window over the ground surface.

Exercises (1 - 3)

1) A child throws a ball from a window that rises 3.6 m from the pavement. What will be its velocity at the moment of contact with the pavement?

2) A ball fell vertically downwards. What is its velocity after 6 seconds from the moment of its falling?

3) A body fell vertically downwards from height 490 m from the surface of the ground find:

a) Time of reaching the ground surface.

b) Its velocity after 5 seconds from starting the motion.

4) A rubber ball fell from a height of 10 meters so it hit the ground and rebounded vertically upward a distance $2\frac{1}{2}$ meters. Calculate the velocity of the ball just after and before hitting the ground.

5) A student practices on kicking football vertically upwards in air, then the ball returns due to the impact of every kick. So, it hits his foot. If the ball takes from the moment of its kicking until colliding with his foot 0.3 seconds.

a) Find the initial velocity.

b) The height that the ball reaches after the student kicked it.

6. A body is projected vertically upwards from the top of a hill of height 9.8 meters with velocity 4.9 m/sec Find:

   a. Velocity of the body at the moment of reaching the bottom of the hill.

   b. The time taken to reach the bottom of the hill.

7. A stone is projected in a well with velocity 4 m/sec vertically downwards so, it reached the bottom of the well after 2 seconds. Find:

   a. The depth of the well.

   b. Velocity of the stone when it collides with bottom of the well.

8. A particle is projected vertically upwards with velocity 14 m/sec from a point at height 350m from ground surface. Find the time that the particle takes to reach the ground surface.

9. A ball is projected upwards from a window. So, it reaches it after 4 seconds from the moment of the projection and it reached the ground surface after 5 seconds from the moment of the projection. Find:

   a. Velocity of the ball's projection.

   b. Maximum height that the ball reached from the point of the projection.

   c. The height of the window from the ground surface.

10. A body is projected vertically upwards from the top of a tower its height 80.5 meters with velocity 8.4 m/sec. Find:

    a. The maximum height that the body reaches from the point of the projection.

    b. The time that the body takes while descending until its velocity become 11.2 m/sec.

    c. The time taken by the body to reach the projection point.

    d. The time taken by the body to reach the ground surface.

11. A ball is projected from the top of the hill of height 140 m vertically upwards; it is found that it covered in the third second a distance 10.5 meters. Find:

    a. The velocity that the ball is projected with.

    b. The maximum height that the ball reached.

    c. The time that the ball takes to reach the ground surface.

Creative thinking:

12. A body fall from a height of 60 meters from the ground surface and at the same moment, another body is projected vertically upwards from the ground surface with velocity 20 m/sec. The two bodies meet after a time interval. Find this time, and then find the distance that the two bodies covered during this time interval. Mention whither the two bodies met each other moving in two opposite directions or in the same directions?

Newton's Laws of Motion

Unit 2

Unit Introduction

Thanks to the discovery of the universal gravitation law by the British scientist, Isaac Newton (1642-1727), who is considered one of the icons of the scientific revolution in the field of modern mechanics, The German scientist, Johann Kepler (1571 - 1630), has stated some mathematical rules which control the motion of the planets around the sun. In addition to the work of the islamic Scientisits which had been translated during the previous centuries. The italian scientist, Galileo Galilit (1564 - 1642), had established the mechanics. He had conducted several experiments on the fallen or thrown bodies and the bodies moving horizontally, Through his experiments, he had discovered a lot of the important properties of the motion of the bodies. Thanks to him it was discovered that the bodies moving on horizontal surfaces without resistance keep moving in a uniform velocity. It is thought that Galileo had discovered the first and second laws of Newton's laws of motion. Isaac Newton had collected his overall researches in a book named (Bernspaa) which means the mathematical principles of the natural philosophy. This book is considered one of the most important scientific books that appeared in the modern age. Newton had formulated his three laws. The newton's law of universal gravitation had clarified the concept that the force can be occured under the action of a distance. Bodies attract each other even if they are not contacting. For example, the Earth attracts the bodies by a force called (the weight force). With respect to the mass, we notice that its static definition does not allow us to identify the mass of objects or bodies but to only compare the masses through the resistance of their weights. The mass can get a dynamic definition through studying the motion of bodies in this unit. You are going to learn the mass, momentum and Newton's laws of motion with application on these laws and you will also learn the motion on a rough or smooth plane

By the end of this unit and by doing all the activities involved, the student should be able to:

- Identify the concept of momentum and its measuring units- the change of momentum.
- Identify Newton's laws (first، second، third).
- Identify the relation between the force and acceleration:
  F= ma (Constant mass - Constant acceleration)
- Apply Newton's laws of motion in daily life siutations
- Motion of a body in a smooth plane (horizontal - inclined).
- Identify the motion on a rough plane (horizontal - inclined)

Key Terms

- Momentum
- Linear momentum
- Mass
- Velocity
- Change of momentum
- Newton's first law
- Inertia
- Inertia principle
- Force
- Newton's second law
- Equation of motion
- Weight
- Newtons third law
- Pressure
- Reaction
- Inclined plane
- Smooth plane
- Rough plane
- Kinetic friction
- Static friction

Unit Lessons

(2 - 1): Momentum

(2 - 2): Newton's first law

(2 - 3): Newton's second law

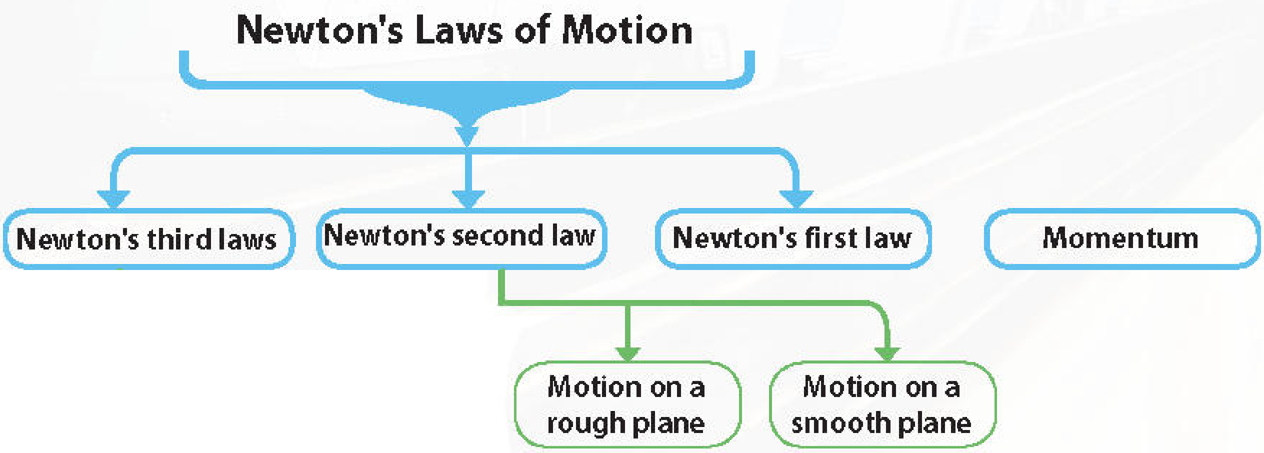
(2 - 4): Newton's third laws and applications on Newton's laws.

Materials

Scientific calculator -

Computer graphics

Unit Planing Guide



Unit Two
2 - 1

Momentum

You will learn

- The concept of momentum
- The measuring units of the momentum
- The change of momentum.

Key terms

- Momentum
- Linear momentum
- Mass
- Velocity
- Change of momentum

Materials

- Scientific calculator.

Think and Discuss

1 What is the action of a big stone placed on the roof of a house? What is the action of shooting a bullet from a barrel of a gun on this roof?

2 What is the action of placing a grain of sand on your palm? What is the action of this grain of sand if it moves in a storm in the direction of a car moving fast towards the storm?

From the examples, notice that:

1 - Shooting a bullet, however its mass is so small, on the roof of the house will lead to the bullet being imbedded in the roof for a distance since the velocity of the bullet is much faster than the velocity of the stone.

2 - The grain of sand, however its mass is very small, can scratch off the windshield glass since it acquired a momentum with respect to the car and the momentum vector of the grain of sand has become extremely strong due to the increase of its relative velocity vector.

Learn

Momentum

The momentum of a moving body is a vector quantity of the same direction of the velocity of this body and its magnitude at a moment is estimated by the product of the mass of this body by its velocity at this moment. The momentum vector is denoted by the symbol $\vec{H}$.

$$\vec{H} = m\,\vec{V}$$

In case of the rectilinear motion, each of $\vec{H}$ and $\vec{V}$ are parallel to the straight line on which the motion occurs. Each of $\vec{H}$ and $\vec{V}$ can be expressed in terms of the algebraic measure for each of them:

$$H = m\,V$$

Since H and V are the two algebraic measures for the momentum vector and velocity respectively.

Measuring units of momentum

The magnitude unit of momentum = mass unit × velocity unit

In the international system of units, the magnitude of momentum is measured in kg. m/sec

i.e.: H (kg. m / sec) = m(kg) × V(m/sec).

Notice that: At the constancy of the mass of the body, H is proportional to V and the relation between them is linear. As a result the momentum in this case is called the linear momentum.

Example Definition of momentum

1. Calculate the momentum of a bike whose mass is 35 kg and moves in a uniform speed of a magnitude 12 m/sec towards East.

Figure (1)

Solution

$\because H = mV$ $\therefore H = 35 \times 12 = 420$ kg. m/sec

The momentum of the bike = 420 kg. m/sec towards East.

Try to solve

1. Calculate the momentum of a train whose mass is 40 tons moving towards the North with a uniform speed of a magnitude 72 km/h.
2. Calculate the momentum of a car whose mass is 800 kg moving towards the Southwest with a uniform speed of a magnitude 126km/h.

The change of momentum

If the two velocity vectors of a moving body at two successive moments t_1 and t_2 respectively are $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ then the change of momentum of the body is determined by the relation:

$$\Delta \overrightarrow{H} = m \Delta \overrightarrow{V}$$

where m is the mass of the moving body, $\Delta \overrightarrow{V}$ is the change occurring in the value of its velocity

$\therefore$ The change of momentum of a body $\Delta H = m(\overrightarrow{v_2} - \overrightarrow{v_1})$

Example The change of momentum

2. A rubber ball of mass 200 gm is let to fall on a horizontal surface from a height of 90 cm to rebound up to a height of 40 cm. Calculate using kg.m/sec unit the magnitude of the change of momentum of the ball as a result of impact.

Solution

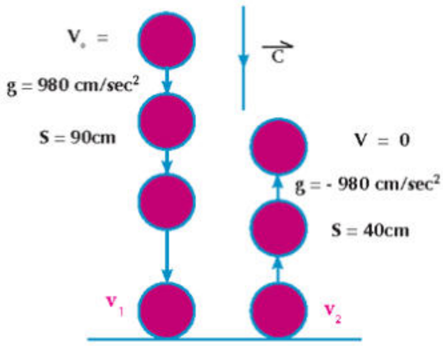


Figure (3)

Let $\overrightarrow{C}$ be the unit vector directed vertically downwards

Studying the motion of the ball in case of falling.

$\because v^2 = V_\circ^2 + 2 g S$

$\therefore V_1^2 = 0 + 2 \times 980 \times 90$

$v_1 = 420$ cm / sec $\therefore \overrightarrow{v_1} = 420 \overrightarrow{C}$

Studying the motion of the ball in case of rebounding.

$\because v^2 = V_\circ^2 + 2 g S$

$\therefore 0 = V_2^2 - 2 \times 980 \times 40$

$v_2 = 280$ cm / sec $\therefore \overrightarrow{v_2} = -280 \overrightarrow{C}$

The change of momentum $\Delta \overrightarrow{H} = m(\overrightarrow{v_2} - \overrightarrow{v_1})$

$$= \frac{200}{1000}(-2.8 - 4.2)\overrightarrow{C} = -1.4\overrightarrow{C}$$

$\therefore$ **the magnitude of the change of momentum** $= 1.4$ kg. m/sec

Try to solve

(4) A stone of mass 800 gm is let to fall from rest for two seconds then it impings with a surface of a pond and sinks down in a uniform velocity to travel 12 meters in three seconds. Find the momentum of the stone as a result of impact against water surface.

Exercises 2 - 1

Choose the correct answer:

(1) The momentum of a bullet whose mass is 100 gm moving at velocity 240m/sec is:

a) 24×10^{-3} gm. m/sec
b) 24kg. m/sec
c) 24×10^{3} gm. m/sec
d) 24×10^{3} kg. m/sec

(2) The momentum of a car whose mass is 2 tons moving in a straight line at velocity 54km/ h is:

a) 108 tons. m/sec
b) 3000kg. m/sec
c) 30000 kg. m/sec
d) 108000 kg. m/sec

(3) A body of a mass 500 gm is let to fall from a height of 4.9 meters on the ground surface, then its momentum as it comes the ground is:

a) 2.45 kg. m/sec
b) 4.9kg. m/sec
c) 2450 kg. m/sec
d) 4900 kg. m/sec

(4) A missile of mass 1 kg is launched at velocity 720 km/h towards a tank of mass 50 tons moving towards the mortar at velocity 20 m/sec, then:

(1) The magnitude of the momentum of the missile with respect to the tank is:

a) 200 kg. m/sec
b) 220 kg. m/sec
c) 10^{7} kg. m/sec
d) 1.1×10^{7} kg. m/sec

(2) The magnitude of momentum of the tank with respect to the missile is:

a) 200 kg. m/sec
b) 220 kg. m/sec
c) 10^{7} kg. m/sec
d) 1.1×10^{7} kg. m/sec

Answer the following questions:

5. A ball of mass 200 gm moves horizontally at a uniform velocity of magnitude 40 m/sec to collide with a vertical wall. If the magnitude of the change of momentum as a result of impact is 12 kg.m/sec, calculate the velocity of rebounding the ball.

6. A body of mass 90 gm is let to fall and after three seconds of falling, the body collides with a viscous liquid surface to imbed in it with a uniform velocity to travel 2.2 meters in half a second. Calculate the change of momentum due to the impact.

7. A rubber body of mass 100 kg moves horizontally with velocity 120 cm/sec. When it collides with a vertical wall and rebounds in a perpendicular direction on the wall, it loses two thirds of the magnitude of its velocity. Calculate the change of momentum of the rubber body due to the impact.

8. From a point under the ceiling of a room of a distance 240 cm, a ball of mass 40 gm is thrown with velocity 980 cm/second vertically upwards to collide with the ceiling and in turn, its momentum changes by a magnitude 0.4 km.m/sec. Find the velocity of rebounding the ball.

9. A rubber ball of mass $\frac{1}{2}$ kg is let to fall from a height of 8.1 meters on horizontal ground, then it vertically rebounds upwards to a height of 3.6 meters after it impings with the ground. Calculate the change of the momentum of the ball due to the impact with the ground.

10. A train wagon of a mass 15 tons moves horizontally with velocity of a magnitude 40 m/sec to collide with a barrier at the end of the railroad, then it rebounds back with velocity 30 m/sec. Calculate the change of its momentum.

Unit Two

2 - 2

Newton's First Law

You will learn

- Newton's first law
- The inertia principle

Key terms

- Newton's first law
- Inertia
- Inertia principle
- Force

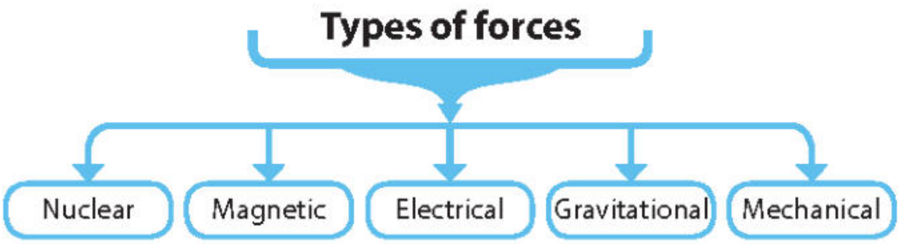
Materials

- Scientific calculator
- Computer graphics

Introduction:

In our daily life, we deal with several types of different forces which may act on the moving bodies to change their velocity. For example, when a person pushes or pulls a car. Forces can also act on the bodies at rest to preserve them in their state of rest for example, the book placed on the office or the picture hanged up on a wall. The action of force may be contact such as pulling the spring or pushing a box or may be acted -at - a distance such as the repultion or attraction of two poles of a magnet. The particle at rest is said to be in an equilibrium state when the resultant of the forces acting on it is equal to zero.

There are several types of forces existing in nature. They are either mechanical, gravitational, electrical, magnetic or nuclear forces. You are going to study the first and second types only in this unit.



To study the mechanical forces, we start to study Newton's laws of motion

Newton's First Law

Within this law, Isaac Newton stated what happens to a body when the resultant of the forces acting on it is equal to zero.

> **Every body preserves in its state of rest or of moving uniformly unless acted upon by an unbalanced external force by an external effect**

From Newton's first law, we notice that:

(1) The body at rest preserves at rest unless acted upon by a force to move it and the body in a uniform motion preserves at motion unless acted upon by a force changes its motion.

(2) In the formulation of the law, the expression "force" means the resultant of all the forces acting upon the body. The force is measured by the newton unit as an honor to Isaac Newton.

(3) The law considers the two states of rest and uniform motion in a straight line in an equivalent position since both states represent the "natural state" of the body when the resultant of the forces acting on the body is equal to zero.

(4) The law shows that the body which is at rest or moving uniformly in a straight line (i.e. when the body is in its normal state) cannot change its state and that's why Newton's first law is named " law of inertia ".

Inertia

From Newton's first law, we can deduce that the bodies naturally endeavor to preserve their states of rest or of moving uniformly in a straight line and this reluctance or resistance to the change is known as inertia.

Inertia principle:

Every body, as much as in its state, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

Activity — The relation between mass and inertia

1. The following activity shows that the mass is the measure to the quantity of the inertia.
2. Bring balls such that one of them is a golf ball of weight about 500 gm.wt and the other is a bowling ball of weight about 5 kg.wt.

Figure (5)

3. Which ball needs a greater force to move?
4. Undoubtedly, the bowling ball requires a force greater to start moving than that of the golf ball.
5. This is because the bowling ball endeavors to preserve its present state (rest position). i.e. its inertia is greater due to its great mass which is approximated ten times of the mass of the golf ball.

Force

Newton's first law includes a definition of the force as it is the effect which changes or triest change the state of the body whether of rest or of moving uniformly forward in a straight line.

Example (Body at rest)

(1) The opposite Figure shows a body at rest and a system of forces acting on it. Find F1 and F2.

Solution

$\because$ The body is at the rest state

$\therefore$ the vertical forces are in equilibrium

$\therefore F_2 + 40 = 70$

$\therefore F_2 = 30$ newton

$\because$ the horizontal forces are in equilibrium

$\therefore F_1 = 20 + F_2$

$\therefore F_1 = 50$ newton

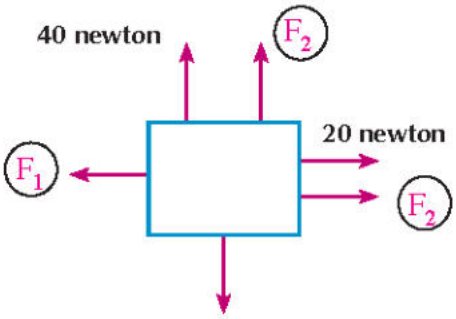


Figure (6)

Try to solve

(1) The opposite Figure shows a body at rest and a system of forces acting on it. Find F_1 and F_2.

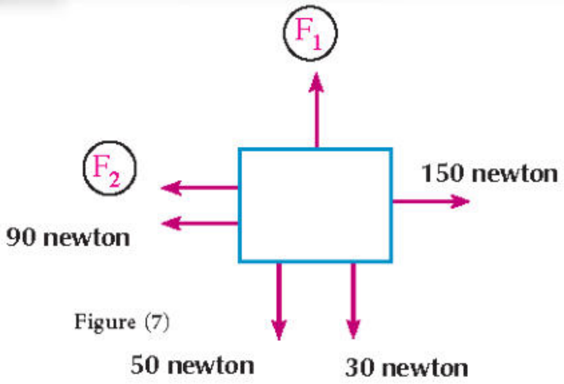


Figure (7)

Example

(Body at rest)

(2) The opposite Figure shows a body moving horizontally in the direction shown with a uniform velocity of magnitude 8m/sec. Find F_1 and F_2.

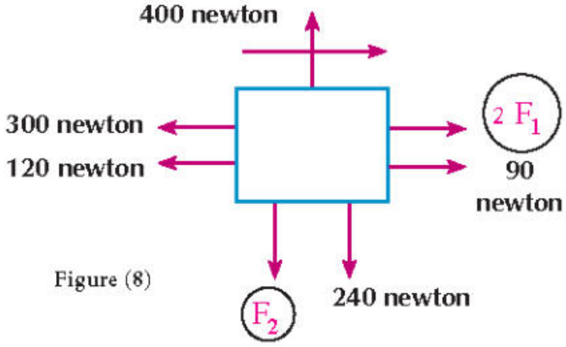


Figure (8)

Solution

$\because$ The body is in a uniform motion state

$\therefore$ The horizontal forces are in equilibrium

$\therefore 2F_1 + 90 = 300 + 120$

$\therefore F_1 = 165$ newton

$\because$ The vertical forces are in equilibrium

$\therefore 240 + F_2 = 400$

$\therefore F_2 = 160$ newton

Try to solve

(2) The opposite Figure shows a body moving vertically upwards with a uniform velocity and a system of forces acting on it. Find F_1 and F_2.

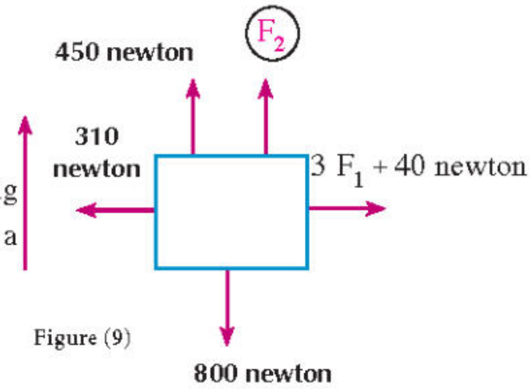


Figure (9)

Example

(3) A train of mass 200 tons moves under the action of a resistance proportional to the square of its velocity. If this resistance is 9.6 kg.wt per ton of the mass of the train when the velocity of the train is 72 km/h, find the maximum velocity for the train if the engine of the train can entrain (pull) it with a uniform force of magnitude 4.32 ton. wt.

Given data

$r_1 = 9{,}6 \times 200$
$= 1920$ sec kg
$v_1 = 72$ km / h


Figure (10)

Solution

Let the resistance $= r_1$ when the velocity of the train is v_1.

The resistance $= r_2$ when the velocity of the train is v_2.

$\because$ The resistance is proportional to the square of its velocity.

$$\therefore \frac{r_1}{r_2} = \frac{V_1^2}{V_2^2}$$

The train reaches the maximum velocity when the resistance is completely equal to the force of pulling the train.

If v_2 is the maximum velocity for the train, then r_2 = 432 tons.wt $\therefore r_2$ = 4320 kg.wt

$$\therefore \frac{r_1}{r_2} = \frac{V_1^2}{V_2^2} \qquad \frac{1920}{4320} = \frac{72 \times 72}{V_2^2} \qquad \therefore v_2 = 108 \text{ km/h.}$$

Try to solve

(3) A train of mass 240 tons is pulled by an engine with a uniform force of magnitude 12 ton.wt If the resistance of the train motion is proportional to the square of the train's velocity and the resistance is 8 kg.wt per ton of the moving mass when the train's velocity is 45 kg/h, calculate the maximum velocity of the train.

Example

(4) A parachutist lands vertically as the parachute is open. If the total weight of the parachutist and his equipments is 90 kg.wt and the air resistance is proportional to the square of its velocity and the landing maximum velocity of the parachutist is 15 km/h, find the air resistance of the parachutist when his velocity is 10 km/h.

Solution

The parachutist reaches the landing maximum velocity when the resistance is equal to the weight of the parachutist and his equipments.

If r_1 is the air resistance when the velocity of the parachutist is v_1

$\therefore r_1$ = 90 kg.wt **when** v_1 = 15 km/ h

If r_2 is the air resistance when the velocity of the parachutist is v_2

$\therefore r_2$ = ؟ **when** v_2 = 10km /h

$\because$ The resistance is proportional to the square of the velocity

$$\therefore \frac{r_1}{r_2} = \frac{V_1^2}{V_2^2} \qquad \therefore \frac{90}{r_2} = \frac{15 \times 15}{10 \times 10}, \quad r_2 = 40 \text{ kg.wt}$$

Figure (11)

Try to solve

(4) A man tied in a parachute lands vertically. If the air resistance is directly proportional to the square of his velocity and if the air resistance is equal to $\frac{4}{9}$ of the total weight of the man and his equipments when his velocity is 12km /h, find the landing maximum velocity of the man.

Exercises 2 - 2

Choose the correct answer:

1. A car of mass 4 tons moves on a horizontal road with a uniform velocity. If the force of the engine is 120 kg.wt, then the resistance of motion per ton of the mass is:

   a. 4 sec tons b. 30 kg.wt c. 120 kg d. 480 kg.wt

2. If a body of mass 20 kg.wt lands with a uniform velocity on an inclined plane to the horizontal with angle of measure 30°, then the resistance of the plane in kg.wt equals:

   a. zero b. 10 c. $10\sqrt{3}$ d. 20

3. A parachutist lands vertically downwards and the air resistance to its motion is proportional to the square of his velocity and v_1 is his velocity when the air resistance is equivalent to $\frac{9}{25}$ of his weight and, v_2 is the landing maximum velocity to the parachutist. Calculate $v_1 : v_2$

   a. 9: 25 b. 25: 9 c. 3: 5 d. 5: 3

Answer the following questions:

4. In each of the following situations, the body is at rest under the action of a system of forces:

a

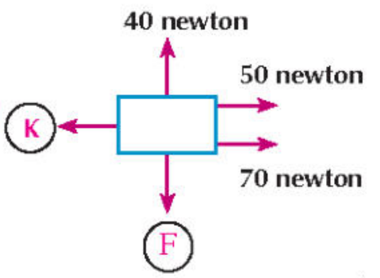


Figure (13)

b

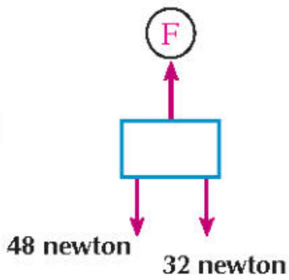


Figure (12)

c

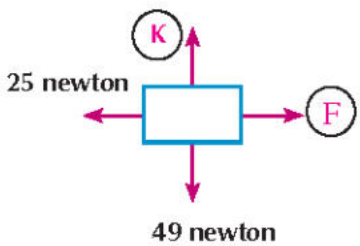


Figure (15)

d

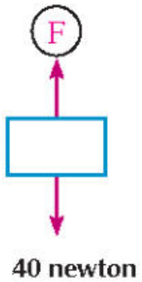


Figure (14)

Find the magnitude of the ungiven force in each case:

5 In each of the following situations, the body moves with a uniform velocity V under the action of a system of forces.

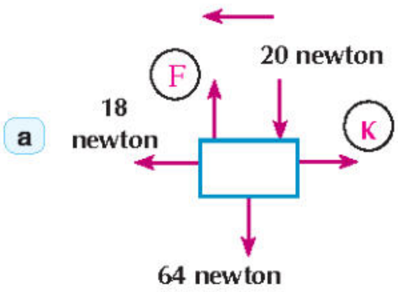


Figure (17)

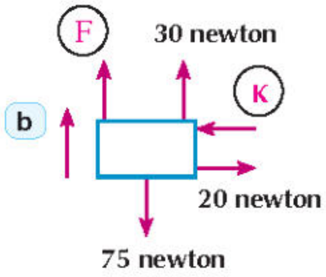


Figure (16)

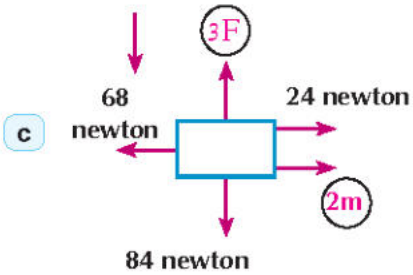


Figure (19)

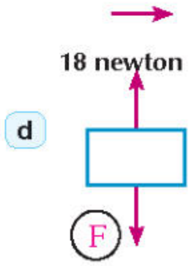


Figure (18)

Find the magnitude of the non-given force in each case:

6 A car of mass 8 tons moves with a uniform velocity under the action of a constant resistance of a magnitude 6 kg.wt per each ton of its mass. What is the force of the car's engine?

7 A train of mass 240 tons moves with a uniform velocity and the force of the train's engine is 4 w tons. Find the magnitude of the resistance per ton of the train's mass?

8 A car of mass 3 tons moves under the action of a resistance proportional to the car's velocity. If this resistance is 8 kg.wt per each ton of the car's mass when its velocity is 36 km/h, find the maximum velocity of the car if the engine pulling force is 120 kg.wt.

9. A train of mass 200 tons moves under the action of a resistance proportional to the square of its velocity. If this resistance is 8 kg.wt per ton of the train's mass when its velocity is 70km/h. Find the maximum velocity of the train if the engine pulls with a constant force of a magnitude 6.4 tons.wt

10. A train of mass 300 tons is pulled by an engine with a constant force of a magnitude 810 kg.wt under the action of a resistance proportional to the square of its velocity. If the maximum velocity of the train is equal to 30m/sec, find the rate of the resistance per ton of the train's mass when its velocity is 90 km/h.

11. The total weight of a parachutist and his equipments is 80 kg.wt and the air resistance to his motion is proportional to the square of his velocity. If this resistance is equal to 45 kg.wt when the velocity of the parachutist is 4.5 km/h, find the maximum velocity which the parachutist can acquire during his landing.

12. The total weight of a soldier and his equipments is 90 kg.wt and the air resistance to his motion is proportional to the square of his velocity. If the landing maximum velocity of the solvdier is 12 km.h, find the air resistance when his velocity is 8km/h.

Newton's Second Law

Unit Two
2 - 3

Think and discuss

From Newton's first law, you know that the resultant of the forces acting on a uniformly moving body vanishes, but if the resultant of the forces acting on a body is not equal to zero, the body moves with an acceleration.

- Is there a relation between the magnitude of the resultant acting on a body and the magnitude of the acceleration of motion?
- Are you able to figure out such a relation?

Learn

1 - Newton's second law

The rate of change of momentum with respect to the time is proportional to the acting force and takes place in the direction in which the force is acting

$\frac{d}{dt}(m\,\vec{V}) \propto \vec{F}$ **i.e.** $\frac{d}{dt}(m\,\vec{V}) = K\,\vec{F}$

(where K is the proportionality constant)

As the mass of body is constant during motion, then:

$m\frac{d\vec{V}}{dt} = K\,\vec{F}$ (where K is the proportionality constant)

$m\,\vec{a} = K\,\vec{F}$

If we define the unit of forces as the force if it acts on a body of mass a unit of masses, it acquires the body the unit of acceleration. By substituting in the equation above, **we find that**:

$1 = K \times 1 \times 1$ $\therefore K = 1$

and the equation above takes the form $m\,\vec{a} = \vec{F}$

This equation is called **the equation of motion of a constant mass body**, it is considered the basic equation in dynamics. It can be applied on all the moving bodies of a constant mass by considering such bodies physical points.

From the equation of motion above, we find that $\vec{F}$ and $\vec{a}$ are in the same direction. If $\vec{a}$ is measured in a certain direction, it is necessary to measure $\vec{F}$ in the same direction so, it is better to write down the equation of motion in the form:

$$m\,\vec{a} = \vec{F}$$

You will learn

- Newton;s second law
- Force units
- Weights and mass

Key terms

- Newton's second law
- Equqtion of motion
- Force
- Mass
- Weight

Materials

- Scientific calculator

If a and F express the algebraic measure of each of $\vec{a}$ and $\vec{F}$ respectively, then the equation of motion of a constant mass body is written in the form:

$$m\,a = F$$

where m is the mass of the moving body, a is the acceleration of motion and F expresses the algebraic measure of the resultant of the system of forces acting on the body. i.e.:

$$m\,a = \Sigma F$$

Units of force and units of mass

As we deduce the equation of motion of a moving body, we choose certain units for each of the force, mass and acceleration until the constant of proportionality is equal to 1 (unity). The equation of motion takes the form m a = F. As a result, when we use the equation of motion, we use the absolute units of force such as newton and dyne.

$m \times a = F$
$1\text{kg} \times 1\text{ m/sec}^2 = 1$ newton
$1\text{gm} \times 1\text{cm/sec}^2 = 1$ dyne

Remember

1 kg.wt = 9.8 newton
1 gm.wt = 980 dyne

Weight and mass

The weight of a body is the Earth's attraction force to the body. If we have a body of mass 1 kg, then its weight according to the equation of motion is equal to 1 kg.wt.

$\because m\,a = F \qquad \therefore 1 \times 9.8 = F \qquad F = 9.8$ newton $= 1$ kg.wt

Example

(1) A force of magnitude 10 newtons acts on a body at rest of mass 8 kg to move it in its direction with a uniform acceleration. Calculate the distance traveled after 12 sec and its velocity.

Solution

$F = 10$ newton $v_0 = 0$

$m = 8$ kg $t = 12$ sec

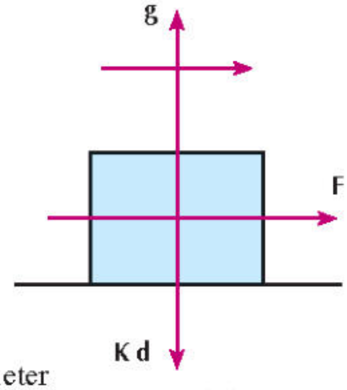


Figure (20)

The equation of motion of the body

$m\,a = F$ $\therefore 8\,a = 10$

$a = \frac{5}{4}$ m/sec

$\because V = v_0 + a\,t$ $\therefore V = 0 + \frac{5}{4} \times 12 = 15$ m/sec

$\because S = v_0\,t + \frac{1}{2}\,a\,t^2$ $\therefore S = 0 + \frac{1}{2} \times \frac{5}{4} \times 144 = 90$ meter

Try to solve

(1) If the last car of a train of mass 24.5 tons is separated from the train when its velocity was 54 km/h to move in a uniform retardation and stopped after 125m, find the magnitude of the resistance acting on the separated car in kg.wt.


Figure (21)

Example

(2) A body of mass 3 kg is let to fall from a height of 10 meters on a sandy ground to sink (embed) for a distance of 5 cm. Find the sand resistance to the body in kg.wt given that the body moves with a uniform acceleration within the sand.

Solution

The phase of free falling

$v^2 = V_0^2 + 2\,g\,S$

$v^2 = 0 + 2 \times 9.8 \times 10$

$V = 14$ m/sec

The phase of sinking (embedding) in sand

$v^2 = v_0^2 + 2a\,S$

$0 = (14)^2 + 2a \times 0.05$

$a = -1960$ m/sec ^{2}

Equation of motion

$m\,a = mg - r$

$3 \times -1960 = 3 \times 9.8 - r$

$\therefore r = 3 \times 9.8 + 3 \times 1960$

$r = 5909.4$ newton

$r = 603$ kg.wt

Try to solve

(2) **A box of mass** 100 kg, is lifted off vertically upwards by a string with a uniform acceleration of magnitude 25 cm /sec^2. Find the force of the tension in the string while neglecting the resistance.

Example

Figure (23)

(3) A train of mass 220 tons moves a long horizontal straight railroad with a uniform velocity of magnitude 29.4 m/sec. During the train's motion, the last car of mass 24 tons is separated and moves with a uniform retardation to stop completely after one minute of the separation moment.

Find:

First: the magnitude of the resistance per ton of the train's mass supposing it is constant.

Second: the magnitude of the force of the train's engine.

Third: The distance between the remaining part of the train and the separated car at the moment the car is completely at rest given that the remaining part of the train moves with a uniform acceleration.

Solution

First: Study the motion of the separated car from the laws of rectilinear motion:

$V = V_{\circ} + a\,t$ | $m = 24$ ton | $v_0 = 29.4$ m/sec

$0 = 29.4 + 60\,a$ | $V = 0$ | $t = 60$ sec

$\therefore a = -0.49$ m/sec^2

From the equation of motion of the separated car

$m\,a = -r$ $\quad 24 \times 1000 \times -0.49 = -r$

$r = 11760$ newton. $\quad \therefore r = 1200$ kg.wt

$r = 50$ kg.wt /ton

the distance traveled by the separated car within a minute after separation

$S = V_{\circ}\,t + \frac{1}{2}\,a\,t^2$

$= 29.4 \times 60 - \frac{1}{2} \times 0.49 \times (60)^2$

$S = 882$ meters

Second: study the motion of the train before separation

$\because$ The train was moving with a uniform velocity before separation $\qquad m = 220$ ton

$\therefore$ The moving force = the total resistances $\qquad$ resistance/ ton = 50 kg.wt

$\therefore F = 50 \times 220 = 11000$ kg.wt

Third: study the motion of the remaining part of the train after the last car has been separated

Equation of motion

$m\,a = F - r$ | $m = 196$ ton

$196 \times 1000\,a = (11000 - 50 \times 196) \times 9.8$ | $F = 11000$ kg.wt

$a = \frac{3}{50}$ m/sec^2 | resistance/ton $= 50$ kg.wt

From the laws of motion

$S = V_\circ t + \frac{1}{2} a t^2$

$= 29.4 \times 60 + \frac{1}{2} \times \frac{3}{50} \times (60)^2$

$S = 1872$ meters

The distance between the remaining part of the train and the separated car at the moment of its rest

$= 1872 - 882$

$= 990$ meters

Critical thinking: Draw a graph representing the distance between the remaining part of the train and the separated car from the separation moment until this cae stops, then find the following from the graph:

a) When is the distance between them equal to 110 meters?

b) The distance between them after 40 seconds from the separation moment.

Try to solve

3) A zeppelin of mass 105 kg, moves vertically downwards with a uniform acceleration of magnitude 98 cm/ sec^2. Find the magnitude of the air rising force acting on the zeppelin in kg. If a body of mass 35 kg is let to fall from the zeppelin when the velocity of the zeppelin was 490 cm/sec, find the distance between the zeppelin and the fallen body after $\frac{20}{7}$ seconds from the separation moment.

Figure (24)

Exercises 2-3

Choose the correct answer:

1. A body of mass 5 kg then its weight is:

   a. $\frac{25}{49}$ newton   b. 5 newton   c. 49 newton   d. 49 kg. wt

2. A body of mass 8 kg moves vertically upwards with a uniform acceleration a under the action of a force acts at the direction of motion of a magnitude 12 kg. wt. Find a in m/ sec^2:

   a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. 4.9   d. 14.7

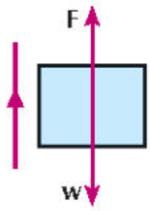


Figure (26)

3. A bullet of mass 7 gm is shot vertically from the barrel of a pistol with velocity 245 m/sec on a vertical barrier of wood to embed in it for 12.25 cm before being at rest. Calculate the wood resistance to the bullet given that it moves in a retarded motion.

   a. 17.15 newton   b. 175 newton   c. 175 kg. wt   d. 1715 kg. wt

4. In each of the following cases; the force F acts upon the body whose mass is m and acquires it a uniform acceleration shown in the figures, both magnitude and direction, find F

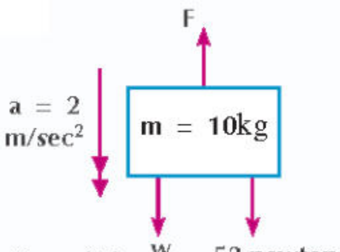


Figure (29)

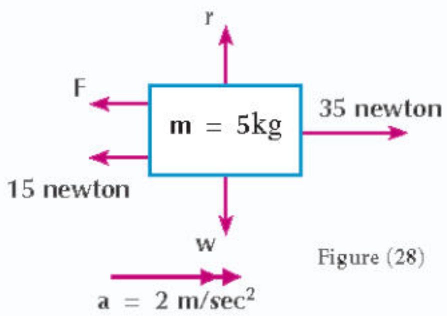


Figure (28)

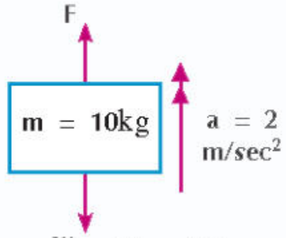


Figure (27)

5. In each of the following cases; the force F acts upon the body whose mass is m and acquires it a uniform acceleration shown in the figures both magnitude and direction, find F

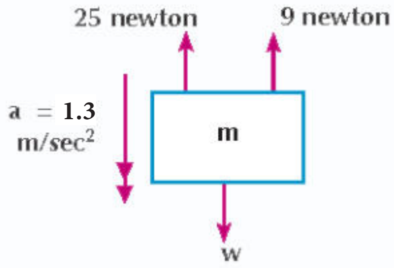


Figure (32)

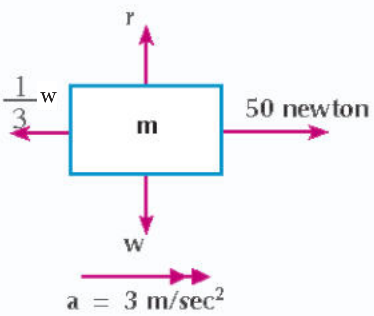


Figure (31)

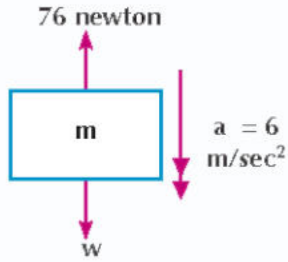


Figure (30)

6. In each of the following cases; the force F acts upon the body whose mass is m and acquires it a uniform acceleration a measured in m/sec^2, find a

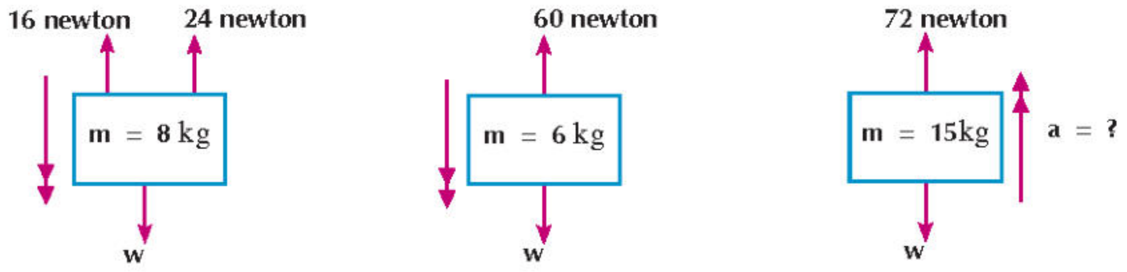


Figure (35) Figure (34) Figure (33)

7. A body mass 150 gm is acted upon by a force of a magnitude 4500 dyne. Find the acceleration resulted.

8. A mass of a magnitude 20 kg is placed on a smooth horizontal plane. A horizontal force of a magnitude F acts on it to move it with a uniform acceleration of a magnitude 49 m/sec. Find F.

9. A car at rest of mass 4.9 tons, is acted upon by a force and its velocity gets 27 km/ h within one minute, find the force acted upon the car in kg.wt.

10. If the force of the engine is equal to 2.5 ton.wt and the mass of the train and the engine is 200 tons and if the train starts to move from rest, find the velocity of the train after half a minute.

11. Find the resistance force of the brakes to the motion of the train in kg.wt per each ton of its mass if the velocity is 72 km/h and the brakes stop the train after it traveled 250 meters, find the time needed for doing that.

12. A man has pushed a car at rest of mass 980 kg with a constant force. If the velocity of the car is 45 cm/sec after 5 seconds, find in kg.wt the force by which the man has pushed the car if the resistance is 50 kg.wt.

13. Find the horizontal force by which an engine of a train of mass 245 tons is pulled to speed up its velocity into 18 km/h after it covers a distance of one kilometer on a horizontal railroad if the resistance force is 4 kg.wt ton.

14. A constant horizontal force of a magnitude 1 ton.wt acts upon a car of mass 4 tons moving on a horizontal road. If the car starts to move from rest and reaches a velocity of 4.9 m/sec in 10 seconds. Find the magnitude of the resistance acting on the car.

15. Find the least acceleration by which a man of mass 75 kg can slide on a survival rope from a fire if the rope cannot stand tension more than 50 kg.wt. Find the velocity of the man after landing 30 meters given that the acceleration is uniform.

16 A bullet of mass 20 gm collides with a constant barrier of wood when its velocity was 700 m/sec to embed in it for a distance of 5 cm. Calculate the resistance of wood supposing it is constant in kg.wt.

17 A body of mass 2 kg is let to fall from a height of 10 meters toward a sandy ground to embed in it for a distance of 5cm. Find the sand resistance supposing it is constant in kg.wt.

18 A train of mass 245 tons (including the engine) moves with a uniform acceleration of a magnitude 15 cm/sec^2 on a horizontal straight rail road. If air resistance and friction are 75 kg.wt per each ton of the train's mass, find the force of the engine in kg.wt. If the last car of the train of mass 49 tons is separated after the train moved from rest for 4.9 minutes, find the time taken by the separated car until it stops.

Unit Two

2 - 4

Newton's Third Law
And Applications On Newton's Laws

You will learn

- The pressure and reaction
- The motion of a body on an inclined plane
- Motion on a rough plane.

Key terms

- Newtons third law
- Pressure
- Reaction
- inclined plane
- Smooth plane
- Rough plane
- Kinetic friction
- Static friction

Materials

- Scientific calculator

Cooperative work

Work with a classmate to bring a pressure scale and place it on the roof of a lift. Then stand on the scale while the lift is at rest and have your classmate record the scale readings as you stand on the pressure scale. Let the lift move upwards and have your classmate record any change occurring in the scale readings. After that, stop the lift and record the readings once more. Then let the lift descend and have your classmate record the readings of the scale when any change occurs in the readings. Repeat this experiment alternately with your classmate. Record the scale readings as you and your classmate stand on the scale in each phase of the scale; at rest, ascending and descending.

Figure (36)

What is your interpretation of the scale readings in each phase?

Learn

1 - Newtons third law

To every action, there is a reaction equal in magnitude and opposite in direction.

2 - Pressure and reaction

When we place a body of mass m on a rested horizontal plane, then the body acts on the plane with a pressure force equal, in this case, to the weight of the body and a force of reaction is generated for the plane acting upon the body completely equal to the pressure exerted by the body on the plane and the two forces are opposite in direction but equal in the magnitude completely. The pressure of the body on the plane changes as the plane moves up or down. The pressure in this case is known as the apparent weight.

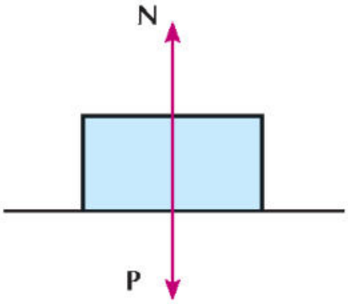


Figure (37)

Motion of a body on a smooth inclined plane

Think and Discuss

If a body of mass m kg is placed on a smooth inclined plane, it inclines to the horizontal with an angle of measure. If this body is acted upon by a force of magnitude F newton in the direction of the line of the greatest slope upwards the plane, then determine the direction of the motion of this body and what acts on the direction of the motion

Motion of a body on a smooth inclined plane

Let a body of mass m moves on a smooth plane inclined to the horizontal with an angle of measure θ under the action of a force of magnitude F acting in the direction of the line of the greatest slope of the plane upwards, then we notice that the body is under the action of the following three forces:

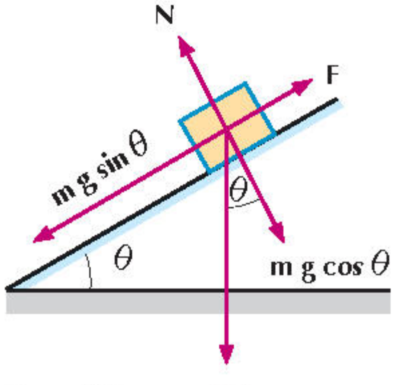


Figure (52)

1 - The given force and acting in the direction of the line of the greatest slope to the plane upwards and its magnitude is F.

2 - The weight of the body acting vertically downwards of magnitude m g.

3 - The reaction of the plane acting in a perpendicular direction to the plane upwards of magnitude N.

By analyzing the weight into two component; one of them is in the direction of the plane downwards and the other is in the perpendicular direction to it.

The component is in the direction of the plane = mg sin θ.

The component is in the perpendicular direction to the plane =mg cos θ then, we have three cases that rely on the comparison among F, mg sin θ with the same unit .

First case: If $F > mg \sin\theta$, then the body moves with a uniform acceleration a upwards the plane and the equation of its motion is $m\ a = F - mg \sin\theta$

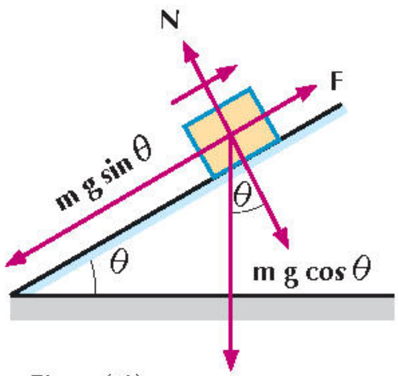


Figure (53)

If the action of the force F is ceased after passing time t from the beginning of the motion, then the body moves upwards the plane (the same previous direction) but with a retarded acceleration a` where $a` = -g \sin\theta$

The body inevitably reaches an instantaneous rest, then it changes the direction of its motion downwards the plane with an increasing acceleration of magnitude g sin θ.

Second case: If $F < mg \sin\theta$ the body moves with a uniform acceleration a downwards the plane and its equation of motion

$m\,a = mg \sin\theta - F$

Third case: If $F = mg \sin\theta$, then the body preserves its rest state on the plane. But if the body has acquired a uniform velocity V in the direction of the plane upwards of downwards, then the body moves on the plane in the direction of $\overrightarrow{V}$ with a uniform velocity according to Newton's first law.

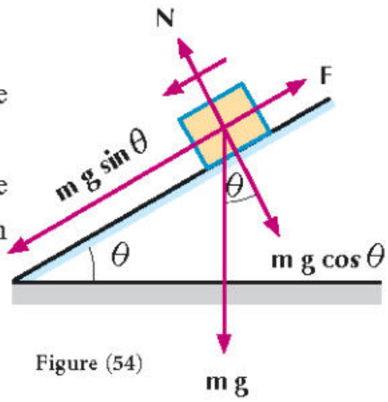


Figure (54)

Example

(1) A body of mass 12 kg is placed on a smooth plane inclines at 30°, to the horizontal. A force of magnitude 88.8 newton acts in the direction of the line of the greatest slope upwards the plane. Find the velocity of this body after 14 seconds from the beginning of the motion. If the force acting on the body is ceased at this moment, find the distance which the body moves on the plane after that until it is at rest.

Solution

$\because F = 88.8$ newton

$\because mg \sin\theta = 12 \times 9.8 \times \frac{1}{2}$

$= 58.8$ newton

$F > mg \sin\theta$

$\therefore$ The body moves upwards the plane with a uniform acceleration a

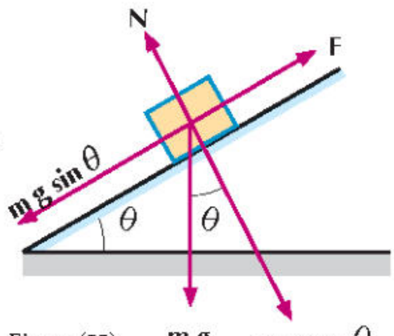


Figure (55)

Equation of motion:

$m\,a = F - mg \sin\theta$

$12\,a = 88.8 - 58.8$

$a = 2.5$ m/sec²

$\because V = V_0 + a\,t = 0 + 2.5 \times 14 = 35$ m/sec

After leasing the action of the force, the body moves in the same previous direction with a uniform retardation a`

Equation of motion:

$K\,a` = -mg \sin\theta$

$a` = -9.8 \times \frac{1}{2} = -4.9$ m/sec²

the body travels a distance S until it reaches the instantaneous rest where

$v^2 = v_0^2 + 2\,a`\,S$

$0 = (35)^2 - 2 \times 4.9\,S$

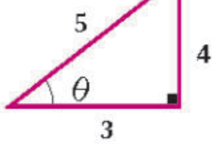
$S = 125$ meters

Try to solve

1) A body of mass 32.5 kg is placed on a smooth plane inclines with an angle of measure θ, where $\cos\theta = \frac{12}{13}$, A force of magnitude 83.5 newtons acts in the direction of the line of the greatest slope to the plane upwards, find the magnitude and direction of the acceleration of motion, then find the velocity of the body after 8 seconds from the beginning of the motion.

Example

2) A body of mass 25 kg is placed on a smooth plane inclines with an angle of measure e where $\tan\theta = \frac{4}{3}$. A horizontal force of magnitude 30 kg.wt, acts in the direction of the plane and its line of action lies in the vertical plane passing through the line of the greatest slope to the plane. Find the acceleration generated acceleration and the magnitude of the reaction force of the plane.



Solution

$F\cos\theta = 30 \times \frac{3}{5} = 18$ kg.wt , $w\sin\theta = 25 \times \frac{4}{5} = 20$ kg.wt

$\because F\cos\theta < w\sin\theta$

$\therefore$ The body moves downwards the plane with a uniform acceleration a where

$m a = mg\sin\theta - F\cos\theta$

$25 a = (20 - 18) \times 9.8$

$a = \frac{98}{125}$ m/sec^2

$g = F\sin\theta + Kd\cos\theta$

$= 30 \times \frac{4}{5} + 25 \times \frac{3}{5} = 39$ kg.wt

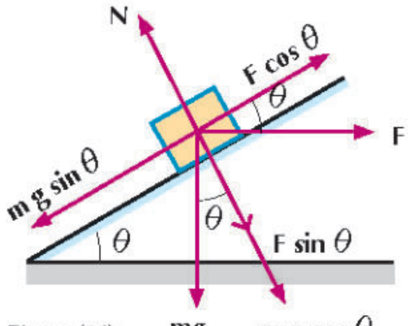


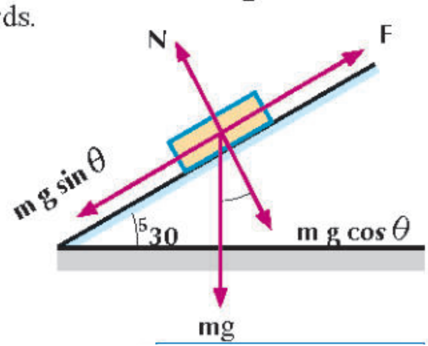
Figure (56)

Try to solve

2) A body of mass 2 kg moves on the line of the greatest slope of a smooth plane inclines at 60° to the horizontal under the action of a force of magnitude 1 kg.wt directed towards the plane and makes an angle of measure 30° to the horizontal upwards. Find the magnitude of the reaction force of the plane on the body and the acceleration of motion.

Example

3) A body of mass 30 kg moves upwards a smooth inclined plane inclined at 30° to the horizontal under the action of a force of magnitude F newtons in the direction of the line of the greatest slope upwards with an acceleration of magnitude 1.5 m/sec^2. Find the acceleration by which this body moves on the same plane under the action of a force of magnitude $\frac{1}{2}$ newton and acts in the direction of the line of the greatest slope upwards.



Solution

The equation motion of the first case

$m a = F - mg\sin\theta$

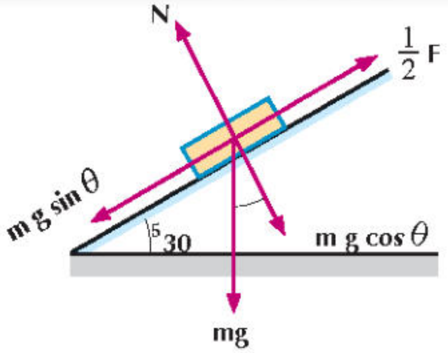
$\therefore 30 \times 1.5 = F - 30 \times 9.8 \times \frac{1}{2}$

then $F = 192$ newton

The equation motion of the second case

$m\,a` = \frac{1}{2} F - mg \sin\theta$

$\therefore 30\,a` = 96 - 30 \times 9.8 \times \frac{1}{2} \qquad \therefore a` = -1.7 \text{ m/sec}^2$



Try to solve

3) A body of mass 200 kg moves upwards a smooth inclined plane inclines at 30° to the horizontal under the action of a force of magnitude F newton in the direction of the line of the greatest slope upwards with an acceleration of magnitude 2m/sec^2. If this force is reduced into a half, then the body moves downwards the plane with an acceleration of magnitude 1.45 m/sec^2 Find the magnitude of $\overrightarrow{F}$.

Motion of a body on a rough plane

Introduction:

In your previous study of friction, you knew that when you try to move a body on a rough plane, the friction force appears as a resistance force acting in an opposite direction to the direction which the body intends to move in this force keeping completely equal. The tangential force acting to move the body increases, the more the friction force increases until it remains equal to this force until it reaches the maximum value. At this time, the body is about to move. If the tangential force acting to move the body increases, it can move the body and the friction force then changes and its value decreases as the body starts to move on that time. The friction force is called the kinetic friction and the coefficient of friction in this case is the coefficient of the kinetic friction.

Learn

Motion on a rough plane

If the body is in equilibrium on a rough plane under the action of a force acting to move it, then the friction force is the static friction force and the coefficient of friction in this case is the coefficient of the static friction μ_x. But if the body moves on a rough plane, then the coefficient of the friction is the coefficient of the kinetic friction μ_k.

Example

1) A rough inclined plane of length 250 cm and height 150 cm, another body at rest is placed on it to slide downwards the plane and the acceleration of motion is equal to 196 cm/sec^2. Find the coefficient of the kinetic friction, then find the velocity of the body after it travels (cuts) 200 cm on the plane.

Solution

$N = m\,g\cos\theta = \frac{4}{5}mg$

$\because$ The body moves downwards with a uniform acceleration

$m\,a = mg\sin\theta - \mu_K N$

$196\,m = \frac{3}{5}mg - \mu_K \times \frac{4}{5}mg$

$196 = \frac{3}{5} \times 980 - \mu_K \times \frac{4}{5} \times 980$

$\therefore \mu_K = \frac{1}{2}$

$\because V^2 = V^2 + 2\,a\,S$

$V^2 = 0 + 2 \times 196 \times 200$

$\therefore V = 280$ cm /sec

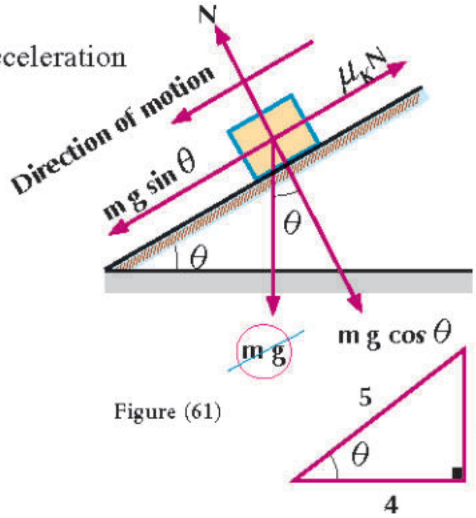


Figure (61)

Try to solve

1. The boxes at a factory are transported by sliding them on an inclined plane of length 15 m and height 9 m. Find the velocity of the box which starts it motion from rest at the top of the plane at the base of the plane if the plane is smooth and the coefficient of the kinetic friction is equal to $\frac{1}{4}$.

Example

2. A body of mass 12 kg. is placed on a rough horizontal plane. If the coefficient of the static friction between the body and the plane is equal to $\frac{\sqrt{3}}{3}$ whereas the coefficient of the kinetic friction is equal to $\frac{\sqrt{3}}{4}$, then calculate the force which make the body about to move, then find the force which make the body move with an acceleration of magnitude $\frac{49\sqrt{3}}{20}$ m/sec² if the force inclines at 30° to the horizontal.

Solution

First: The force makes the body about to move

$N + F\sin 30 = W$

$N = (12 - \frac{1}{2}F)$ kg.wt

$\because F\cos 30 = \mu_S N$

$\therefore \frac{\sqrt{3}}{2}F = \frac{\sqrt{3}}{3}(12 - \frac{1}{2}F)$

$3F = 24 - F$

$4F = 24$

$F = 6$ kg. wt

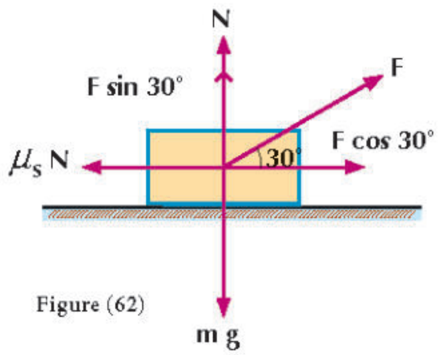


Figure (62)

Second: The force moves the body with an acceleration of magnitude $\frac{49\sqrt{3}}{20}$ m/sec^2

$\because$ N = mg - F sin 30 i.e. N = (12 × 9.8 - $\frac{1}{2}$ F) newton

$\because$ m a = F cos 30 - μS N

$$12 \times \frac{49\sqrt{3}}{20} = F \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}(12 \times 9.8 - \frac{1}{2} F)$$

$$12 \times \frac{49\sqrt{3}}{20} = \frac{5\sqrt{3}}{8} F - 3\sqrt{3} \times 9.8$$

F = 94.08 newton

Try to solve

1. In the previous example, calculate the magnitude of the force F if the force acting on the body is horizontal.

Example

3. A body of weight 800 newtons is placed on a rough inclined plane inclines at 25° to the horizontal and the coefficient of the static friction between the body and the plane is equal to 0.35 and the coefficient of the kinetic friction is equal to 0.25. Find the force in each of the following cases:

a F makes the body about to move upwards the plane.

b F is the minimum force that moves the body upwards the plane.

c F prevents the body from sliding.

where F acts in the direction of the line of the greatest slope upwards the plane.

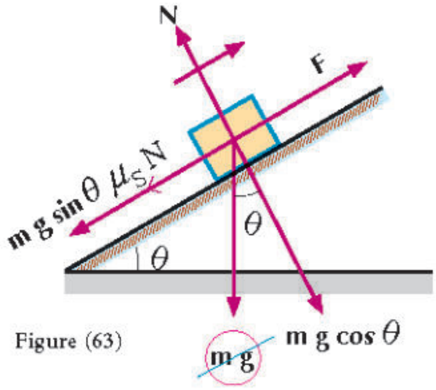


Figure (63)

Solution

a $\because$ The body is about to move upwards the plane

g = mg cos θ

F = μ_S N + mgsin θ

= 0.35 × mg cos θ + mg sin θ

= 0.35 × 800 cos 25 + 800 sin 25

= 591.86 newton

b The minimum force moves the body upwards the plane

F = μ_S N +mg sin θ

= 0.25 × mg cos θ + mg sin θ

= 0.25 × 800 cos 25 + 800 sin 25

$\simeq$ 519.36 newton

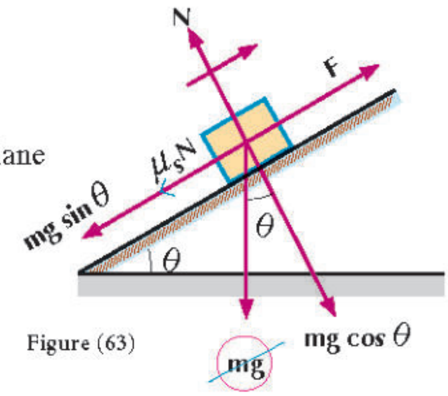


Figure (63)

c The force F prevents the body from sliding

$N = mg\cos\theta$

$F + \mu_s N = mg\sin\theta$

$F + 0.35 \times mg\cos\theta = mg\sin\theta$

$F = 800\sin 25 - 0.35 \times 800\cos 25$

$F = 84.33$ newton

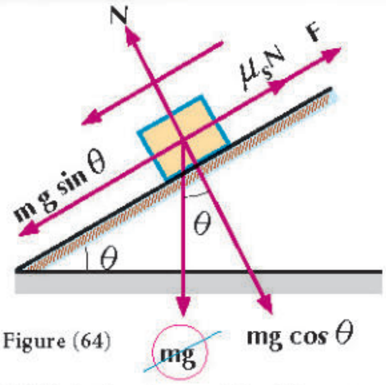


Figure (64)

Try to solve

2 In the previous example, calculate the magnitude of the force F if it is horizontal in all cases.

Exercise (2 - 4)

Complete the following:

1 **In the Figure drawn**: the mass of the body placed on the smooth plane is 2 kg and starts to move from rest under the action of the force F whose magnitude is 1.5 kg.wt

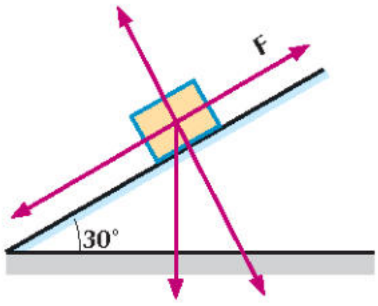


Figure (58)

a The acceleration of motion =.................... m/ sec^2 and its direction is.....................

b The velocity of the body after 4 seconds of starting to move

c The reaction of the plane =.................... kg.wt.

2 **In the Figure drawn:** the mass of the body placed on a smooth plane is m = 12 kg. It starts to move from rest under the action of the force F whose magnitude is 8 kg.wt.

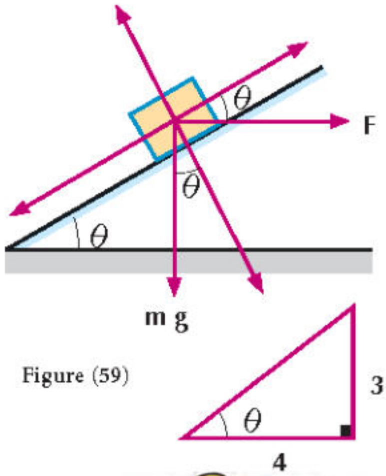


Figure (59)

a The acceleration of motion =.................... m/sec, and its direction is.....................

b The distance which the body travels on the plane in 3 seconds from the beginning of the motion is.................... meters

c The reaction of the plane =.................... kg.wt

Choose the correct answer:

3 A cyclist and his cycle of mass 85 kg move with a uniform acceleration of magnitude 0.5 m/sec^2, then the force used to occur this acceleration is:

a 42.5/ kg.wt b 42.5 newton

c 170 kg.wt d 170 newton.

4. A car on a road of negligible resistances with acceleration of magnitude 1.47 m/sec^2. If the force of the engine is 150 kg.wt, then the mass of the car is equal to:

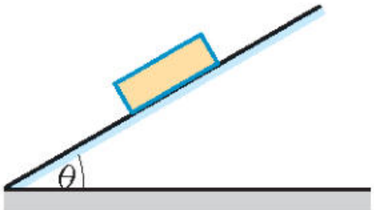
a) 102 kg   b) 100 kg

c) 1000 kg   d) 220.5 kg.

5. If a body moves on a smooth inclined plane and inclines with an angle of measure θ under the action of its weight only, then its acceleration of motion is equal to:

a) g   b) g cos θ

c) g sin θ d) zero.



6. If a body moves on a smooth inclined plane under the action of its weight only, then its acceleration is only based on:

a) its mass   b) its weight

c) the angle of inclination of the plane   d) the reaction of the plane.

Answer the following questions:

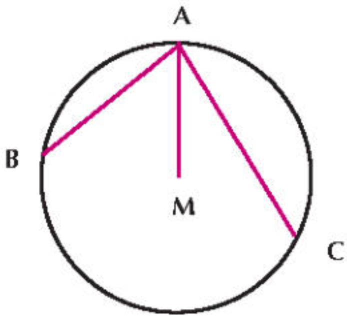
7. A body of mass 10 kg is placed on a smooth plane inclines with an angle of sine $\frac{3}{5}$ to the horizontal. A force of magnitude 80 newtons acts in the direction of the line of the greatest slope upwards. Find the magnitude and direction of the acceleration generated and the magnitude of the reaction of the normal.

8. A body of mass 1 kg is placed on a smooth plane inclines at 30° to the horizontal. A force of magnitude 10 newtons acts in the direction of the line of the greatest slope of the plane upwards. Find the acceleration of motion and the reaction of the plane on the body.

9. A body of mass 16 kg is placed on a smooth plane inclines at 45°, to the horizontal. A horizontal force of magnitude 24 newtons acts toward the plane and its line of action lies in the vertical plane passing through the line of the greatest slope to the plane. Find the magnitude of the acceleration of the motion and the reaction of the plane.

10. **Creative thinking:**

In the opposite Figure: $\overline{AM}$ is a vertical radius, $\overline{AB}$, $\overline{AC}$ are two chords representing two smooth rods in the circle where AC > AB. Two beads slid from rest from point A such that one bead on the chord $\overline{AB}$ to reach B after time t_1 and the other bead on the chord $\overline{AC}$ to reach C after time t_2, find the value of the ratio $t_1 : t_2$.



11 In each of the following figures, a body of mass 5 kg is placed on a rough horizontal plane and the coefficient of the kinetic friction between the body and the body μ_k, calculate μ_k in each case, F is the friction force.

a

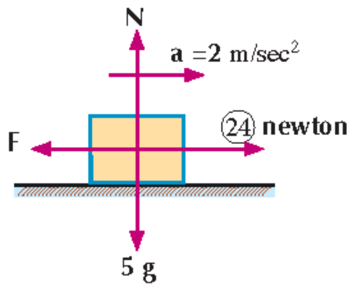


Figure (65)

b

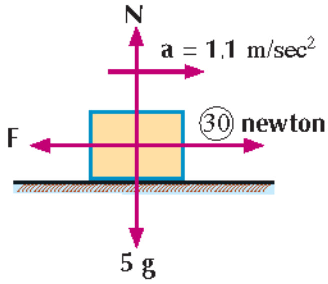


Figure (66)

c

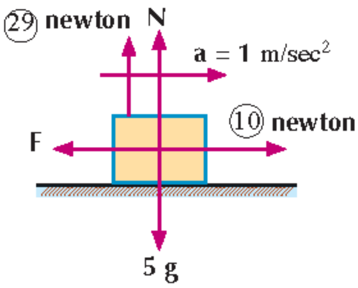


Figure (67)

d

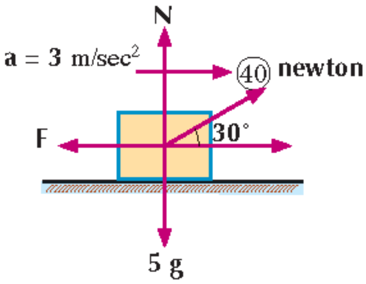


Figure (68)

d

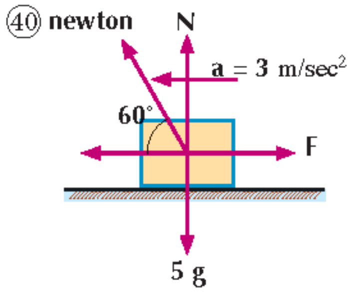


Figure (69)

f

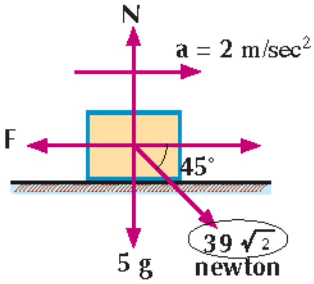


Figure (70)

g

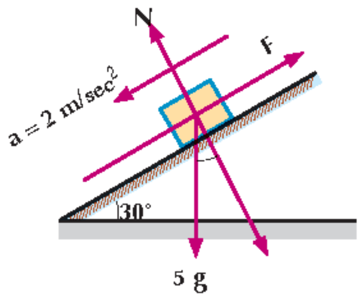


Figure (71)

h

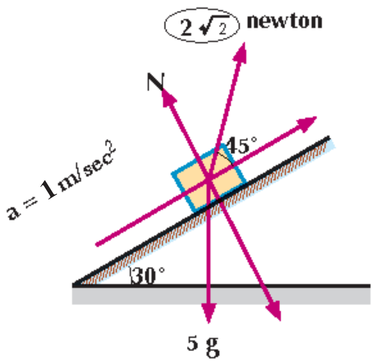


Figure (72)

12) A body of mass 1 ton is needed to drag into a rough plane inclines with an angle of measure θ where $\tan\theta = \frac{3}{4}$ to the horizontal by a force parallel to the plane in the direction of the line of the greatest slope upwards. Find the coefficient of the kinetic friction between the body and the plane if the least force needed to move this body on the plane is of magnitude 1400 kg. wt.

13) A body of mass 2 kg is placed on a rough horizontal plane and the coefficient of the kinetic friction between the body and the plane is $\frac{1}{2}$, Find the horizontal force making the body move with a uniform acceleration a where:

a) $a = 5 m/sec^2$ b) $a = 1 m/sec^2$

14) A body of weight 10 kg.wt is placed on a rough horizontal plane. A force of magnitude 37 newtons acts on this body to move it on the horizontal plane with a uniform acceleration of magnitude $\frac{5}{4}$ m/sec^2, Find the coefficient of the kinetic friction between the body and the plane.

15) A body of mass 2 kg is placed on a rough horizontal plane and inclines at 30° to the horizontal. A horizontal force of magnitude 20 newtons acts on the body towards the plane, then the body moves in a uniform velocity. Find the coefficient of the kinetic friction between the body and the plane.

16) A body slides on a rough plane inclind at 45° to the horizontal. If the coefficient of the kinetic friction between the body and the plane is equal to $\frac{3}{4}$, prove that the time taken by the body to travel any distance is equal to twice the time taken by the body to travel the same distance if the plane is smooth supposing the body starts to slide from rest in both cases.