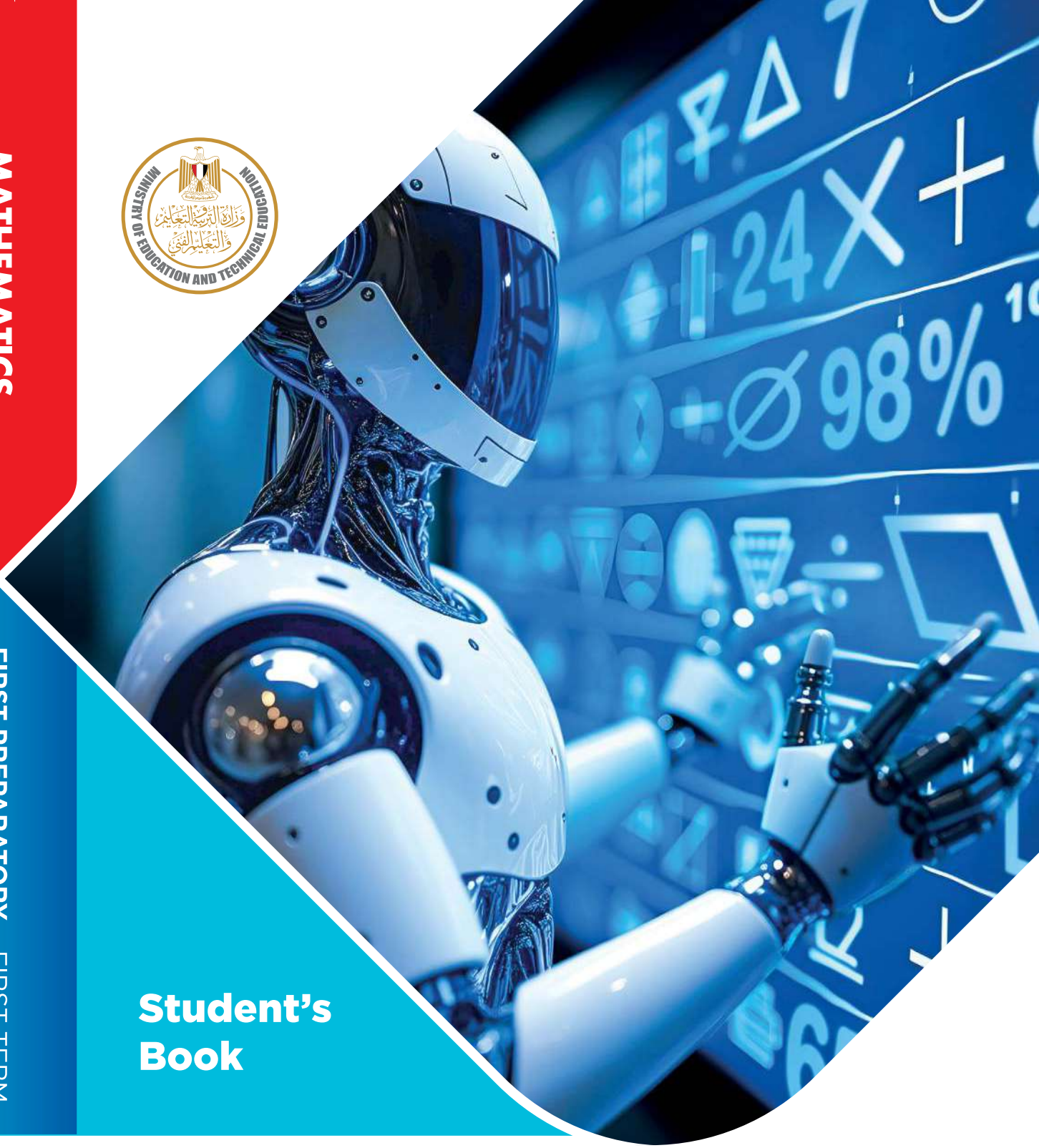
MATHEMATICS

FIRST PREPARATORY FIRST TERM



Student's Book

MATHEMATICS

First Preparatory - First Term

2025-2026

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Student's Book

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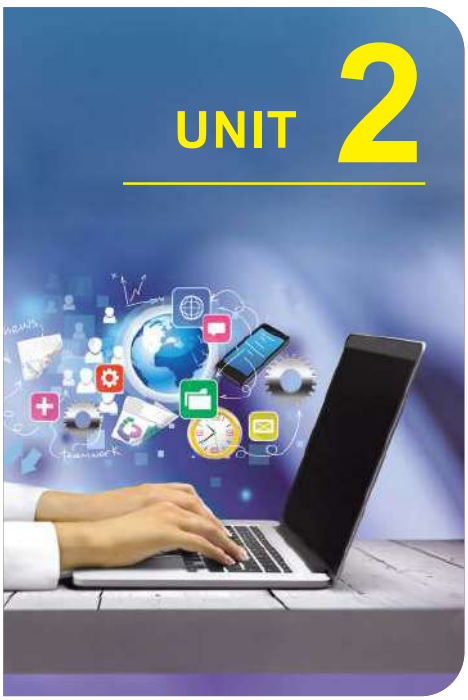
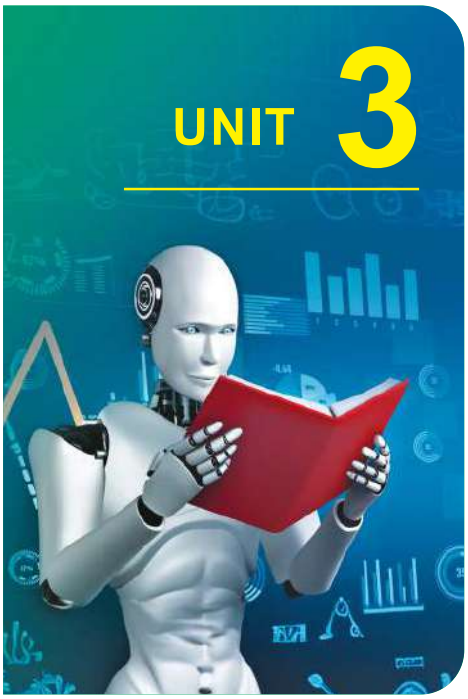


Algebra



Statistics

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Introduction

In the framework of the continuous development of the education system, and as part of the development plan that began with the kindergarten stage and continues until the end of the secondary stage, according to Egypt's Vision 2030 and the Ministry of Education's development plan, this book has been prepared to complete and implement the development processes of educational curriculums for the preparatory stage to create a qualitative leap in the way Egypt's students are prepared to deal with future challenges.

We are pleased to present this mathematics textbook for the first preparatory grade for the first term, which includes a set of tasks and tools that help students solve mathematical problems. This aims to achieve learning outcomes that keep pace with the challenges of the 21st century, whose structure is reinforced by the rapid revolution in information and communication technology.

The current curriculum aims to bring about a qualitative leap in the teaching and learning of mathematics. It also aims to provide an appropriate means to achieve the general educational goals in an integrated manner that aligns with the following:

- Emphasizing the positive role of the student in the learning process.
- Included knowledge curriculum, skills, values, positive trends needed for education, good citizenship, productive work and active participation in sustainable development programs.
- Including of modern positive trends in curriculum building, such as critical thinking skills, problem-solving skills, self-learning skills, cooperative learning, and effective communication with knowledge sources.
- Developing performance skills by focusing on self-learning and cooperative work.
- Achieving integration between mathematics and other subjects across different educational stages.
- Providing students with the opportunity to choose activities that suit their abilities, tendencies, and needs.

Asking Almighty God to make this book beneficial and hope that it will be part of a great effort to elevate Egypt to the ranks of advanced countries, ensuring a great future for all students. We wish you a very successful academic year full of achievements and success.

May God grant success,
The Authors

UNIT 1

Numbers and Their Operations

Unit Lessons

Percentage, Ratio, and Proportion

1-1 Proportion

1-2 Applications of Ratio and Proportion

Number Sets

1-3 Sets and Their Operations

Operations on Numbers

1-4 Operations on Integers

1-5 Operations on Rational Numbers

Artificial Intelligence (AI) is one of the latest fields and forms of technological development.
It can be used to enhance teaching and learning techniques.

▶ **Can we develop (AI) applications to solve mathematical problems in our daily life?**

Issues and Life Skills

- Mathematical Communication
- Mathematical Understanding
- Creative/Critical Thinking
- Information Technology (IT)
- Sustainable Development
- Globalization

Values

- Respect
- Responsibility
- Perseverance
- Endurance
- Belonging
- Justice

Lesson

1 - 1

Proportion

■ Learning Outcomes

- Learn the concept of proportion
- Write a proportional relationship
- Identify proportional and non-proportional relationships
- Solve proportions using cross multiplication
- Represent proportional relationships graphically
- Employ proportion in solving real-life problems

■ Vocabulary

- Ratio
- Proportion
- Cross Multiplication

■ Previous Skill

You can find a ratio equivalent to another ratio by multiplying or dividing both of its terms by the same number, except zero.

Example:

$\frac{2}{3} \overset{\times 2}{=} \frac{4}{6}$

$\frac{160}{2} \overset{\div 2}{=} \frac{80}{1}$

Get Ready!

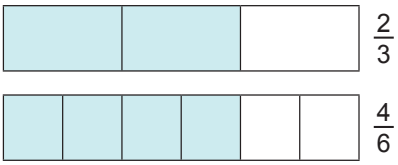
Khaled visited the Cairo International Book Fair to buy a children's book series for his kids.

Given that the price of three books is 96 LE, and Khaled bought 18 books, how much did Khaled pay for the books?

In this lesson, you will learn about the concept and properties of proportion, which can help you solve such real-life problems.

Think & Discuss!

Ahmed and Amal have two identical rectangles. Ahmed divided his rectangle into three equal parts and colored two of those parts. Amal divided her rectangle into six equal parts and colored four of those parts.



Is $\frac{2}{3} = \frac{4}{6}$?

Learn!

Proportion Concept: A proportion is an equality of at least two ratios or two rates.

Examples:

Equality of two ratios (proportion)

$\frac{2}{3} = \frac{4}{6}$

Equality of two rates (proportion)

$\frac{80 \text{ km}}{1 \text{ hour}} = \frac{160 \text{ km}}{2 \text{ hours}}$

How to write a proportion

Assume the following proportion $\frac{a}{b} = \frac{c}{d}$

Then the quantities a, b, c and d respectively are called proportional quantities.

The proportion can be expressed as follows:

$$\underbrace{a : b}_{\text{Extremes}} = \underbrace{c : d}_{\text{Means}}$$

a and d are called **extremes**, and b and c are called **means**.

■ Quick Revision

- A ratio is one of the methods used to compare between two quantities of the same kind.
- The ratio between the quantities a and b can be expressed by any of the following methods: a to b, $a:b$ or $\frac{a}{b}$

Cross Multiplication Property

For each of the following proportions, find the products of extremes and means, and compare them.

$\frac{2}{5} = \frac{8}{20}$, $\frac{24}{32} = \frac{3}{4}$, $\frac{6}{18} = \frac{2}{6}$

What do you conclude?

Property

If $\frac{a}{b} = \frac{c}{d}$ then $a \times d = b \times c$ (cross multiplication)

If $a \times d = b \times c$ then a , b , c and d are proportional.

Identifying Proportion Using Cross Multiplication

Example 1

Identify which of the following ratio pairs represents a proportion.

1 $\frac{4}{7}$, $\frac{12}{21}$ 2 $\frac{6}{10}$, $\frac{10}{25}$

1 $\frac{4}{7} \stackrel{?}{=} \frac{12}{21}$

Use cross multiplication $4 \times 21 \stackrel{?}{=} 7 \times 12$

$84 = 84$

Therefore, $\frac{4}{7}$ and $\frac{12}{21}$ represent a proportion

2 $\frac{6}{10} \stackrel{?}{=} \frac{10}{25}$

Use cross multiplication $6 \times 25 \stackrel{?}{=} 10 \times 10$

$150 \neq 100$

Therefore, $\frac{6}{10}$ and $\frac{10}{25}$ do not represent a proportion

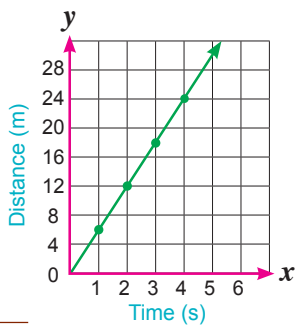
Identifying Proportion Using Graphical Representation

Example 2

The opposite table illustrates the distance in meters (m) covered by Bassem in one of his running exercises. Determine whether the distance in meters (m) is proportional to the time in seconds (s).
Explain your answer.

Covered Distances	
Time (s)	Distance (m)
1	6
2	12
3	18
4	24

Represent on a coordinate plane the ordered pairs that express the relation between time and distance. The points of the ordered pairs lie on a straight line passing through the origin, thus, the distance in meters is proportional to the time in seconds.



Guide

- If the points do not lie on a straight line, or lie on a straight line does not pass through the origin, then the relationship does not represent a proportion.

Diversified Strategies

- In Example 1, you can solve problem (1) by simplifying the ratio $\frac{12}{21}$

$\frac{12}{21} \overset{\div 3}{=} \frac{4}{7}$

- Could you solve problem (2) by simplifying ratios?

Self-Evaluation (1)

Identify which of the following represents a proportion.

1 $\frac{4}{8} \stackrel{?}{=} \frac{10}{20}$

2 $\frac{30}{42} \stackrel{?}{=} \frac{25}{40}$

Using Technology

Information provided in Example 2 can be represented graphically using different software and applications, such as:
GeoGebra , Excel

Self-Evaluation (2)

Use Excel to represent the data in the table below graphically, then determine whether they express proportional relationship or not, and justify.

Price of pies including delivery fees	
Number of Pies	Price (LE)
1	25
2	45
3	65
4	85

Solving Proportion

Example 3

Solve the proportion: $\frac{12}{16} = \frac{21}{x}$

Solving the proportion means finding the unknown value in the proportion.

Write the proportion	$\frac{12}{16} = \frac{21}{x}$
Use cross multiplication	$x \times 12 = 16 \times 21$
Multiply	$12x = 336$
Divide by 12	$\frac{12x}{12} = \frac{336}{12}$
Simplify	$x = 28$

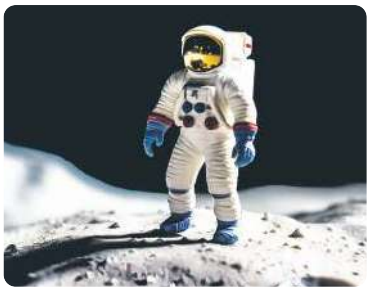
Example 4

Science: The weight of a body on earth is 90 Newtons.

If its weight on the moon is 15 Newtons.

What is the weight of another body on the moon if its weight on earth is 60 Newtons?

Let x be the weight of the other body on the moon.



Then, $\frac{15}{90} = \frac{x}{60}$	The proportion
$x \times 90 = 15 \times 60$	Cross multiplication
$90x = 900$	by Multiplying
$\frac{90x}{90} = \frac{90}{90}$	Dividing by 90
$x = 10$	by Simplifing

Hence, the weight of the other body on the moon is 10 Newtons.

Mental Math

▸ You can simplify the ratio $\frac{12}{16}$ before using cross multiplication

$\frac{12}{16} = \frac{3}{4}$ (÷ 4 above, ÷ 4 below)

Then, the proportion becomes as follows $\frac{3}{4} = \frac{21}{x}$

▸ You can solve the previous proportion using mental math directly

$\frac{3}{4} = \frac{21}{x}$ (× 7 above, × 7 below)

Then, $x = 28$

Self-Evaluation (3)

Solve each of the following proportions:

1. $\frac{2}{5} = \frac{x}{20}$
2. $a : 36 = 25 : 20$

Enriched Information

Units of measuring weights are different from units of measuring mass.

Examples of units of measuring weight are Newton - Dyne

While,
Ton - Kilogram - Gram
are units of measuring mass.

Self-Evaluation (4)

If a car covers a distance of 320 km in 4 hours, how much time would it take the car to cover another distance of 480 km if it travels at the same rate?

Collaborative Activity

Collaborate with one of your friends and use the Internet to discover the relationship between the weight of a body on earth and its weight on the moon.

Use your research to calculate the weight of your friend on the moon, and let your friend calculate your weight on the moon.

Lesson Assessment

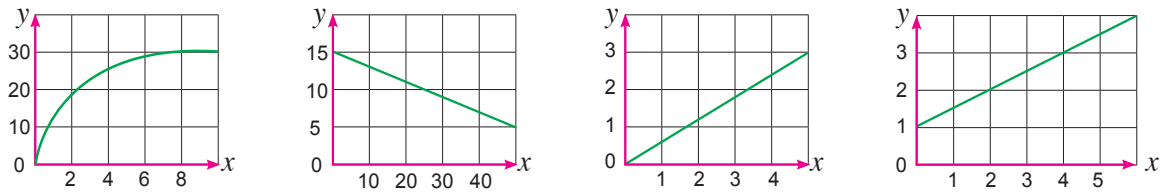
First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. Which of the following represents a proportion?
   (a) The price of 3 kg of bananas is 54 LE, and the price of 5 kg of bananas is 80 LE.
   (b) 10 goals out of 12 attempts, and 12 goals out of 18 attempts.
   (c) Reading 3 books in 2 months, and reading 9 books in 6 months.
   (d) 144 pulses in 2 minutes, and 210 pulses in 3 minutes.

2. Magdy can run 75 meters in 25 seconds. If he maintained his speed, which proportion you can use to find the time (x) he needs to run 300 meters?
   (a) $\frac{75}{25} = \frac{x}{300}$ (b) $\frac{75}{25} = \frac{300}{x}$
   (c) $\frac{25}{x} = \frac{300}{75}$ (d) $\frac{75}{x} = \frac{300}{25}$

3. Which of the following graphs represents a proportional?



▸ **In each of the following, are the quantities in proportion? If yes, write the proportion.**

4. 5 , 8 , 15 , 24

5. 12 , 27 , 16 , 18

▸ **Which of the following represents a proportion?**

6. $\frac{2}{5} \stackrel{?}{=} \frac{4}{10}$

7. $\frac{3}{4} \stackrel{?}{=} \frac{6}{9}$

8. $\frac{15}{25} \stackrel{?}{=} \frac{30}{60}$

9. $\frac{5}{4} \stackrel{?}{=} \frac{7.5}{6}$

Second Applying Scientific Concepts

▸ **Solve each of the following proportions:**

10. $\frac{3}{4} = \frac{x}{20}$

11. $\frac{15}{x} = \frac{30}{12}$

12. $a : 16 = 5 : 4$

13. $\frac{1}{3} = \frac{2}{b+1}$

▸ **Complete each of the following proportions:**

14. $\frac{6}{8} = \frac{\quad}{\quad}$

15. $\frac{20}{25} = \frac{36}{\quad}$

16. $\frac{48}{72} = \frac{\quad}{15}$

17. $\frac{12}{\quad} = \frac{\quad}{15}$

18. **Saving:** The opposite table shows how much Ibrahim saves within a certain number of months. Are the amounts saved proportional to the number of months?

Amount (LE)	300	600	900	1200
Number of Months	2	4	6	8

Third Analysis and Subjects Integration

19 **Shopping:** Omar bought 8 apples for 60 LE. How many apples of the same type can he buy for 105 LE?

Eating fruits daily provides the body many benefits. Fruits are rich in vitamins your body needs.

Regular car maintenance reduces environmentally harmful carbon emissions.

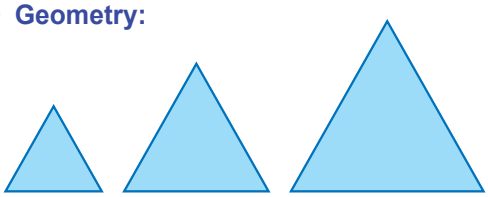
20 **Science:** A car uses 5 litres of petrol to cover a distance of 40 km. How much petrol would the car need to cover the distance of 128 km if it travels at the same rate?

21 **Culture:** Eman reads 10 pages in 40 minutes. How much time, in hours, would it take her to read a book of 120 pages if she reads at the same rate?

Reading helps enhance imagination, broaden cultural awareness, and gain knowledge.

Creative Thinking

22 **Geometry:**



The triangles above are equilateral.
Does the relationship between the perimeter and the side length in the equilateral triangles represent a proportion? Explain your answer.

23 If 15% of x equals 20% of y, then which of the following is equal to $x : y$?

(a) 5 : 4

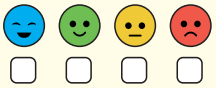
(b) 3 : 2

(c) 4 : 3

(d) 3 : 4

***Evaluate** your understanding!*

How well do you understand proportion? Tick the right box



Lesson

1 - 2

Applications of Ratio and Proportion

Learning Outcomes

- Learn the scale drawings.
- Calculate the scale drawings.
- Calculate the real length.
- Calculate the length in the drawing.
- Solve problems involving scale drawings.
- Learn the concept of proportional division.
- Apply ratio and proportion to solve real-life problems, including scale drawings, proportional division, sales, and purchases.

Vocabulary

- Scale Drawings
- Length in Drawing
- Real Length
- Magnification
- Minimization
- Proportional Division
- Percentage

Link to Life

GPS apps use scale drawings to determine distances on maps and estimate the journey's time based on the car's speed.

Get Ready!

The Iconic Tower is one of the most significant projects in Egypt's New Administrative Capital. It is the tallest building in Africa.

Sameh sketched a picture of the Iconic Tower, where its length measured 38.5 cm. Given that the ratio of the Tower's length in the picture to its real length is 1:1,000. What would you estimate as the height of the real Tower?

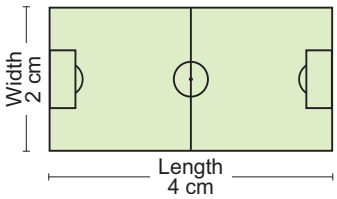
In this lesson, you will learn some concepts related to ratio and proportion, such as scale drawing, which will help you solve similar real-life problems.

The Iconic Tower

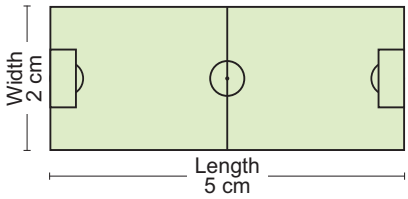
Think & Discuss!

Magdy and Amal each drew a rectangle representing a football pitch with a length of 20 m and a width of 10 m. Which one of them represented the pitch correctly? Explain your answer.

Amal's Drawing



Magdy's Drawing



In which rectangle do you find that the ratios:

$\frac{\text{Length in the drawing}}{\text{The pitch's real length}}$ and $\frac{\text{width in the drawing}}{\text{the pitch's real width}}$ represent a proportion?

Learn!

First Scale Drawings

The scale drawing is fundamental to understand and draw maps. It is an essential and effective tool for engineering drawings and model building.

Scale Drawing Concept:

Scale Drawing is the ratio of the dimension in the drawing or model to the dimension in reality.

$$\text{Scale Drawing} = \frac{\text{Length in drawing}}{\text{Real length}}$$

Example 1

Geography: If the scale drawing of a map is 1:600,000 and the distance between two points on the map is 4.5 cm, find the real distance between the two points.

Since scale drawing $= \frac{\text{Distance on map}}{\text{Real distance}}$

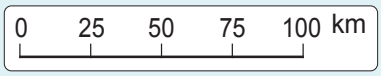
Therefore, $\frac{1}{600{,}000} = \frac{4.5 \text{ cm}}{\text{Real distance}}$

$1 \times \text{Real distance} = 600{,}000 \times 4.5 \text{ cm}$ (cross products)

$\text{Real distance} = 2{,}700{,}000 \text{ cm}.$

$= \frac{2{,}700{,}000}{100 \times 1{,}000} = 27 \text{ km}$

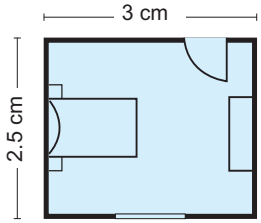
▸ **Scale drawing can be expressed on maps as follows :**



This means that every 1 cm on the map represents 25 km in reality.

Example 2

Geometry: The opposite figure represents a room model, where every 1 cm in the model represents 1.2 m in reality.
What is the room's real area?



Scale drawing $= \frac{\text{Length in drawing}}{\text{Real length}}$

$\frac{1 \text{ cm}}{1.2 \text{ m}} = \frac{3 \text{ cm}}{\text{The room's real length}}$

The room's real length $= 3 \times 1.2 = 3.6$ m

$\frac{1 \text{ cm}}{1.2 \text{ m}} = \frac{2.5 \text{ cm}}{\text{The room's real width}}$

The room's real width $= 2.5 \times 1.2 = 3$ m

The room's real area $= 3.6 \times 3 = 10.8 \text{ m}^2$

Quick Revision

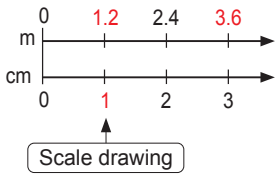
- 1 kilometer = 1,000 meters
- 1 meter = 10 decimeters = 100 centimeters
- 1 centimeter = 10 millimeters

Technology

You can use a calculator to convert between metric measurement units.

Diversified Strategies

- You can use the double number line to find the room's length.



- Use the double number line to find the room's width.

Self-Evaluation (1)

Samar took a magnified picture of a tiny insect. If the insect's length in the picture is 12 cm, and its real length is 3 mm, find the scale drawing.

Note

- If the scale drawing <1, it is a minimization.
- If the scale drawing >1, it is a magnification.

Self-Evaluation (2)

A model of a piece of land was designed as a rectangle of dimensions 6 cm and 5 cm where every 1 cm in the model represents 5 m in reality. What is the perimeter of the land for building a surrounding fence?

Second Proportional Division

Proportional Division Concept:

Proportional division is dividing something such as money, a piece of land, or weight, etc. into two or more parts based on a specified ratio.

Proportional division is used in many real-life applications, including dividing profits, distributing inheritance, and other practical applications.

Example 3

Science: If the ratio between the number of hydrogen atoms (H) to oxygen atoms (O) in water molecule is 2:1, how many atoms of each of them are there in a drop of water containing 1,200,000 atoms?

400,000	400,000	400,000

Ratio of hydrogen to oxygen is 2:1

Sum of parts $= 2 + 1 = 3$

Part value $= \frac{1,200,000}{3} = 400,000$ atoms.

Number of oxygen atoms $= 1 \times 400,000 = 400,000$ atoms.

Number of hydrogen atoms $= 2 \times 400,000 = 800,000$ atoms.

Example 4

Business: Three persons invested in the construction of a factory. The first person paid 9,000,000 LE, the second person paid 6,000,000 LE, and the third person paid 7,500,000 LE.
At the end of the first year, the profits amounted to 2,250,000 LE, and the profits were distributed based on each person's capital contribution. Calculate the share of the first year's profit for each person.

The profits were distributed according to each person's capital contribution ratio.

First		Second		Third	
9,000,000	:	6,000,000	:	7,500,000	(÷100,000)
90	:	60	:	75	(÷15)
6	:	4	:	5	

Profit distribution ratio $= 6 : 4 : 5$

Sum of parts $= 6 + 4 + 5 = 15$

Part value $= \frac{2,250,000}{15} = 150,000$ LE

The first persons's share $= 6 \times 150,000 = 900,000$ LE

The second persons's share $= 4 \times 150,000 = 600,000$ LE

The third persons's share $= 5 \times 150,000 = 750,000$ LE

Technology

You can use a calculator to perform the calculations in this example and make sure the results are correct.

Self-Evaluation (3)

If a man wants to distribute 3,600 LE between two people in a ratio of 3:2, what would be the share of each one?

Diversified Strategies

- You can calculate the number of atoms of each of (O) and (H) without calculating the part value.
  - Number of oxygen atoms (O) $= 1 \times \frac{1,200,000}{3}$
  - Number of hydrogen atoms (H) $= 2 \times \frac{1,200,000}{3}$

Entrepreneurship

There are many projects in Egypt that aim to boost the Egyptian economy and achieve sustainable development, including projects for developing factories and industrial facilities such as steel, petrochemicals, and car manufacturing.

Self-Evaluation (4)

Three people invested in a project with a total capital of 500,000 LE. The ratio of their investment contributions was 2:5:3. Calculate how much capital each one contributed.

Third Percentage Applications

First : Calculating Deduction (Discount):

When the price of an item decreases at a rate of d%, then:

Deduction value = Item price × d% = Item price × $\frac{d}{100}$

The item's price after deduction = Original price − Deduction value

Finding the price after deduction or discount

Example 5

If the price of a smart phone displayed in a store is 12,750 LE, and the store offered a 8% discount rate. What is the phone's price after discount?

Discount value = 12,750 × $\frac{8}{100}$ = 1,020 LE

Phone's price after discount = 12,750 − 1,020 = 11,730 LE

Self-Evaluation (5)

A store offers a 20% discount rate on all items. What is the sale price of a shirt whose price before discount is 420 LE?

Finding the original price before discount

Example 6

Sandy received a 15% discount rate on a pair of sneakers from one of the stores. She paid 340 LE. What was the original price of the sneakers?

Percentage of the sale price to the original price is:

100% − 15% = 85%

$P \times \frac{85}{100} = 340$

$P = 340 \div \frac{85}{100}$

$P = 340 \times \frac{100}{85} = 400$

Therefore, the original price = 400 LE

Diversified Strategies

Ratio → $\frac{85}{100} = \frac{340}{P}$ ← Sale price / ← original price

Use cross products

$P \times 85 = 340 \times 100$

Divide by 85

$P = \frac{340 \times 100}{85} = 400$

Self-Evaluation (6)

If a jacket costs 498 LE after a 17% discount rate of its original price, find the original price.

Second : Calculating Addition (Increase):

When the price of an item increases due to the addition of a tax, commission, or profit at a rate of t%, then:

Increase value = Item price × t% = Item price × $\frac{t}{100}$

The item's price after the increase = Original price + Increase value

■ Common Mistake

In the past, we used to say, "the sales tax is **14%**," but this statement is not accurate. **14%** represents the tax rate, whereas **72.8 LE** is the actual tax amount, as demonstrated in Example (7).

■ Self-Evaluation (7)

If a car tyre costs 3,200 LE, how much is it after adding a 10% tax rate?

Finding the tax value

Example 7

Omar subscribed to a home Internet service for 520 LE per month, along with a 14% tax rate applied to the service fee. What is the tax value and the total monthly subscription cost that Omar pays ?

Tax value $= 520 \times \frac{14}{100} = 72.8$ LE

Monthly subscription $= 520 + 72.8 = 592.8$ LE

■ Diversified Strategies

You can calculate the monthly subscription cost paid by Omar without calculating the tax value as follows:

Percentage of the paid amount to the service fee is **100% + 14% = 114%**

$\frac{114}{100} = \frac{m}{520}$ ← Paid amount ← Service fee

$m = 114 \times \frac{520}{100} = 592.8$ LE

Collaborative Activity

Work with your classmates to measure the dimensions of your classroom. Use these dimensions and choose an appropriate scale drawing and create a reduced sketch of your classroom.

Lesson Assessment

First Measuring Conceptual Understanding

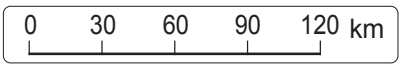
▸ **Choose the correct answer from the given ones:**

(1) If the length in the drawing is 2 cm and the real length is 6 m, what is the scale drawing?
(a) 1:3 (b) 1:30 (c) 1:300 (d) 1:3,000

(2) If the scale drawing is 1:1,000, and the length in the drawing is 2.5 cm, what is the real length in meters?
(a) 0.25 meter (b) 25 meters
(c) 2.5 meters (d) 250 meters

(3) If the real length is 90 meters and the scale drawing is 1:10,000, what is the length in the drawing in centimeters?
(a) 0.09 cm (b) 90 cm
(c) 9 cm (d) 0.9 cm

(4) If the scale drawing of a map is



and the distance between two cities on the map is 3 cm, what is the real distance between them?
(a) 30 km (b) 1,200,000 cm
(c) 90 km (d) 90,000 cm

▸ **Spot the Mistake:**

(5) An amount of 960 LE was divided between two people in the ratio of 3:5.
Which of Noor and Maram followed the correct method of solution? Discuss.

Noor's Solution

The first one share $= \frac{3}{5} \times 960$

The second one share $= \frac{5}{3} \times 960$

Maram's Solution

The first one share $= \frac{3}{8} \times 960$

The second one share $= \frac{5}{8} \times 960$

Second Applying Scientific Concepts

▸ **Choose the correct answer from the given ones:**

6 If an item price was reduced from 1,500 LE to 1,200 LE, what is the deduction rate?

(a) 3% (b) 15%

(c) 20% (d) 30%

7 A piece of land with an area of 36 feddans was divided between two persons in the ratio of 7:2. Which of the following could represent the share of either person?

(a) 4 feddans (b) 14 feddans

(c) 18 feddans (d) 28 feddans

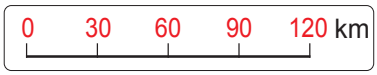
▸ **Answer the following questions:**

8 The ratio between two numbers is 2:5. If the smaller number is 48, what is the larger number?

9 Three persons shared 7,200 LE in the ratio of 3:4:5. Calculate the share of each one.

10 Omar earns twice as much as Khaled. If the sum of their wages is 36,000 LE, what is the wage for each of them?

11 Use the scale drawing below to find the real distance between Cairo and Alexandria.



Third Analysis and Subjects Integration

12 **Geography:** If the real distance between Cairo and Damietta is 200 km and the distance between them on the map is 5 cm, find the map's scale drawing.

13 **Maps:** If the scale drawing of a map is 1:4,000,000, and the real distance between two cities is 350 km, find the distance between the two cities on the map.

14 **Public Domain:** A model of the Cairo Tower was created at a scale drawing of 1:200. Given that the height of the Cairo Tower is 187 meters, what is the length of the model tower in centimeters?

Time management helps you succeed and reach your goals.

15 **Education:**

If the number of students in a school is 576, and the ratio of girls to boys is 5:3, find the increase of the number of girls than boys.

We are responsible for maintaining public buildings.

16 **Alloys:** Medals are made of a bronze alloy consisting of three metals, copper, tin, and zinc, in the ratio of $\frac{1}{5} : \frac{1}{18} : \frac{1}{45}$. Given that the mass of one medal is 425 grams, calculate the mass of each of the three metals.

17 **Inheritance:** A man passed away, leaving 150,000 LE for his wife, two sons, and one daughter. If his wife's share is $\frac{1}{8}$ of the total amount, and the remaining sum is to be distributed among his children, what is the share of each son and the daughter, considering the ratio of a son to a daughter is 2:1?

18 **Geometry:** If the ratio among the side lengths of a triangle with a perimeter of 135 cm is 7:5:3, find the length of its longest side.

19 A meal costs 150 LE, and 18% of the cost is added as the meal's profit. What is the meal's sale price ?

20 **Shopping:** Mariam wants to buy a vacuum for 6,800 LE. If the discount rate is 12%, find what Mariam will pay to buy the vacuum.

21 **E-commerce:** An online store receives a commission rate of 1.5% for marketing seller products. If the store sold products for 42,000,000 LE within a week, calculate the total commission amount for this week.

22 If a TV price after deduction rate of 16% is 12,600 LE, what is the TV price before deduction?

Creative Thinking

23 **Geometry:** If figure (1) represents a minimization of figure (2) at a scale drawing, and width of room B is $\mathcal{X}$ cm, find the value of $\mathcal{X}$.

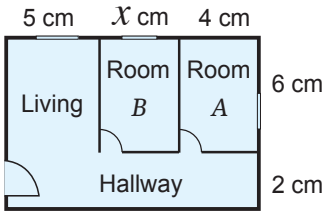


Fig. (1)

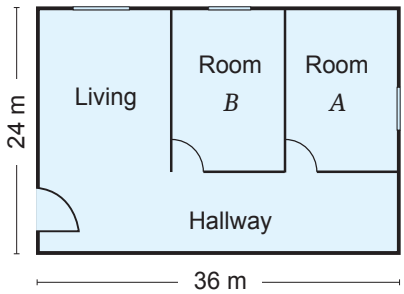
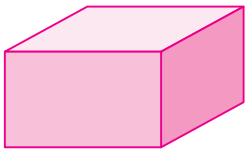


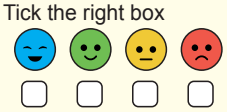
Fig. (2)

24 The sum of a cuboid's edge lengths is 1,440 cm. Given that the dimensions of the cuboid are in the ratio of 3:4:5, calculate its volume.



***Evaluate** your understanding!*

How well do you understand applications of ratio & proportion? Tick the right box



Lesson

1 - 3 Sets and Their Operations

Learning Outcomes

- Learn the concept of sets.
- Express sets using the listing method.
- Express sets using the set-builder method.
- Represent sets using Venn diagrams.
- Learn the "belongs to" and "subset" symbols.
- Learn the "intersection" and "union" symbols.
- Find the union or intersection of two sets.
- Represent the operations of union and intersection of two sets using Venn diagrams.

Vocabulary

- Set
- Venn Diagram
- Listing Method
- Set-Builder Method
- Belong to
- Subset
- Intersection
- Union

History of Maths

One of the uses of Venn diagram is to represent sets and show the relationships between them.
It was developed by the English mathematician John Venn (1834 - 1923) in the nineteenth century A.D.

Self-Evaluation (1)

1. Write the set of digits of the number 4540 using the listing method.
2. Write the set {2,3,5} using the set-builder method.

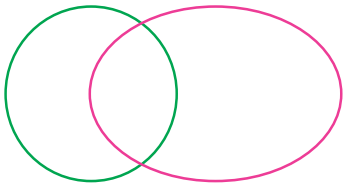
Get Ready!

Omar created a set of the Arab countries whose flags contain a crescent moon, and Salwa created a set of the Arab countries whose flags contain one star at least. What is the set of the common Arab countries between them?

In this lesson, you will learn the concept of sets and the related concepts and operations, which will help you solve such real-life problems.

Think & Discuss!

Numbers 4, 6, 8 and 12 are divisible by 2, and numbers 6, 9 and 12 are divisible by 3. Could you place all the numbers mentioned above in the opposite Venn diagram?



What do the numbers that you placed in the common area of the Venn diagram represent?

Learn!

Set Concept

Set Concept:

A set is a collection of well-defined objects, which are completely determined. These objects are called elements of the set, and they are written inside the braces { } without repetition. The order does not matter.

Expressing Sets:

Sets can be expressed using the listing method, the set-builder method, or Venn diagrams.
For example, if A is the set of natural numbers less than 4, then A can be expressed using any of the following methods:

Listing Method	Set-builder Method	Venn Diagram
$A = \{0, 1, 2, 3\}$	$A = \{x : x \in N, x < 4\}$ where N is the set of natural numbers.	A • 0 • 1 • 2 • 3

Types of Sets

Finite Set	Infinite Set	Empty Set (Null Set)
A set that has a finite number of elements. For example: $X = \{2, 3, 5\}$, containing 3 elements.	A set that has an infinite number of elements. For example : $N = \{0, 1, 2, 3, ...\}$	A set that has no elements and is denoted by the symbol ϕ or $\{\ \}$. For example: The set of integers between 1 and 2

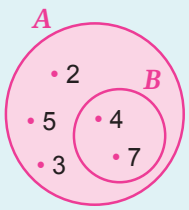
Inclusion (Subset)

If $A = \{2, 3, 4, 5, 7\}$, and $B = \{4, 7\}$,

then $4 \in A$, $4 \in B$ and $7 \in A$, $7 \in B$.

i.e. Every element of set B belongs to set A.

Hence, B is a subset of A and written as $B \subset A$.



You can express this as:

B is a subset of A, or A contains B.

When A and B are represented by a Venn diagram, draw B inside A.

Example 1

Which of the following sets is a subset of $X = \{1, 3, 4, 5, 6\}$?

1 $A = \{3 , 5 , 9\}$ 2 $B = \{1 , 5 , 6\}$

1 $9 \in A$ and $9 \notin X$, thus $A \not\subset X$ "The symbol $\not\subset$ is read as not subset"

2 Every element of B is an element of X, hence, $B \subset X$

Example 2

Write all the subsets of the set $A = \{4 , 6 , 8\}$, and mention their number.

The subsets of the set A are:

$\{4\}$, $\{6\}$, $\{8\}$

$\{4 , 6\}$, $\{4 , 8\}$, $\{6 , 8\}$

$\{4 , 6 , 8\}$, ϕ

The number of subsets of $A = 2^3 = 8$

■ Solution Guide

- The number of subsets of an n-element set is 2^n
- The empty set "ϕ" and the set A itself are called improper subsets.

■ Self-Evaluation (2)

Give an example of each of the following:

1 Finite set.

2 Infinite set.

3 Empty set.

■ Note

The symbol $\in$ is read as "belongs to" and is used to express that an element belongs to a set.

The symbol $\notin$ is read as "does not belong to" and is used to express that an element does not belong to a set.

■ Self-Evaluation (3)

If $X = \{2 , 9 , 3 , 5\}$, which of the following sets is a subset of X?

1 $A = \{1 , 5 , 9\}$

2 $B = \{2 , 3\}$

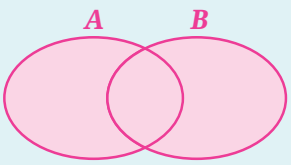
■ Self-Evaluation (4)

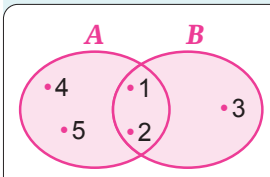
Find the number of subsets of a set that contains 5 elements.

Operations on Sets

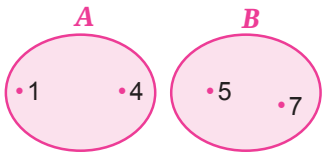
First : The Union of Two Sets

$A \cup B$ equals the set of elements which belongs to set A **or** set B.

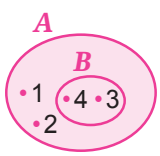




$A \cup B = \{1, 2, 3, 4, 5\}$



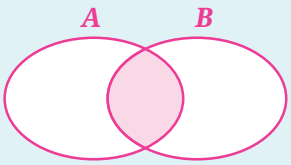
$A \cup B = \{1, 4, 5, 7\}$

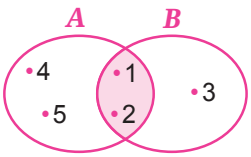


$A \cup B = \{1, 2, 3, 4\}$

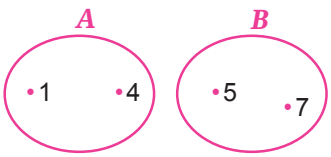
Second : The Intersection of Two Sets

$A \cap B$ equals the set of common elements between set A **and** set B.

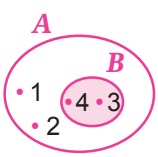




$A \cap B = \{1, 2\}$



$A \cap B = \phi$



$A \cap B = \{3, 4\}$

Example 3

If A is the set of digits of the number 243,
and B is the set of digits of the number 2234.
Is $A = B$?
What is the result of each of $A \cup B$ and $A \cap B$?

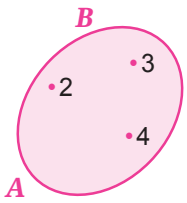
$A = \{2, 4, 3\}$, and $B = \{2, 3, 4\}$

Since A and B contain the same elements,

Therefore, $A = B$,

Thus:

- $A \cup B = \{2, 3, 4\}$
- $A \cap B = \{2, 3, 4\}$



■ Think & Discuss

For any two sets A and B:

- If $B \subset A$,
  Does $A \cup B = A$?
  Does $A \cap B = B$?
- What is the result of
  $A \cap \phi$?
  $A \cup \phi$?

■ Note that

If A and B are two non-empty sets and $A \cap B = \phi$, then A and B are disjoint (non-intersecting) sets.

■ Self-Evaluation (5)

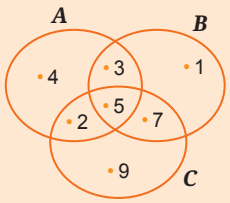
If $\{1, 3, x\} = \{7, y, 1\}$, what is the value of:
$x - y$?

■ Equal Sets

If every element in A belongs to B, and every element in B belongs to A, then $A = B$
Therefore:

- $A \cup B = A = B$
- $A \cap B = A = B$

Self-Evaluation (6)



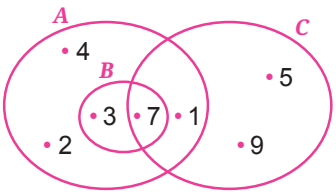
From the above Venn diagram, find:

1. $A \cap B$
2. $A \cup C$
3. $A \cap B \cap C$
4. $A \cap (B \cup C)$

Example 4

From the opposite Venn diagram, find:

1. $A \cap B$
2. $B \cup C$
3. $A \cap B \cap C$
4. $A \cup (B \cap C)$

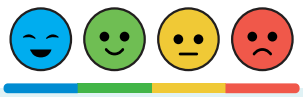


1. $A \cap B = \{3 , 7\} = B$
2. $B \cup C = \{3 , 7 , 1 , 9 , 5\}$
3. $A \cap B \cap C = \{7\}$
4. $A \cup (B \cap C) = \{1 , 2 , 3 , 4 , 7\} \cup \{7\} = \{1 , 2 , 3 , 4 , 7\}$

Collaborative Activity

Collaborate with one of your friends in the class, using Egyptian Knowledge Bank, and draw a Venn diagram using one of the computer application for drawing diagrams. Place on it the names of the Arab countries whose flags contain a crescent moon, or at least one star, or both.

Lesson Assessment



First — Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. If $x \in \{2 , 5 , 8\}$, what is the value that cannot be x?
   (a) 2 (b) 3 (c) 5 (d) 8

2. If $x \notin \{3 , 4 , 1\}$, which of the following can be x?
   (a) 1 (b) 2 (c) 3 (d) 4

3. If $A = \{2 , 5 , 8\}$, which of the following is correct?
   (a) $\{2\} \in A$ (b) $\{3\} \notin A$
   (c) $\{5\} \subset A$ (d) $\{8 , 5\} \not\subset A$

4. If $A = \{4 , 2 , 7\}$ and $B \subset A$, which of the following can be the set B?
   (a) $\{4 , 6\}$ (b) $\{1 , 3 , 5\}$
   (c) $\{4 , 3 , 7\}$ (d) $\{7 , 2\}$

5. If $A = \{8 , 9 , 6\}$, and $B = \{2 , 6 , 7\}$, which set expresses $A \cup B$?
   (a) $\{6\}$ (b) $\{2 , 7\}$
   (c) $\{8 , 9\}$ (d) $\{2 , 6 , 7 , 8 , 9\}$

6. If $A = \{8 , 9 , 6\}$, and $B = \{2 , 6 , 7\}$, which set expresses $A \cap B$?
   (a) $\{6\}$ (b) $\{2 , 7\}$
   (c) $\{8 , 9\}$ (d) $\{2 , 6 , 7 , 8 , 9\}$

7. If $A = \{5 , 7\}$, what is the number of all subsets of set A?
   (a) 2 (b) 4 (c) 6 (d) 8

Second Applying Scientific Concepts

▸ **Choose the correct answer from the given ones:**

(8) If $\{3, 6, 7, x\} = \{6, y, 3, 5\}$, what is the value of $y - x$?

(a) -12 (b) -2 (c) 2 (d) 12

(9) If $\{4, 5, 7\} \subset \{3, x, 4, y, 8\}$, what is the value of $x + y$?

(a) 5 (b) 7 (c) 8 (d) 12

▸ **Use the listing method to write each of the following sets:**

(10) A is the set of odd natural numbers less than 12

(11) B is the set of negative integers.

(12) C is the set of the digits of the number 12332551

(13) $X = \{x : x \in Z, x > -3\}$, where Z is the set of integers.

▸ **Answer the following questions:**

(14) Find all the proper subsets of $\{2, 3, 5\}$

(15) If $A = \{1, 2, 3, 4\}$, and $B = \{3, 4, 5, 6\}$, find: (1) $A \cup B$ (2) $A \cap B$

(16) If $A = \{5, 2, 3, 7\}$, and $B = \{2, 7\}$, find: (1) $A \cup B$ (2) $A \cap B$

(17) If $A = \{5, 2, 3, 7\}$, $B = \{4, 5, 7\}$, and $C = \{7, 2\}$,
find: (1) $A \cap B \cap C$ (2) $A \cup B \cup C$ (3) $(A \cup B) \cap C$

Third Analysis and Subjects Integration

(18) **Education:**

Some students joined three school activities sets:

- Press set $(A) = \{$Sameh, Yasser, Mona, Shaimaa$\}$.
- Theatre set $(B) = \{$Youssef, Mazen, Sameh, Mona, Sally$\}$.
- Community service set $(C) = \{$Rami, Huda, Sameh, Mazen$\}$.

(1) Represent the three sets in one Venn diagram.

(2) Find: $A \cap B \cap C$, $B \cup C$ and $A \cap B$

(3) Write using the listing method:

- The set of students who joined any of the three activities.
- The set of students who joined at least two activities.

Participation in school activities develops personality, foster teamwork, and work spirit.

Creative Thinking

▸ **Choose the correct answer from the given ones:**

(19) The number of subsets of any set cannot be .....

(a) 16 (b) 36 (c) 64 (d) 128

(20) If the number of elements of $A \cup B$ is 5, then the number of elements of A cannot be .....

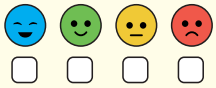
(a) 3 (b) 4 (c) 5 (d) 6

(21) If the number of elements of $A \cap B$ is 5, then the number of elements of A cannot be .....

(a) 4 (b) 5 (c) 6 (d) 11

***Evaluate** your understanding!*

How well do you understand sets and their operations ? Tick the right box



Lesson

1 - 4

Operations on Integers

■ Learning Outcomes

- Find the sum or difference of two integers.
- Apply the properties of addition and subtraction to solve problems involving integers.
- Find the product of two integers.
- Find the quotient of two integers (If possible).
- Learn the properties of multiplication and division of integers.

■ Vocabulary

- Addition
- Subtraction
- Multiplication
- Division
- Closure
- Commutative
- Associative
- Additive Identity
- Multiplicative Identity
- Additive Inverse (Opposite)

■ History of Maths

Al-Khwarizmi, a mathematician who lived in Baghdad (780 – 850 A.D.), authored several math books.
His works played a crucial role in spreading Hindu numerals and the concept of zero all over the world.

Get Ready!

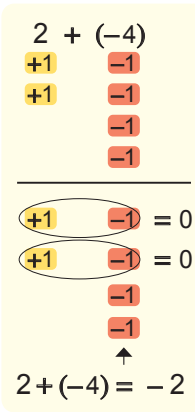
The highest point in Egypt is the peak of Mount Catherine, located in South Sinai, at 2,620 meters above sea level. On the other hand, the lowest point in Egypt is located at Qattara Depression, lying at a depth of 134 meters below sea level. What is the difference between these two points in meters?

Mount Catherine, South Sinai

In this lesson, you will learn how to perform operations on integers, which will help you solve such real-life problems.

Think & Discuss!

Consider that the card +1 represents Positive One and the card –1 represents Negative One. Then, an addition problem such as $2 + (-4)$ can be represented by the opposite figure.



Activity:

- Collaborate with your classmates and use the counting cards to find each of the following:

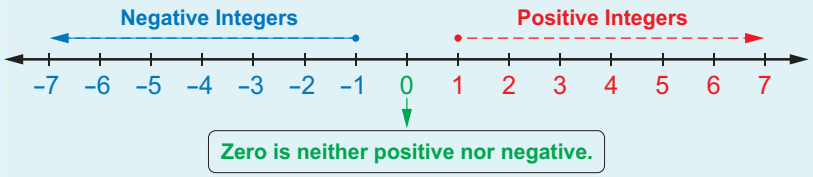
$3 + 5$, $-4 + (-2)$, $5 + (-2)$, $-6 + 4$

- Could you use the counting cards to find each of the following?

$5 - 3$, $2 - 6$, $-3 - 2$, $-2 - (-3)$

Learn!

- The set of integers consists of positive integers, zero and negative integers.
- The set of integers $(Z) = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$



- The set of integers is an ordered set increasing on the number line from left to right. For example:

$3 > -2$, $0 > -4$, $-3 > -5$

First Addition of Integers

Using the number line strategy for addition

Example 1

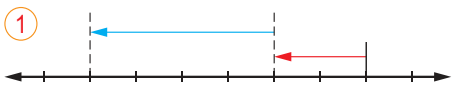
Find the result of each of the following:

1 $-2+(-4)$ 2 $-2+5$

Number Line Strategy

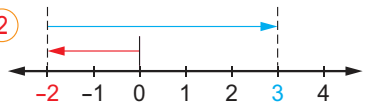
- Always start at 0.
- Move right to add a positive integer.
- Move left to add a negative integer.

1



$-2+(-4)=-6$

2



$-2+5=3$

Using the absolute value strategy for addition

Example 2

Find the result of each of the following:

1 $13+7$ 2 $-13+(-7)$

3 $-13+7$ 4 $13+(-7)$

Solution Guide

$|7|=7$, $|-7|=7$

$|13|=13$, $|-13|=13$

1 **Since the integers are positive:**

Therefore, the sum will have the same positive sign (and find the sum of the absolute values)

$13+7=20$

2 **Since the integers are negative:**

Therefore, the sum will have the same negative sign (and find the sum of the absolute values)

$-13+(-7)=-20$

3 **Since the integers have opposite signs, and:**

$|7|<|-13|$

Therefore, the sum will have the negative sign (and find the difference of the absolute values)

$-13+7=-6$

4 **Since the integers have opposite signs, and:**

$|13|>|-7|$

Therefore, the sum will have the positive sign (and find the difference of the absolute values)

$13+(-7)=6$

Properties of Addition of Integers

Let a, b and c be three integers, the following properties hold for addition:

1 **Closure:** The sum of any two integers is an integer $(a+b \in Z)$.
2 **Commutative:** $a+b=b+a$
3 **Associative:** $(a+b)+c=a+(b+c)$
4 **Existence of an additive identity, which is 0:** $a+0=0+a=a$
5 **Existence of additive inverses:** $a+(-a)=(-a)+a=0$

Self-Evaluation (1)

Using the number line, find the sum of each of the following:

1 $-5+(-3)$
2 $4+(-7)$

Absolute Value Strategy

- To find the sum of two integers with the same sign, add the absolute values of both integers, and the sum will have the same sign.
- To find the sum of two integers with opposite signs, find the difference of the absolute values of both integers, and the difference will take the sign of the integer with the larger absolute value.

Self-Evaluation (2)

Using the absolute value, find the sum of each of the following:

1 $-6+14$
2 $-6+(-14)$
3 $6+14$
4 $6+(-14)$

Second Subtraction of Integers

The subtraction of b from a is the same as the addition of a to the additive inverse of b.

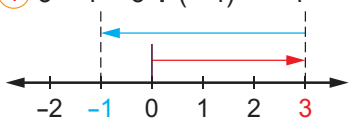
$$a - b = a + (-b)$$

Using the number line strategy for subtraction

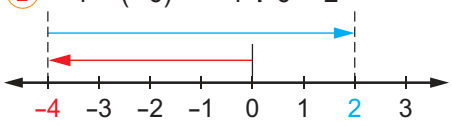
Example 3

Find the result of each of the following: 1 $3 - 4$ 2 $-4 - (-6)$

1 $3 - 4 = 3 + (-4) = -1$



2 $-4 - (-6) = -4 + 6 = 2$



Third Multiplication of Integers

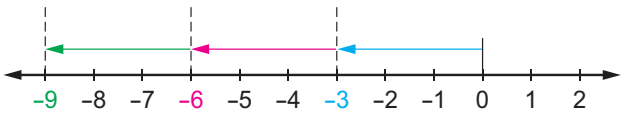
Multiplication is a repeated addition.

Using the number line strategy for multiplication

Example 4

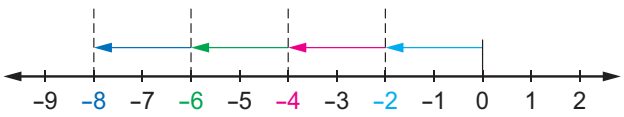
Find the product of each of the following: 1 $3 \times (-3)$ 2 $(-2) \times 4$

1 $3 \times (-3) = (-3) + (-3) + (-3)$



$3 \times (-3) = -9$

2 $(-2) \times 4 = (-2) + (-2) + (-2) + (-2)$



$(-2) \times 4 = -8$

Properties of Multiplication of Integers

Let a, b and c be three integers, the following properties hold for multiplication:

1 **Closure:** The product of any two integers is an integer ($a \times b \in Z$).
2 **Commutative:** $a \times b = b \times a$
3 **Associative:** $(a \times b) \times c = a \times (b \times c)$
4 **Existence of a multiplicative identity, which is 1:** $a \times 1 = 1 \times a = a$
5 **Distributing multiplication on addition and subtraction:**
$a \times (b + c) = a \times b + a \times c$, $a \times (b - c) = a \times b - a \times c$

■ Think with your classmates

Does the subtraction of integers have the same properties as the addition of integers? Discuss.

■ Self-Evaluation (3)

Find the resulf of each of the following:

1 $6 - 10$
2 $-4 - 8$
3 $-5 - (-12)$

■ Note that

- When multiplying two integers with the same sign, the product is positive.
- When multiplying two integers with opposite signs, the product is negative.
- When multiplying any number by zero, the product is zero.

■ Self-Evaluation (4)

Find the product of each of the following:

1 $5 \times (-3)$
2 $(-2) \times 3$
3 $(-2) \times (-7)$
4 $-(-4) \times (-5)$

Think with your classmates

Does the division of integers have the same properties as the multiplication of integers? Discuss.

Self-Evaluation (5)

Find the quotient of each of the following:

1. $(-33) \div (-11)$
2. $54 \div (-9)$
3. $(-16) \div 4$

Fourth Division of Integers

Example 5

Find the quotient of each of the following:

1. $25 \div (-5)$
2. $(-27) \div 9$
3. $(-32) \div (-4)$

1. $25 \div (-5) = -5$
2. $(-27) \div 9 = -3$
3. $(-32) \div (-4) = 8$

Note that

- When dividing two integers with the same sign, the quotient is positive.
- When dividing two integers with opposite signs, the quotient is negative.
- When dividing any number by zero, the result is undefined.

Collaborative Activity

Using different sources (e.g. the Internet, your school library, ... etc.), collaborate with your classmates under the supervision of your teacher to consolidate your understanding of the sign rules in multiplication and division of integers.

Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. Which of the following sums has a positive sign?
   (a) $19 + (-26)$ (b) $-35 + 17$
   (c) $-25 + (-12)$ (d) $40 + (-18)$

2. Which of the following products has a negative sign?
   (a) $-3 \times (-9)$ (b) -2×5
   (c) $0 \times (-5)$ (d) $-1 \times (-1)$

3. Which of the following is equivalent to the subtraction operation: $-5 - (-8)$?
   (a) $5 - 8$ (b) $8 - 5$
   (c) $5 + 8$ (d) $-5 - 8$

4. Which of the following is equal to -5?
   (a) $0 \div (-5)$ (b) $1 \div (-5)$
   (c) $-25 \div (-5)$ (d) $-25 \div 5$

▸ **Verify each of the following statements.**

5. The additive inverse of an integer is not always negative.

6. The product of a positive integer and a negative integer is always negative.

▸ **Spot the mistake in each of the following, and make the necessary corrections.**

7. $9 + (-12) = 21$

8. $-3 + (-5) = -2$

9. $-13 + (-4) = 17$

Second Applying Scientific Concepts

▸ **Find the result of each of the following:**

(10) $6 + (-4)$ (11) $-4-5$ (12) -3×7 (13) $15 \div (-3)$

(14) $7 + (-7)$ (15) $18 - 40$ (16) $-37 + 59$ (17) $-72 \div (-9)$

(18) $9 - (-105)$ (19) $-17 - (-24)$ (20) $-65 \div (-13)$ (21) $420 \div (-15)$

▸ **If $a = 15$, and $b = -5$, find the result of each of the following:**

(22) $|9 - a|$ (23) $|3 - b|$ (24) $a - b$ (25) $|15 \div b|$

(26) $|b \times a|$ (27) $b - (-a)$

▸ **Use the addition or multiplication properties to find the result of each of the following:**

(28) $-6 + (-13) + 6$ (29) $-4 \times (-19) \times 25$ (30) -9×102

Third Analysis and Subjects Integration

(31) **Tourism:** In Saint Catherine City in South Sinai, the maximum temperature on a winter day was 11 degrees Celsius. Then, it dropped by 15 degrees Celsius overnight to be the lowest temperature degree in this city. Find this temperature degree using addition.

Saint Catherine in Winter

Egypt boasts numerous unique tourist and historical monuments that attract visitors from around the whole world.

Water is the source of life on Earth. Therefore, we must conserve water by using it wisely, and not polluting it with garbage.

(32) A submarine was moving vertically under water. When it reached a depth of 84 meters, it started to rise again. Find its location after 20 minutes if it is moving upward at a speed of 4 meters per minute.

Creative Thinking

(33) When would the equation $a + |a| = 0$ be correct ?

(34) If a and b are two integers, such that $a - b = 5$, and $3 - (-b) = -7$, find: $a + b$

(35) When Mazen multiplied two integers together, the product was (-28). What are the possible two integers that Mazen could have multiplied together?

***Evaluate** your understanding!*

How well do you understand operations on integers?
Tick the right box

Lesson

1 - 5

Operations on Rational Numbers

Learning Outcomes

- Learn different forms of a rational number.
- Find the sum or difference of two rational numbers.
- Apply addition and subtraction properties to solve rational numbers problems.
- Find the product of two rational numbers.
- Find the quotient of two rational numbers (if possible).
- Learn multiplication and division properties of the rational numbers.
- Utilize mathematical operations properties to solve problems involving rational numbers.

Vocabulary

- Rational Number
- Multiplicative Inverse
- Recurring Decimal.

Get Ready!

Ras Mohamed Nature Reserve is one of the world's most stunning diving destinations, boasting abundant coral reefs and rare marine creatures.

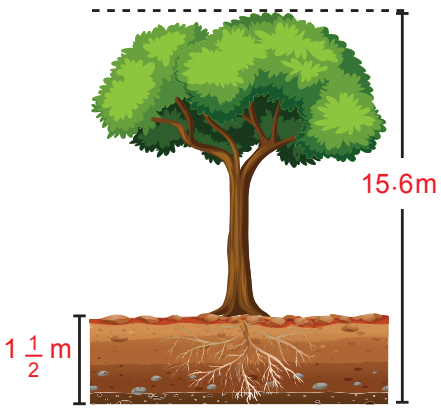
A man dives $12\frac{1}{2}$ meters in the water. Then, he rose 4.75 meters. At this point, what is the man's depth in water?

In this lesson, you will learn how to perform operations on rational numbers, which will help you solve such real-life problems.

Think & Discuss!

A tree root grows $1\frac{1}{2}$ meters below the ground.

Can you find the height of the tree above the ground surface?



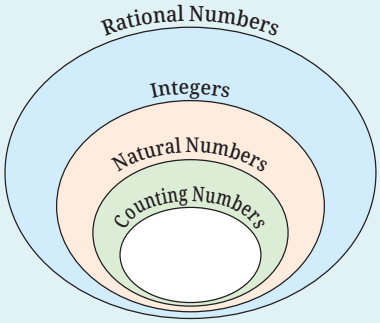
Learn!

The Set of Rational Numbers

- The set of rational numbers (Q) is the set of all numbers that can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

$$Q = \left\{ \frac{a}{b} : a \in Z , b \in Z , b \neq 0 \right\}$$

- Based on the above definition: The set of counting numbers, the set of natural numbers, and the set of integers are all subsets of the set of rational numbers. Discuss.



The different Forms of a Rational Number

Think

How can you use division to find the decimal equivalent of a rational number?

- **Expressing a rational number in the form of a percentage:**

  For example: $\frac{1}{2} = 50\%$, $2\frac{1}{4} = 225\%$

- **Expressing a rational number in the form of a finite decimal:**

  For example: $\frac{2}{5} = 0.4$, $1\frac{3}{4} = 1.75$

- **Expressing a rational number in the form of an infinite (recurring) decimal:**

  For example: $\frac{2}{9}$ can be expressed in the form of an infinite decimal,

  $0.222...$ or $0.\overline{2}$ (which is read as 0.2 recurring)

  The three dots mean that the decimal is infinite.

  The line above one digit or more means that these digits are repeating.

First Addition of Rational Numbers

- **If $\frac{a}{c}$ and $\frac{b}{c}$ are two rational numbers with equal denominators, then:**

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

- **If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers with different denominators, then:**

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Example 1

Find the result of each of the following in the simplest form:

1) $\frac{-5}{8} + \frac{3}{8}$ 2) $\frac{1}{4} + (-\frac{1}{3})$ 3) $2.6 + 2.\overline{6}$

1) $\frac{-5}{8} + \frac{3}{8} = \frac{-5+3}{8} = \frac{-2}{8} = \frac{-1}{4}$

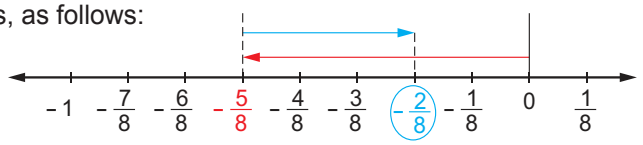
2) $\frac{1}{4} + (-\frac{1}{3}) = \frac{3+(-4)}{12} = \frac{-1}{12}$

3) $2.6 + 2.\overline{6} = 2\frac{6}{10} + 2\frac{2}{3} = 2\frac{3}{5} + 2\frac{2}{3} = 2\frac{9}{15} + 2\frac{10}{15}$

$= 4\frac{19}{15} = 5\frac{4}{15}$

Diversified Strategies:

You can use the number line strategy to find the sum of two rational numbers, as follows:



$$\frac{-5}{8} + \frac{3}{8} = -\frac{2}{8}$$

Using Technology

You can use the calculator to convert a rational number in the form of $\frac{a}{b}$ to a recurring decimal, and vice versa, as follows:

- To express the fraction $\frac{2}{9}$ in the form of an infinite (recurring) decimal, enter $\frac{2}{9}$ on your calculator. Press the [=] key, and then [S⇔D]. The calculator will display 0.22222222, which is written as $0.\overline{2}$
- Conversely, to express the recurring decimal $0.\overline{2}$ in the form of $\frac{a}{b}$, enter 0.22222222 on your calculator till you have full screen of 2s, then press the [=] key. The calculator will display $\frac{2}{9}$

Self-Evaluation (1)

Find the result of each of the following:

1) $\frac{3}{7} + (\frac{-6}{7})$

2) $|\frac{-8}{9}| + 2\frac{1}{9}$

3) $-\frac{1}{5} + \frac{2}{3}$

4) $0.\overline{4} + 1\frac{2}{3}$

Using properties of addition of rational numbers

Example 2

Find the sum of each of the following:

1) $\frac{-3}{5} + \frac{2}{15} + \frac{8}{5}$ 2) $\frac{-7}{4} + \frac{2}{5} + \frac{7}{4}$

1) $\frac{-3}{5} + \frac{2}{15} + \frac{8}{5} = \frac{-3}{5} + \frac{8}{5} + \frac{2}{15}$ (Commutative property)

$= (\frac{-3}{5} + \frac{8}{5}) + \frac{2}{15}$ (Associative property)

$= 1 + \frac{2}{15} = 1\frac{2}{15}$

2) $\frac{-7}{4} + \frac{2}{5} + \frac{7}{4} = \frac{-7}{4} + \frac{7}{4} + \frac{2}{5}$ (Commutative property)

$= (\frac{-7}{4} + \frac{7}{4}) + \frac{2}{5}$ (Associative property)

$= 0 + \frac{2}{5} = \frac{2}{5}$ (Additive identity property)

Properties of addition of rational numbers

Addition of rational numbers has the following properties:

1) Closure
2) Commutative
3) Associative
4) Existence of an additive identity (0)
5) Existence of additive inverses

Self-Evaluation (2)

Apply the properties of rational numbers addition to find the sum of each of the following:

1) $\frac{-2}{7} + 1 + \frac{4}{14}$
2) $1\frac{1}{4} + 0.6 + \frac{-1}{4}$

Think with your classmates

Does the subtraction of rational numbers have the same properties of the addition of rational numbers? Discuss.

Second Subtraction of Rational Numbers

The subtraction of $\frac{c}{d}$ from $\frac{a}{b}$ is the same as the addition of $\frac{a}{b}$ to the additive inverse of $\frac{c}{d}$

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(\frac{-c}{d}\right)$$

Example 3

Find the result of each of the following:

1) $2\frac{1}{2} - 3\frac{1}{4}$
2) $-3\frac{2}{3} - 2\frac{5}{6}$
3) $35\% - |\frac{-3}{5}|$
4) $0.36 - \frac{2}{25}$

1) $2\frac{1}{2} - 3\frac{1}{4} = \frac{5}{2} - \frac{13}{4} = \frac{10}{4} - \frac{13}{4} = \frac{-3}{4}$

2) $-3\frac{2}{3} - 2\frac{5}{6} = -3\frac{4}{6} - 2\frac{5}{6} = -5\frac{9}{6} = -5\frac{3}{2} = -6\frac{1}{2}$

3) $35\% - |\frac{-3}{5}| = \frac{35}{100} - \frac{3}{5} = \frac{7}{20} - \frac{3}{5} = \frac{7}{20} - \frac{12}{20} = \frac{-5}{20} = \frac{-1}{4}$

4) $0.36 - \frac{2}{25} = \frac{36}{100} - \frac{2}{25} = \frac{9}{25} - \frac{2}{25} = \frac{7}{25}$

Self-Evaluation (3)

Find the result of each of the following:

1) $\frac{3}{5} - (-\frac{1}{2})$
2) $4\frac{7}{10} - 1\frac{3}{5}$
3) $25\% - 0.1\overline{6}$

Third Multiplication of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Example 4

Find the product of each of the following:

1. $\frac{4}{3} \times \frac{6}{7}$
2. $\frac{1}{2} \times 6\frac{2}{7}$
3. $-\frac{3}{5} \times (-0.8\overline{3})$
4. $\left|-1\frac{3}{4}\right| \times (-2\frac{2}{7})$

1. $\frac{4}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{7} = \frac{8}{7}$
2. $\frac{1}{2} \times 6\frac{2}{7} = \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{44}^{22}}{7} = \frac{22}{7} = 3\frac{1}{7}$
3. $-\frac{3}{5} \times (-0.8\overline{3}) = -\frac{\cancel{3}^{1}}{\cancel{5}_{1}} \times (-\frac{\cancel{5}^{1}}{\cancel{6}_{2}}) = \frac{1}{2}$
4. $\left|-1\frac{3}{4}\right| \times (-2\frac{2}{7}) = 1\frac{3}{4} \times (-2\frac{2}{7})$
$= \frac{\cancel{7}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{-16}^{-4}}{\cancel{7}_{1}} = -4$

Mental Math

Problem 2 can be solved using the distributive property, as follows:

$\frac{1}{2} \times 6\frac{2}{7} = \frac{1}{2}(6 + \frac{2}{7})$
$= \frac{1}{2}(6) + \frac{1}{2}(\frac{2}{7})$
$= 3 + \frac{1}{7} = 3\frac{1}{7}$

Note that

The multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{b}{a}$ (where : $a \neq 0$).

For example: • The multiplicative inverse of $\frac{3}{8}$ is $\frac{8}{3}$

• The multiplicative inverse of $-\frac{1}{2}$ is -2

Fourth Division of Rational Numbers

Division of rational numbers is multiplying the dividend by the multiplicative inverse of the divisor.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, where $\frac{c}{d} \neq 0$

Example 5

Find the quotient of each of the following:

1. $\frac{-2}{3} \div \frac{4}{9}$
2. $-3\frac{3}{4} \div (-2\frac{1}{4})$
3. $0.6\overline{3} \div 1\frac{3}{11}$

1. $-\frac{2}{3} \div \frac{4}{9} = \frac{\cancel{-2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{4}_{2}} = \frac{-3}{2}$
2. $-3\frac{3}{4} \div \left(-2\frac{1}{4}\right) = -\frac{15}{4} \div -\frac{9}{4} = \frac{\cancel{-15}^{5}}{\cancel{4}_{1}} \times \frac{\cancel{-4}^{1}}{\cancel{9}_{3}} = \frac{5}{3}$
3. $0.6\overline{3} \div 1\frac{3}{11} = \frac{7}{11} \div \frac{14}{11} = \frac{\cancel{7}^{1}}{\cancel{11}_{1}} \times \frac{\cancel{11}^{1}}{\cancel{14}_{2}} = \frac{1}{2}$

Note that

- When multiplying zero by any rational number, the product is zero.
- The number zero does not have a multiplicative inverse, since the division by zero is undefined.

Properties of multiplication of rational numbers

Multiplication of rational numbers has the following properties

1. Closure
2. Commutative
3. Associative
4. Existence of a multiplicative identity (1)
5. Existence of multiplicative inverses
6. Distributing multiplication on addition

Self-Evaluation 4

Find the product of each of the following:

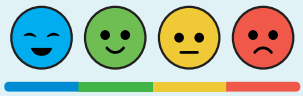
1. $0.3 \times (-\frac{4}{9})$
2. $-3\frac{1}{4} \times (-1\frac{1}{2})$
3. $25\% \times 2\frac{1}{2}$

Self-Evaluation 5

Find the quotient of each of the following:

1. $\frac{4}{7} \div \frac{12}{7}$
2. $(\frac{-1}{2})^2 \div (-3\frac{3}{4})$
3. $\frac{1}{8} \div 0.\overline{1}$

Lesson Assessment



First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) **Marine Creatures:** A dolphin dived to a depth of $3\frac{1}{4}$ meters underwater, then it dived a further $2\frac{1}{2}$ meters. Which of the following does not express the dolphin's position relative to the water surface?

(a) $-3\frac{1}{4}+(-2\frac{1}{2})$ (b) $-3\frac{1}{4}+|-2\frac{1}{2}|$ (c) $-3\frac{1}{4}-2\frac{1}{2}$ (d) $-(3\frac{1}{4}+2\frac{1}{2})$

(2) **Economy:** If the stock price of a company listed on the Egyptian exchange increased by 3.25 LE, and then if decreased by 2.75 LE, which of the following expresses the change in the stock price at that moment?

(a) $-3.25+2.75$ (b) $3.25-2.75$ (c) $3.25+2.75$ (d) $-3.25-2.75$

(3) Which of the following expressions has the same product as $\frac{-3}{8}\times\frac{8}{3}$?

(a) $2\frac{1}{4}\times\frac{4}{9}$ (b) $\frac{-2}{5}\times3\frac{1}{2}$ (c) $-\frac{1}{4}\times(-4)$ (d) $2\frac{1}{2}-3{\cdot}5$

(4) Which of the following operations has the same result of $2\frac{2}{3}\div(-1\frac{3}{7})$?

(a) $2\frac{2}{3}\times(-1\frac{7}{3})$ (b) $2\frac{2}{3}\times1\frac{3}{7}$ (c) $-2\frac{2}{3}\times\frac{7}{10}$ (d) $-1\frac{3}{7}\times2\frac{2}{3}$

(5) What is the multiplicative inverse of $-3\frac{1}{2}$?

(a) $-\frac{7}{2}$ (b) $-2\frac{1}{3}$ (c) $\frac{-2}{7}$ (d) $\frac{2}{7}$

Second Applying Scientific Concepts

▸ **Find the result of each of the following:**

(6) $\frac{5}{8}+(\frac{-7}{8})$ (7) $-3.2+(-1.\overline{3})$ (8) $-\frac{4}{5}\div4\frac{2}{5}$

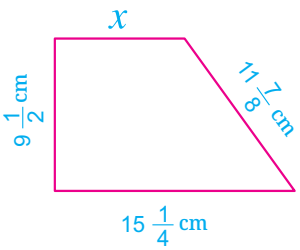
(9) $\frac{-3}{4}\times(-2\frac{2}{5})$ (10) $-2.125+2.6+(-7\frac{21}{24})$

▸ **Find the value of x in each of the following:**

(11) $\frac{-5}{12}-(-\frac{7}{6})=\frac{1}{6}+x$ (12) $\frac{2}{7}-(\frac{-11}{21})=\frac{11}{21}-x$

▸ **Answer the following questions:**

(13) **Geometry:** If the perimeter of the opposite figure is 44 cm, find the value of x in centimeters.



(14) **Spot the mistake:**

Ahmed calculated the sum of $\frac{1}{3}$ and $\frac{3}{4}$ as follows :

$\frac{1}{3} + \frac{3}{4} = \frac{1+3}{3+4}$

Find the mistake in his calculation and correct it.

Third Analysis and Subjects Integration

(15) If a shark ascends 124.1 meters from a depth of 152.5 meters below the water surface, where is the shark position relative to the water surface now?

Regular plumbing system maintenance saves water and keeps buildings safe.

(16) A plumber has a pipe 64 $\frac{5}{8}$ decimeters long. He first cuts off a part of 2 $\frac{7}{8}$ decimeters from the end of the pipe, and then he cuts off another 1 $\frac{3}{8}$ decimeters. How long is the remaining part of the pipe ?

Sports teach us perseverance and facing difficulties.

(17) A mountaineer climbs a mountain with a height of 1 $\frac{1}{2}$ kilometers. What is the height of the mountaineer above the ground surface if he climbs $\frac{2}{3}$ of the mountain's height ?

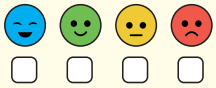
(18) The length of a fabric roll is 7 $\frac{1}{2}$ meters. If the fabric roll is to be divided into smaller pieces, each of length 30 centimeters, how many pieces will you get?

Creative Thinking

(19) **Open Question:** Write a question for adding two rational numbers with different denominators, such that their sum equals $-\frac{3}{8}$

***Evaluate** your understanding!*

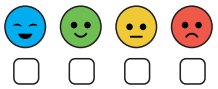
How well do you understand operations on rational numbers ? Tick the right box



Unit One Assessment

Evaluate your understanding!

How well do you understand Unit One? Tick the right box



▸ Choose the correct answer from the given ones:

1. Which of the following scale drawings is equivalent to "every 1 cm in the drawing represents 6.5 km in reality"?
   (a) 1 : 6,500,000 (b) 1 : 6.5
   (c) 1 : 650,000 (d) 1 : 6,500

2. If $x \notin \{2, 5, 7\}$, which of the following can be a value of x?
   (a) 1 (b) 2 (c) 5 (d) 7

3. Which of the following is equal to $\frac{3}{5} + \frac{-2}{3}$?
   (a) $\frac{1}{8}$ (b) $\frac{1}{15}$
   (c) $\frac{-6}{15}$ (d) $\frac{-1}{15}$

4. A piece of land with an area of 63 feddans was divided between two persons in the ratio of 4:5. Which of the following could represent the share of either person?
   (a) 9 (b) 45 (c) 28 (d) 30

▸ Complete each of the following with the correct answer:

5. If $\frac{14}{x} = \frac{y}{7}$, then $xy =$ .....
6. If $A = \{7, 8, 0\}$, then the number of all subsets of A is .....
7. Hesham packed 100 cardboard boxes in 2.5 hours. It would take Hesham ..... hours to pack 160 boxes, working at the same rate.
8. If 13,500 tourists represent 12% of a tourist group, then the number of tourists in the entire group is ..... tourists.

▸ Answer the following questions:

9. If $A = \{1, 5, 3, 7\}$, and $B = \{1, 2, 5, 9\}$. Find each of : $A \cup B$ and $A \cap B$
10. Alaa purchased a mobile phone for 6,750 LE and sold it for 7,776 LE. Calculate the percentage of Alaa's profit.
11. Spot the mistakes and make the necessary corrections.
    ① $-8 - (-3) = -11$ ② $\frac{-5}{3} \times \frac{9}{15} = 1$ ③ $\frac{5}{8} + \frac{-3}{7} = \frac{2}{15}$ ④ $48 \div (-8) = 6$
12. If the magnification ratio of an insect's image is 30 : 1 , and the insect's real length is $2\frac{1}{2}$ mm, Find the length of the insect in the image in centimeters.

Unit One Activity

▸ **Activity Aim:** To employ Artificial Intelligence (AI) capabilities in understanding scale drawing and to create a cover or background for a Maths book.

▸ **Implementation Steps:** Work in pairs and try to complete the following steps:

1. Investigate a range of AI applications related to image editing.
2. Find an image suitable for a Maths book cover.
3. Use one of the chosen AI applications to magnify the selected image at two different scale drawings.
4. Print the resulting images at both scales.
   Choose one as your final cover and justify your decision.
5. Write a report that discusses the importance of AI applications in the learning process.

UNIT 2

Algebra

Unit Lessons

Digital technology and algebra share a mutual relationship; the progress of one influences the other. Technology facilitates the creation of virtual learning environments, and software can also be used to solve algebraic equations.

▶ **Can digital technology apps be further developed to solve more complex algebraic equations?**

Issues and Life Skills		Values	
- Mathematical Communication	- Sustainable Development	- Responsibility	- Belonging
- Language & Translation	- Creative Thinking	- Work	- Respecting Others

Lesson

2 - 1 Mathematical Expressions and Formulas

Learning Outcomes

- Represent variables using symbols.
- Learn algebraic term and algebraic expression.
- Learn mathematical expression and mathematical formula.
- Differentiate between mathematical expression and mathematical formula.
- Learn equation and inequality.
- Simplify and evaluate algebraic expression at a specific value.
- Learn about temperature measuring units (Celsius and Fahrenheit) and the relationship between them.

Vocabulary

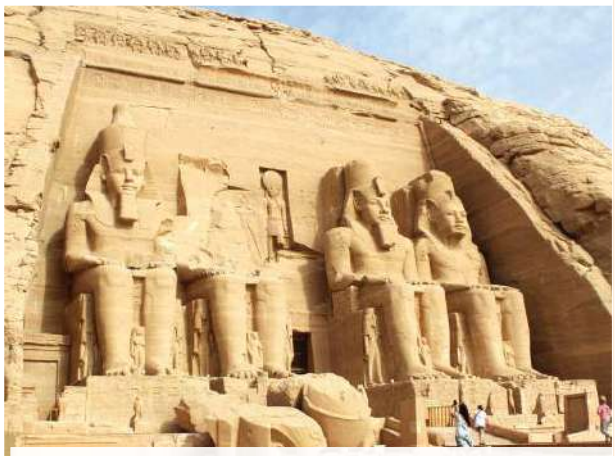
- Term
- Mathematical Expression
- Variable
- Constant
- Equation
- Inequality
- Mathematical Formula
- Coefficient

Note that

A variable is a symbol that represents an unknown quantity,
e.g. x, y, z, ... etc.

Get Ready!

Salwa is planning to spend her vacation in Aswan, exploring its tourist attractions, such as Abu Simbel Temple. To make a decision about the best time to visit, she used a weather app on her smartphone. She found that the temperature in Aswan reaches $95°F$.



Abu Simbel Temple in Aswan

Could you help Salwa convert this Fahrenheit temperature ($°F$) to Celsius ($°C$) using the mathematical formula: $C = (F - 32) \times \frac{5}{9}$

Now, do you think it is the best time for Salwa to visit Aswan?

In this lesson, you'll learn essential concepts in algebra, including mathematical expression, formula, and equation, which will empower you to solve such real-life problems.

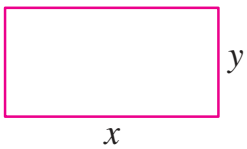
Do you know ?!

In the International System of Units (IS), temperature is measured using three units: Celsius, Fahrenheit, and Kelvin. Each of them can be converted to the others.

Think & Discuss!

Which of the following mathematical expressions represents the perimeter of the opposite rectangle?

$2x + 2y$ | $x + y$ | xy



Learn!

Using symbols to represent unknown numbers or variables

Mathematical Expression:

1. Numerical Mathematical Expression: It consists of one or more numbers with one or more arithmetic operations ($+$, $-$, $\times$ or $\div$).
   For examples: 5 , $4 + 2$, $2 \times (5 - 7)$, or $4 \div 7 \times (3 - 2)$, ... etc.
2. Algebraic Mathematical Expression (Algebraic Expression): It consists of (one or more variables) or (a combination of numbers and variables) with one or more arithmetic operations ($+$, $-$, $\times$ or $\div$).
   For examples: xy , $2 \times n$, $\frac{m}{4}$, or $x^2 + 3x - 1$, ... etc.

■ Note that

When a variable is multiplied by another variable, or a number is multiplied by a variable, the multiplication sign (×) can be omitted:

- $\ell \times w$ can be written as ℓw
- $2 \times n$ can be written as $2n$

Equation:

It consists of two mathematical expressions with an equal sign (=) between them.

For examples: $2x = 5$, $\frac{x}{3} = 5$, $3y - 4 = 2y$, … , etc.

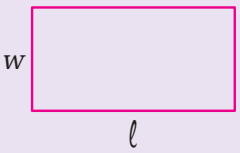
Inequality:

It consists of two mathematical expressions with one of the inequality signs ($<$, $>$, $\leq$, $\geq$) between them.

For examples: $x < -2$, $2x - 1 > 3$, $x + 1 < -2x$, … , etc.

Mathematical Formula:

It represents a fact, rule, or principle using mathematical form, such as equation, inequality, or other mathematical forms.



For example: The area of a rectangle (A) = Length (ℓ) × Width (w)

$$A = \ell \times w$$

■ Self-Evaluation (1)

1. Write the algebraic expression that represents subtracting 5 from 3 times of the number x.
2. Write the mathematical formula expressing the volume (V) of a cube whose edge length is x.

Example 1

Express each of the following in a mathematical form:

1. The amount of money Ahmed paid for 5 cinema tickets, the price of each is x LE, and 3 popcorn cups, the price of each is y LE, after receiving a 20 LE discount on the total amount.
2. The speed at which a car can travel on the (Cairo - Alexandria) desert road must not exceed 120 km/h.
3. Double the number x added to 5 is equal to 1.
4. The area of a square (A) with side length (ℓ).

1. The amount Ahmed paid can be expressed by the algebraic expression: $5x + 3y - 20$
2. The speed at which the car can travel can be expressed by the inequality: $x \leq 120$
3. Double the number x added to 5 is equal to 1 can be expressed by the equation: $2x + 5 = 1$
4. The relation between the area of a square and its side length can be expressed by the mathematical formula: $A = \ell^2$

Algebraic Terms

• Algebraic Term:

An algebraic term is the product (or quotient) of a non-zero number and at least one variable. The number is called the coefficient of the algebraic term.

Examples:

Algebraic Term	Algebraic Term	Algebraic Term
$3\ a$	$-2\ x\ y$	$\frac{1}{4}\ m$
Coefficient Variable	Coefficient Variable Variable	Coefficient Variable

• Like Algebraic Terms:

They are terms with the same variables and raised to the same powers even though their coefficients are different.

- $-3\ x^2$ and $4\ x^2$ are like terms.
- $4y$ and $\frac{1}{2}\ y^2$ are unlike terms because the variable y in both terms is raised to a different power.
- $2\ n^2\ m$ and $2\ n\ m^2$ are unlike terms because the variables m and n in both terms are raised to different powers.

Example 2

Identify the like terms (if exist) in each of the following groups:

1. $2x\ ,\ 5y\ ,\ 3x$
2. $a^2\ ,\ -a\ ,\ 3\ a^3\ ,\ 5$
3. $5y\ ,\ 3xy\ ,\ -2yx\ ,\ 3x$
4. $4x\ ,\ 5y\ ,\ -3x\ ,\ 2y$

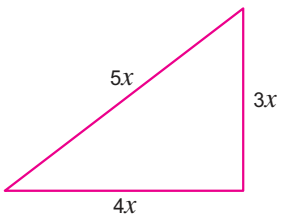
1. $2x$ and $3x$ are like terms.
2. There are no like terms.
3. $3xy$ and $-2yx$ are like terms.
4. $4x$ and $-3x$ are like terms, $5y$ and $2y$ are like terms.

Note that
$xy = yx$

Addition of Like Algebraic Terms

Example 3

Find the mathematical expression representing the perimeter of the opposite triangle, and then determine its numerical value at: $x = 1$



The perimeter of the triangle can be represented by the mathematical expression:
$4x + 3x + 5x = (4 + 3 + 5)x = 12x$
The numerical value of the triangle's perimeter at $x = 1$ is $12 \times 1 = 12$

■ Remember

The perimeter of a triangle is the sum of the lengths of its sides.

■ Note that

- An algebraic term must contain at least one variable.
- An algebraic term is a mathematical algebraic expression that does not involve addition (+) or subtraction (–).

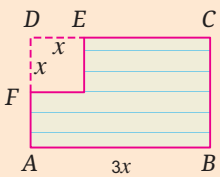
■ Note that

The terms that do not contain any variables, and contain numbers only are called constant terms and they are like terms, such as: $2, -\frac{3}{5}, 1$, ... etc.

■ Self-Evaluation (2)

Group the like terms:
$4x\ ,\ 5y\ ,\ 2xy\ ,\ -3x\ ,\ -5xy\ ,\ 8y\ ,\ 6x^2$

■ Self-Evaluation (3)



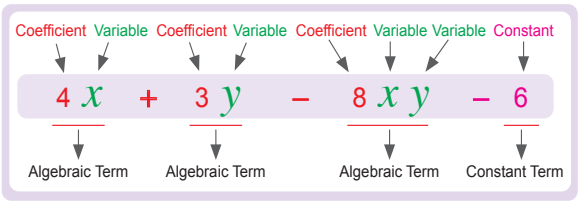
The figure shows a rectangular piece of paper from which a square is cut off. Write the mathematical expression representing the length of $\overline{EC}$.

Example 4

The algebraic expression $4x + 3y - 8xy - 6$ consists of four terms, write:

1. The terms of the expression
2. The constant term
3. The coefficient of xy
4. The variable whose coefficient is 3

1. The terms of the expression are $4x$, $3y$, $-8xy$, and -6
2. The constant term is -6
3. The coefficient of xy is -8
4. The variable whose coefficient is 3 is y



Simplification of Algebraic Expression

- An algebraic expression is in its simplest form when it does not have any like terms.
- To write an algebraic expression in its simplest form, add the like terms together after removing the brackets (if exist) using the distributive property.

Example 5

Simplify each of the following algebraic expressions:

1. $-x + 2y - 8y + 5x + 7$
2. $2(3x - 4) - 3(x - 2)$

1. $-x + 2y - 8y + 5x + 7$

$= -x + 5x + 2y - 8y + 7$ (Commutative property)

$= (-x + 5x) + (2y - 8y) + 7$ (Associative property)

$= (-1 + 5)x + (2 - 8)y + 7$ (Add the coefficients of the like terms)

$= 4x - 6y + 7$ (Write the expression in the simplest form)

2. $2(3x - 4) - 3(x - 2)$

$= 6x - 8 - 3x + 6$ (Use the distributive property in expanding the brackets)

$= 6x - 3x - 8 + 6$ (Commutative property)

$= (6x - 3x) + (-8 + 6)$ (Associative property)

$= 3x - 2$ (Write the expression in the simplest form)

Self-Evaluation (4)

The algebraic expression $5x - 7y + 1$ consists of three terms, write:

1. The terms of the expression
2. The constant term
3. The variable with the smallest coefficient
4. The coefficient of x

Remember: Properties of Operations

1 Commutative property

In addition:
$a + b = b + a$

In multiplication:
$a \times b = b \times a$

2 Associative property

In addition:
$(a + b) + c = a + (b + c)$

In multiplication:
$(a \times b) \times c = a \times (b \times c)$

3 Distributive property

In addition:
$a(b + c) = a(b) + a(c)$

In subtraction:
$a(b - c) = a(b) - a(c)$

Self-Evaluation (5)

Simplify each of the following algebraic expressions:

1. $2x + 4y + x - 7y$
2. $5a + 2a - b + 3b$
3. $13x - 7 + 8x + 19$
4. $3(x - 7) - 5(2x - 5)$

Think with your classmates

What are the steps of simplifying algebraic expression?

Addition of Algebraic Expressions

Example 6

A person has distributed an amount of money among three other persons. The first person has received $(x + 5)$ LE, the second person has received $(2x + 3)$ LE, and the third person has received $(3x - 1)$ LE.
Express in the simplest form the total amount of money has been distributed.
If $x = 20$, what is the value of this amount?

The amount distributed by that person can be expressed as follows:

$$x + 5 + 2x + 3 + 3x - 1 = x + 2x + 3x + 5 + 3 - 1$$
$$= (1 + 2 + 3)x + (5 + 3 - 1)$$
$$= 6x + 7$$

At $x = 20$, the amount of money is:

$6 \times 20 + 7 = 120 + 7 = 127$ LE.

Self-Evaluation (6)

Omar read a book over three days. On the first day, he read x pages. On the second day, he read 31 pages. On the third day, he read $(2x + 17)$ pages. Write, in the simplest form, the algebraic expression representing the number of pages Omar read, and then find the number of pages Omar read at $x = 20$.

Subtraction of Algebraic Expressions

Example 7

The price of a children's car model is $5x$ LE. A shopkeeper purchases it with a discount of 10 LE and assembles it, then he sells it for $(6x + 7)$ LE. Write the algebraic expression for the shopkeeper's profit.
If $x = 40$, what is the profit?

- The purchase price is $5x - 10$
- The sale price is $6x + 7$
- The profit is calculated as the sale price minus the purchase price

$$(6x + 7) - (5x - 10) = 6x + 7 - 5x + 10 = 6x - 5x + 7 + 10$$
$$= (6 - 5)x + 7 + 10 = x + 17$$

Thus, the shopkeeper's profit is $(x + 17)$ LE

At $x = 40$, the profit is $40 + 17 = 57$ LE

Self-Evaluation (7)

Mohammed decided to read a book of $(7x + 31)$ pages. He read $(4x + 17)$ pages over three days.
How many pages are left for Mohammed to read to finish the book?

Note that

$-(x + y) = -x - y$
$-(x - y) = -x + y$
$-(-x + y) = x - y$
$-(-x - y) = x + y$

Collaborative Activity

Collaborate with one of your friends and use the internet to find out the temperature at which water freezes and the temperature at which it boils in Fahrenheit. Then, apply the mathematical formula $C = \frac{5}{9}(F - 32)$ to convert both temperatures to Celsius, where F represents Fahrenheit and C represents Celsius.

Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) What is the mathematical expression for subtracting (-2) from x?

(a) $x-2$ (b) $2-x$

(c) $-2-x$ (d) $x+2$

(2) What is the mathematical formula representing the area (A) of a parallelogram with a base length (ℓ) and its corresponding height (h)?

(a) $A=\frac{1}{2}\ell h$ (b) $A=\ell+h$

(c) $A=\ell h$ (d) $A=\frac{\ell}{h}$

(3) What is the suitable equation to find the side length of an equilateral triangle whose perimeter is 12 cm?

(a) $x+3=12$ (b) $3x=12$

(c) $2x=12$ (d) $x=12$

(4) What is the inequality expressing that the suitable height (n) cm of a person should not be less than180 cm to join one of the sports?

(a) $n<180$ (b) $n>180$

(c) $n\leq 180$ (d) $n\geq 180$

(5) Which of the following pairs of terms are like terms?

(a) $2x$, $-2x^2$ (b) $3a$, $8a$

(c) $7x$, 7 (d) x^2 , y^2

(6) Which of the following is equal to $5a$?

(a) $3+2a$ (b) $2+3a$

(c) $2a+3a$ (d) $5+a$

Second Applying Scientific Concepts

▸ **Express each of the following mathematically:**

(7) Sally's age 5 years ago, if she is now x years old.

(8) The arithmetic mean of the two numbers x and y is not less than 18.

(9) The perimeter (p) of a triangle, if the lengths of its sides are a, b and c.

(10) A rectangle with dimensions x and y, whose area is 36 square centimeters.

(11) Bassem has x one-pound banknotes, y five-pound banknotes, and z ten-pound banknotes.

▸ **Write the following algebraic expressions in their simplest forms :**

(12) $7m-2n-7m+1$ (13) $-2n+3(n-1)$ (14) $3(2x-5)-4(x-6)$

▸ **Find the value of each of the following expressions, given that $a=8$, $b=3$, $d=4$, $f=-1$:**

(15) $\frac{-a}{4}$ (16) b^2-2f (17) $af+3d$

(18) **Write the missing coefficients to make the following equation true:**

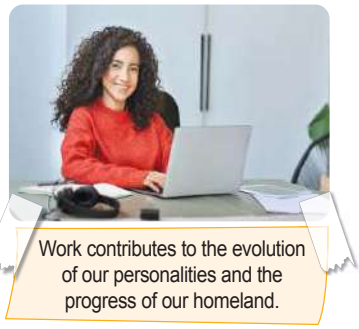
$10x + 6y - \square x + \square y = 3x + 8y$

(19) Temperature is measured in Fahrenheit ($°F$) or Celsius ($°C$).

Apply the mathematical formula $F = 1.8\,C + 32$

to convert 30 $°C$ to Fahrenheit.

Third **Analysis and Subjects Integration**



Work contributes to the evolution of our personalities and the progress of our homeland.

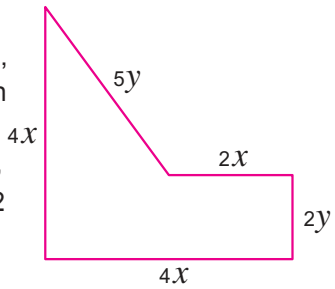
(20) **Work:** Nada is paid x LE per hour in her job.

Write a mathematical expression for each of the following:

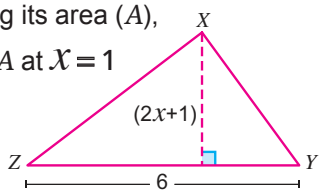
(1) How much does Nada earn if she works for 8 hours?

(2) If Nada got a raise of 3 LE per hour,
what would she earn in 5 hours after the new raise?

(21) **Geometry:**
Write, in the simplest form, a mathematical expression expressing the perimeter of the opposite shape (P), then find P at $x = 3$, $y = 2$

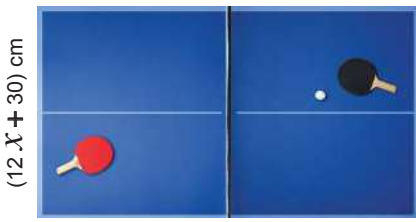


(22) **Geometry:** XYZ is a triangle.
Write the mathematical formula expressing its area (A), then find A at $x = 1$



(23) **Education:** The number of girls in a school is $(8x + 15)$ and the number of boys is $(7x - 10)$
Write the mathematical expression expresses the increase in the number of girls than the number of boys in the school.

(24) **Sports:** The opposite figure shows a table-tennis table.
Write a mathematical expression, in its simplest form, for the table's perimeter, and then find the value of the perimeter (P) at $x = 10$.

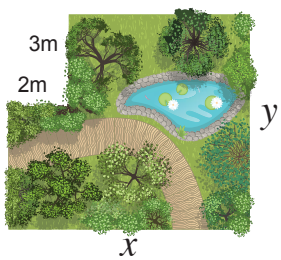


Creative Thinking

(25) The opposite figure represents a garden layout.

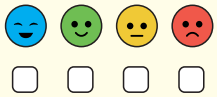
(1) Write a mathematic expression to express the area of the garden.

(2) Write, in the simplest form, an algebraic expression representing the garden's perimeter.



***Evaluate** your understanding!*

How well do you understand mathematical expressions and formulas?
Tick the right box



Lesson

2 - 2

Linear Equations

■ Learning Outcomes

- Form a linear equation in one variable.
- Recognize the relationship between the substitution set and the solution set.
- Solve a linear equation in one variable within the sets of natural numbers, integers, and rational numbers.

■ Vocabulary

- Linear Equation
- Solution
- Substitution Set
- Solution Set

■ Enriched Information

Linear equations are graphically represented in the plane by a straight line. And each of their variables is raised to the power of one.

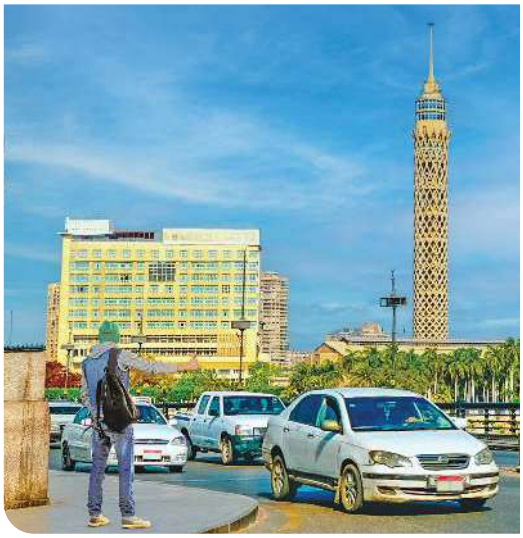
■ Using Technology

You can explore the graphical representation of a linear equation on a plane using the GeoGebra application.

Get Ready!

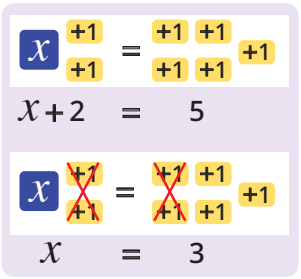
Hani took a taxi from his home to school. The starting fare for the taximeter was 10 LE, and an extra charge of 5.50 LE for each kilometer covered. If Hani paid 87 LE for his journey, how many kilometers did the taxi cover during this ride?

In this lesson, you'll learn how to form and solve a linear equation in one variable, enabling you to solve real-life problems, such as the above problem.



Think & Discuss!

Assume that the card +1 represents positive 1 , then the equation: $x + 2 = 5$ can be modelled as in the opposite figure.



Activity:

Collaborate with your classmates and use the cards to model and solve each of the following equations:

$x + 3 = 4$ | $x - 2 = 3$ | $x - 1 = -3$ | $x + 5 = 2$

Learn!

Equation Concept: An equation is a mathematical statement expressing that two mathematical expressions are equal.

Examples:

Equations in One Variable (Unknown)

- $x + 2 = 5$ (Linear Equation)
- $4x^2 + 1 = 4$ (Quadratic Equation)
- $x^3 + x = 2$ (Cubic Equation)

Linear Equations

- $x + 5 = -2$ (Equation in One Variable)
- $x + 2y = 5$ (Equation in Two Variables)

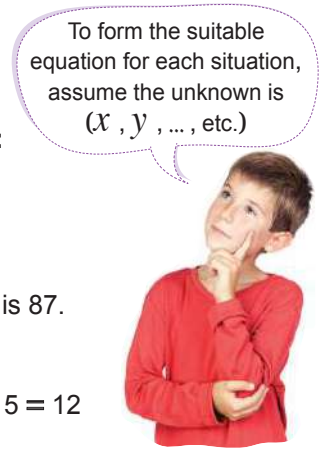
In this term, you will study only the linear equations in one variable.

Forming Equation

Example 1

Represent each of the following situations in the form of a suitable equation:

1. When 5 is added to a number, the result is (-3).
2. When 15 is subtracted from twice a number, the result is 12.
3. The sum of three consecutive odd numbers is 87.



① $x + 5 = -3$ ② $2x - 15 = 12$

③ $y + (y + 2) + (y + 4) = 87$

Solving Equation

- **Substitution Set**
  The set of potential values for the unknown in the equation.
- **Solution Set**
  The set of values that belong to the substitution set and satisfy the equality of the two sides of the equation.

Solving Equations Using the Substitution Method

Example 2

Find the solution set of the equation:

$2x + 1 = 7$, if the substitution set is $\{2, 3, 4\}$

The values of x in the substitution set	2	3	4
Substitute the values of x into the equation $2x + 1 = 7$	$2(2) + 1 \stackrel{?}{=} 7$ $5 \neq 7$	$2(3) + 1 \stackrel{?}{=} 7$ $7 = 7$	$2(4) + 1 \stackrel{?}{=} 7$ $9 \neq 7$
Are both sides of the equation equal?	✗	✓	✗

Therefore, the equation has a solution at $x = 3$,

i.e., the solution set of the equation is $\{3\}$

Note that The solution set of an equation is a subset of the substitution set.

Properties of Equality

The substitution method above becomes almost impossible when the substitution sets are infinite as N, Z, Q. So, we need simpler techniques to find solutions. That's why studying the properties of equality is essential: it allows us to find **an equivalent equation** of the original equation where the variable is isolated on one side of the equation.

■ Self-Evaluation (1)

Represent each of the following situations in the form of an equation:

1. When 3 is added to a number, the result is 0
2. When (-8) is subtracted from three times a number, the result is (-5)
3. The sum of two consecutive even numbers is 54

■ Self-Evaluation (2)

Find the solution set of the equation $3x + 2 = 17$, if the substitution set is $\{3, 4, 5, 6\}$

■ Note that

Linear equations in one variable that have the same solution are called equivalent equations.
For example:

$2x + 7 = 19$

$2x = 12$

$x = 6$

are equivalent equations.

If A, B and C are three numbers, then these numbers have the following properties:

■ Property 1

The same number can be added to both sides of an equation, and both sides remain equal.

1 Addition Property:

If $A = B$ then $A + C = B + C$

For example: If $X - 1 = 3$ then $X - 1 + 1 = 3 + 1$ thus $X = 4$

■ Property 2

The same number can be subtracted from both sides of an equation, and both sides remain equal.

2 Subtraction Property:

If $A = B$ then $A - C = B - C$

For example: If $X + 2 = 3$ then $X + 2 - 2 = 3 - 2$ thus $X = 1$

■ Property 3

Both sides of an equation can be multiplied by the same number, and both sides remain equal.

3 Multiplication Property:

If $A = B$ then $A \times C = B \times C$

For example: If $\frac{1}{3}X = 3$ then $3 \times \frac{1}{3}X = 3 \times 3$ thus $X = 9$

■ Property 4

Both sides of an equation can be divided by the same number (other than 0), and both sides remain equal.

4 Division Property:

If $A = B$ then $\frac{A}{C} = \frac{B}{C}$ (where $C \neq 0$)

For example: If $4X = 20$ then $\frac{4X}{4} = \frac{20}{4}$ thus $X = 5$

Solving Equations Using the Properties of Equality

Example 3

Find the solution set of the equation $2X + 7 = 3$

If the substitution set is: 1 Z 2 N

$2X + 7 = 3$

$2X + 7 - 7 = 3 - 7$ (subtracting 7 from both sides)

$2X = -4$

$\frac{2X}{2} = \frac{-4}{2}$ (dividing both sides by 2)

$X = -2$

1 If the substitution set is Z, $-2 \in Z$, then the solution set of the equation in Z is $\{-2\}$

2 If the substitution set is N, $-2 \notin N$, then the solution set of the equation in N is ϕ

■ Diversified Strategies

You can solve the equation by (Reversing the operation)

$2X + 7 = 3$

$2X = 3 - 7$

$2X = -4$

$X = \frac{-4}{2} = -2$

■ Self-Evaluation (3)

Find the solution set of each of the following equations:

$X + 12 = 7$

$3X + 11 = 9$

$2y - 5 = -2$

$9 - 2b = 7$

If the substitution set is:

1 Z 2 Q

Self-Evaluation (4)

Find in Q the solution set of each of the following equations:

1. $3(x-5) = -18$
2. $3(x-1) + 4 = 3$
3. $4(x-3) = 2(x+4)$
4. $5(x-3) = 2(x-1)$

Verifying the Solution

You can verify the solution of equation (1) as follows:
Write the original equation
$2(x-3) = 8$
Substitute x with 7
$2(7-3) \stackrel{?}{=} 8$
$2 \times 4 \stackrel{?}{=} 8$ simplify
$8 = 8$ ✓ the statement is true.
i.e., the solution is 7

▸ Can you verify the solution of equation (2) ?

Self-Evaluation (5)

Hazem has a wooden strip of length 180 centimeters. He intends to create a rectangular picture frame where one of the dimensions exceeds the other by 15 centimeters.

Write the equation expressing this situation and solve it to determine the frame's dimensions.

Example 4

Find in Q the solution set of each of the following equations:

(1) $2(x-3) = 8$ (2) $2(x+3) = 3(1-x)$

(1) $2(x-3) = 8$

$2x - 6 = 8$ (Distributive property)

$2x - 6 + 6 = 8 + 6$ (Add 6 to both sides)

$2x = 14$ (Simplify the equation)

$\frac{2x}{2} = \frac{14}{2}$ (Divid both sides by 2)

$x = 7 \quad , \quad 7 \in Q$

therefore, the solution set $= \{7\}$

(2) $2(x+3) = 3(1-x)$

$2x + 6 = 3 - 3x$ (Distributive property)

$2x + 3x = 3 - 6$ (Associating the like terms)

$5x = -3$ (Simplify each side)

$x = -\frac{3}{5} \quad , \quad -\frac{3}{5} \in Q$ (Find the value of x)

therefore, the solution set $= \{-\frac{3}{5}\}$

Diversified Strategies

You can solve equation (1) by dividing both sides by 2 without using the distributive property

$2(x-3) = 8$

$\frac{2(x-3)}{2} = \frac{8}{2}$

$x - 3 = 4$

$x - 3 + 3 = 4 + 3$

$x = 7$

Example 5

Culture: Four persons purchased tickets to the Egyptian Museum in Cairo. They also purchased souvenirs for 500 LE, if the total cost is 620 LE, write the equation expressing this situation.
What is the price of one ticket?

Assume the price of one ticket is x LE,
therefore, the price of four tickets is $4x$ LE

$4x + 500 = 620$

$4x = 620 - 500$

$4x = 120$

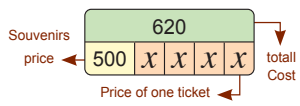
$x = \frac{120}{4}$

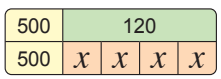
$x = 30$

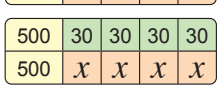
Thus, the price of one ticket is 30 LE

Diversified Strategies

The equation can be solved using bar model, where x is the price of one ticket.







The price of one ticket x is 30 LE

Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) Two consecutive numbers whose sum is 29. Which of the following equations expresses that?

(a) $x + x + 2 = 29$ (b) $x + x + 1 = 29$

(c) $x + x - 1 = 28$ (d) $x + x + 1 = 30$

(2) Ziad is x years old now. Seven years ago, he was 18 years old. Which of the following equations represents this situation?

(a) $x + 7 = 25$ (b) $x - 7 = 11$

(c) $x + 7 = 18$ (d) $x - 7 = 18$

(3) Which of the following equations does not have a solution in Z?

(a) $6x = 12$ (b) $6x = 15$

(c) $6x = 18$ (d) $6x = 24$

(4) Which of the following represents the solution of the equation $2(x - 5) = 0$ in Q?

(a) 0 (b) 5

(c) -5 (d) 10

▸ **Spot the mistake:**

(5) Ahmed and Noha solved the equation: $2x + 7 = 16$

Which one of them followed the correct solution method? Discuss.

Ahmed's Solution

$2x + 7 = 16$

$2x + 7 - 7 = 16 - 7$

$2x = 9$

$\frac{2x}{2} = \frac{9}{2}$

$x = 4.5$

Noha's Solution

$2x + 7 = 16$

$\frac{2x}{2} + 7 = \frac{16}{2}$

$x + 7 = 8$

$x + 7 - 7 = 8 - 7$

$x = 1$

Second Applying Scientific Concepts

▸ **Find the solution set of each of the following equations in N:**

(6) $-\frac{2}{5} + a = \frac{3}{5}$ (7) $4 + 5x = 9$ (8) $2(x + 7) = 10$

▸ **Find the solution set of each of the following equations in Z:**

(9) $2x - 5 = -17$ (10) $4 - 3x = 19$ (11) $4(x - 1) = 3(x + 1)$

▸ **Find the solution set of each of the following equations in Q:**

(12) $\frac{1}{3}x + 3 = 12$ (13) $2x + 5 = 12 + 3x$ (14) $7 = 2(x + 3)$

▸ **Answer the following questions :**

(15) If $\frac{m}{3} = 7$, then what is the value of $m - 19$?

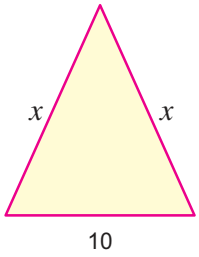
(16) If $-7k = 28$, then what is the value of $3k + 6$?

(17) A teacher of mathematics wrote an integer on the board, and then he wrote another number, which was 17 less than twice the first number. If the sum of both numbers is 112, what number did the teacher initially write?

(18) If my mother's age now is three times my age, and she is also 24 years older than me, how old is each of us now?

Third Analysis and Subjects Integration

19 **Geometry:** If the perimeter of the opposite triangle equals 34, then what is the value of x ?



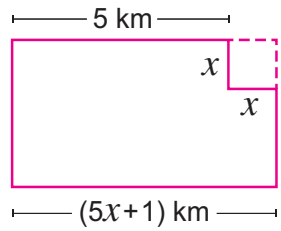
Saving enables people to deal with unexpected emergencies in the future.

20 **Saving:** In a money jar, there are 42 twenty-pound banknotes and fifty-pound banknotes. If the total amount of money in the jar is 1,800 LE, then how many twenty-pound banknotes in the jar?

21 **Sports:** A rectangular-shaped football pitch has a length less than twice its width by 15 meters. If the pitch's perimeter is 330 meters, calculate its dimensions.

Creative Thinking

22 In light of the State's focus on expanding agricultural land, a rectangular desert plot has been reclaimed. From this plot, a square area with side length x km has been devoted to create a farm for development of animal wealth. What is the value of x in kilometres?



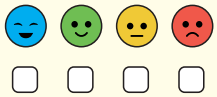
Expanding the agricultural land helps solve problems caused by climate change.

Discipline and punctuality demonstrate respect for yourself and others.

23 **Employment:** A company announced job vacancies, allocating two minutes for each person to apply. While Salma, who came to apply for one of the jobs, waited in the line, there were $(x + 2)$ people ahead of her and $(3x - 4)$ people behind her. If the total number of people in the line was 35, how long did Salma have to wait to reach the front of the line?

***Evaluate** your understanding!*

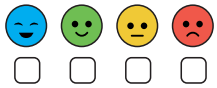
How well do you understand linear equations?
Tick the right box



Unit Two Assessment

Evaluate your understanding!

How well do you understand Unit Two? Tick the right box



▸ Choose the correct answer from the given ones:

1. What is the suitable equation to determine the price of one shirt if you purchased three shirts of the same type from an online store, and the total cost was 490 LE after adding 40 LE for shipping?
   (a) $x + 40 = 490$ (b) $3(x + 40) = 490$
   (c) $x + 120 = 490$ (d) $3x + 40 = 490$

2. What is the algebraic expression equivalent to the following expression: $2x - 3 - 4x + 1$?
   (a) $2x - 2$ (b) $-2x + 2$
   (c) $-6x - 4$ (d) $-2 - 2x$

3. Which of the following equations is equivalent to the equation: $2n + 1 = 3$?
   (a) $n + 2 = 6$ (b) $2n = 4$
   (c) $2n = 2$ (d) $n + 1 = \frac{3}{2}$

4. Which inequality expresses that Bassem requires a minimum of 10 gigabytes per month to accomplish his work online?
   (a) $x < 10$ (b) $x > 10$
   (c) $x \leq 10$ (d) $x \geq 10$

▸ Complete each of the following with the correct answer:

5. The value of the expression $(4x - 10)$ at $x = -1$ equals .....
6. The solution set in N of the equation $2(3x - 1) = 10$ is .....
7. If Marawan has x ten-pound banknotes in addition to 1 one-hundred-pound banknote, and the total amount of Marawan's money is 330 pounds , then $x =$ .....
8. A parallelogram with a base length of 12 cm , its corresponding height is $(3x - 1)$ cm and its area is 60 squared centimeters, then $x =$ .....

▸ Answer the following questions:

9. Write in the simplest form, the expression: $2(n - 3m) - 3(2n - 1)$, and then find its value at: $m = -2$, $n = 2$
10. Find in Q the solution set of the equation: $2(\frac{1}{2}x - 3) = 4x$
11. Write an algebraic term like the algebraic term (x^2), where its coefficient is -3. Then, find the sum of both terms.
12. Write the mathematical formula that expresses the total area (A) of a cube, where one of its faces has an area equal to x^2

Unit Two Activity

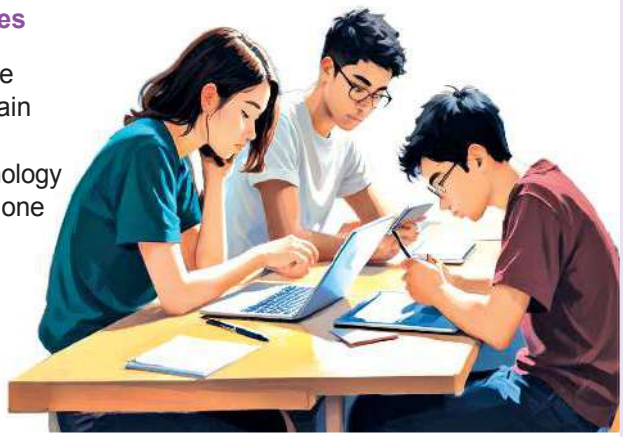
Linear Equations in our Lives

Gather data from everyday real-life scenarios, which can be expressed using linear equations in one variable, and explain how to graph them using GeoGebra app.

▸ **Activity Aim:** To foster teamwork skills through digital technology while enhancing your understanding of linear equations in one variable and graphing them.

▸ **Implementation Steps:**

1. Form a team of three members to carry out the activity.
2. Have an agrement on everyday scenarios that can be expressed using linear equations in one variable.
3. Write these scenarios into linear equations in one variable.
4. Solve the resulting linear equations in one variable and verify your solution.
5. Use the GeoGebra app to graph these equations.



UNIT 3

Statistics

Unit Lessons

Machine Learning is used in analyzing data to extract the information it contains, which helps in making decisions and improving performance in various areas.

▶ Can Machine Learning algorithms be developed to solve statistical problems in our daily lives?

Issues and Life Skills

- Information Technology
- Creative Thinking
- Predicting
- Decision Making

Values

- Responsibility
- Work
- Justice
- Persistence

Lesson

3 - 1

Organizing Data

Learning Outcomes

- Recall the different methods of organizing data.
- Learn how to create a frequency table with intervals.
- Learn how to represent data using a stem-and-leaf plot.
- Distinguish between misleading and Non-misleading graphs.

Vocabulary

- Collecting Data
- Organizing Data
- Frequency Table with Intervals
- Dot Plot
- Bar Graph
- Histogram
- Stem-and-Leaf Plot
- Box Plot

Activity

Give examples of statistical and non-statistical questions.

Get Ready!

The following numbers are the numbers of goals scored by a football team in 34 matches of the Egyptian League in one season.

1	1	4	2	2	3	1	2	0
1	3	2	0	3	1	3	1	2
3	3	2	2	2	1	3	3	0
1	2	4	4	1	0	0		

How can this data be organized to make it easy to interpret and analyze? What is the most effective way to represent this data graphically?

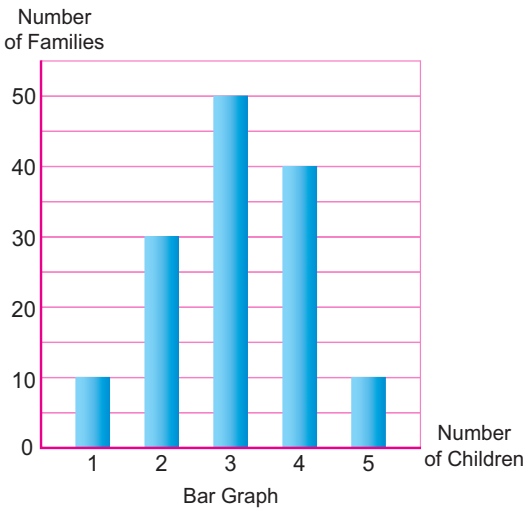
In this lesson, you will learn how to organize data into frequency tables and display it using various types of graphical charts.

Think & Discuss!

▸ **How many children are there in your family?**

You can simply answer this question with 1, 2, or whatever the number of children in your family is. Therefore, this question is a non-statistical question.

▸ **How many children are there in the families of the first preparatory students in your school?**



Bar Graph

To answer this question, you need to collect and organize data. Hence, this question is a statistical question.

▸ The bar graph shown above demonstrates the number of children in the families of the first preparatory students in one of the schools.

Can you determine the number of families with fewer than 5 children?

■ Collecting Data

You can collect data using several methods such as observation, personal interviews, and surveys

Learn!

Statistics

Statistics is a branch of mathematics that involves collecting, organizing, presenting, analyzing, and interpreting data to make a decision.
It has applications in various fields such as medicine, economics, social sciences, etc.

How do you handle data?

1. Identify the question or problem.
2. Collect data.
3. Organize and present data.
4. Analyze and interpret data.
5. Make prediction and decisions.

Some types of graphical charts:

1. Dot plot.
2. Bar graph.
3. Histogram.
4. Stem-and-leaf plot.
5. Box plot.

You studied most of these charts in your primary stage.

■ Self-Evaluation (1)

The following data represents laptop sales in a computer shop in six months.

July	8
August	6
September	7
October	8
November	8
December	7

Represent this data in a suitable graphical chart.

Example 1

A teacher recorded the marks of 25 students in a quiz as follows:

6, 5, 6, 3, 4, 6, 7, 5, 6, 3, 8, 6, 6, 7, 7, 6, 7, 8, 4, 6, 8, 7, 5, 6, 5

1. Represent this data using a dot plot and a bar graph.
2. How many students got at least 7 marks?
3. What is the mark that most of the students got?

To answer such questions, it is important to organize the data first.
One method of data organizing is to arrange it in a list in ascending order as follows :

3, 3, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8

This data can be organized into a frequency table as follows :

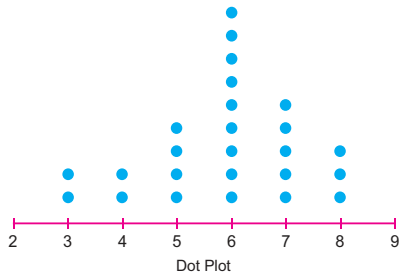
Mark	3	4	5	6	7	8
Frequency	2	2	4	9	5	3

■ **Critical Thinking**

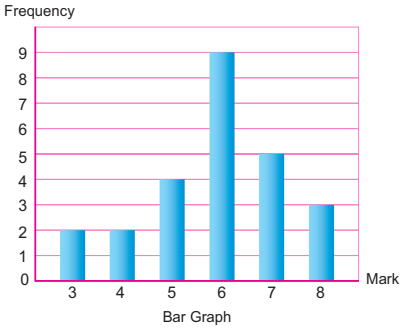
Deduce the full mark of the teacher's quiz in Example 1. Based on this mark, conclude the decision that the teacher would make regarding her students.

Using the previous table, create the dot plot and bar graph to represent this data as follows:

1



Dot Plot



Bar Graph

2 The number of students who got at least 7 marks is $5 + 3 = 8$

3 The mark that most of the students got (mode) is 6

■ **Note that**

Both the dot plot and bar graph display the actual values of data.

Frequency Table with Intervals and its Representation

If the amount of data is large, the data is organized into equal intervals or groups, from which the frequency table with intervals is formed.

■ **Self-Evaluation** 2

The following data represents the masses in kilograms of a group of 30 students:
70 , 43 , 48 , 72 , 53 , 81 , 76 , 54 , 58 , 64 , 51 , 53 , 75 , 62 , 84 , 67 , 72 , 80 , 88 , 65 , 60 , 43 , 53 , 42 , 57 , 61 , 55 , 75 , 82 , 71

1 Organize this data in a table using equal intervals 40 –, 50 –, ...

2 Which interval has the greatest number of students?

3 How many students have masses less than 70 kg?

4 Represent the data in a histogram.

Example 2

A judo trainer at a club weighed all the judo players before they joined one of the judo championships. The masses of a group of 40 players in kilograms were as follows:

41	85	74	86	65	62	100	95	77	82
50	83	77	93	73	72	69	73	87	50
98	66	47	100	50	89	78	70	75	95
80	78	83	81	72	75	48	63	58	56

1 Organize this data and represent it using a histogram.

2 How many judo players have masses of 81 kg or more?

1 To form a frequency table with intervals, do the following:

- Find the range:
  the greatest value minus the smallest value, $100 - 41 = 59$

i.e. the range is approximately 60.

- Divide 60 into a suitable number of intervals, let's say 6 intervals. Thus, the length of each interval = 10.

For example, the first interval represents the masses from 41 kg till less than 51 kg, and it is written as (41 –)

Therefore, the intervals are: 41 – , 51 – , 61 – , 71 – , 81 – , 91 –

- Write down the intervals and the frequency in the above table, then remove the tally column to obtain the following table:

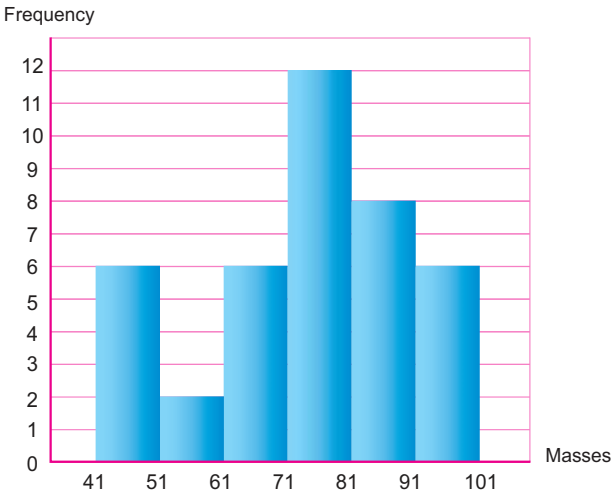
Masses	Tally	Frequency
41 –	𝍸 I	6
51 –	II	2
61 –	𝍸 I	6
71 –	𝍸 𝍸 II	12
81 –	𝍸 III	8
91 –	𝍸 I	6

■ Previous Knowledge

In a frequency table with intervals, because the intervals have the same length. Therefore, they are when represented using a histogram, the columns should have the same width with no gaps between them.

Masses	41 –	51 –	61 –	71 –	81 –	91 –
Frequency	6	2	6	12	8	6

▸ Represent this data using a histogram.



■ Note that

The histogram does not display the actual values of data.

② The number of players who have masses of 81 kg or more
= 8 + 6 = 14 players.

Stem-and-Leaf Plot

A stem-and-leaf plot is a chart that presents numerical data arranged in order. Each number is split into two parts (Stem - Leaf).

For example: For a number that consists of two digits (41), the tens are represented by the stem (4), and the ones are represented by the leaf (1). This can be demonstrated by the following example:

■ Self-Evaluation ③

The following data represents the temperatures of a city over three weeks:

21	41	42	26	25
25	43	24	25	19
18	41	17	40	38
33	32	29	33	28
34				

Draw a stem-and-leaf plot, then deduce the median and the mode.

Example 3

If the number of weekly hours that 27 students spend using the Internet are as follows:

14	35	27	21	20	46	20
40	31	18	28	11	44	33
18	32	21	19	43	15	34
41	21	16	26	32	30	

1. Organize and represent this data using a stem-and-leaf plot.
2. From the stem-and-leaf plot, find the mode, the median, the first quartile, and the third quartile.
3. Represent the data using a box plot.

■ Note that

If the data contains decimal numbers, such as: 12.7, 9.4, 10.5, … The leaves are the decimal parts, like 7, 4, 5, and the stems are the whole number parts, like 12, 9, 10. The key is written as follows:
12|7 represents 12.7

1 Representing the data using a stem-and-leaf plot:

- The smallest number is 11, and the greatest number is 46.
- Draw a vertical line.
- On the left, write "Stems" and list the following numbers below it: 1, 2, 3, 4.
- On the right, write "Leaves" represented by the units, arranged in ascending order in each row.
- Write the stem-and-leaf plot key.

Stems	Leaves
1	1 4 5 6 8 8 9
2	0 0 1 1 1 6 7 8
3	0 1 2 2 3 4 5
4	0 1 3 4 6

Key	2|6 means 26 hours

■ Note that

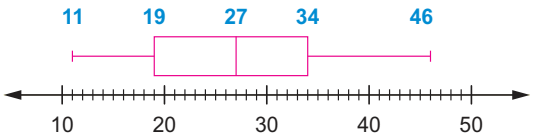
The stem-and-leaf plot displays the actual values of data arranged in order.

2 The mode is the most repeated number in a row.
For example, 1 is repeated 3 times in the second row; thus, the mode = 21.
Since the values are arranged in ascending order, the median = 27, the first quartile = 19, and the third quartile = 34.

■ Previous Knowledge

In a box plot, data is distributed along a number line by determining the smallest value, the greatest value, the median, the first and third quartiles.

3 Representing the data using a box plot.



■ Note that

The box plot does not display the actual values, but it makes it easier to find the median and the first and third quartiles.

Misleading Graphs

■ Beware of Misleading Graphs

Carefully read graphs and understand what they represent to avoid being misled by their depiction.

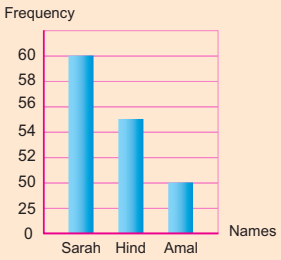
Graphs can be an effective tool for illustrating data; however, sometimes they can be misleading if not used properly or if they do not display data correctly, which can adversely impact deductions.

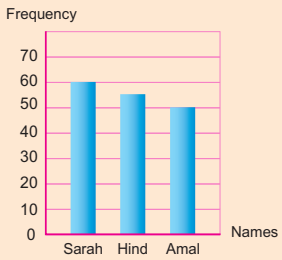
Here are some common ways in which graphs can be misleading:

1. If the vertical axis does not start with zero.
2. If an unequal scale is used on the vertical axis.

Self-Evaluation (4)

Both of the following graphs represent the masses of Sarah, Hind, and Amal in kilograms. Which graph could be misleading, and why?





Example 4

Both of the following graphs represent the number of post offices in the governorates of Fayoum, Beni Suef, Luxor, and Kafr El Sheikh for the year 2021/2022. Which graph could be misleading, and why?

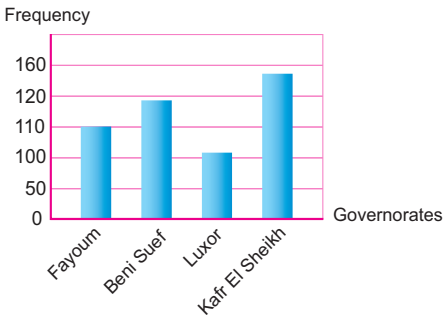


Fig. (1)

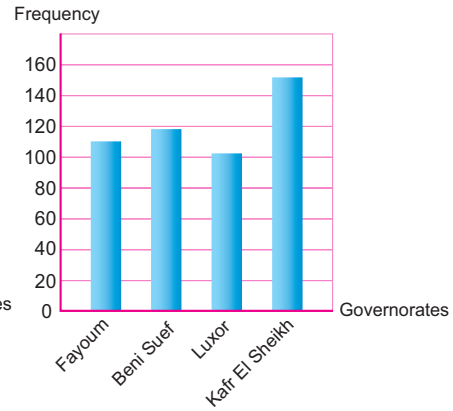


Fig. (2)

The graph in Figure (1) is clearly misleading because the vertical axis has unequal scale, which makes it appears as the number of post offices in Beni Suef is almost double the number of post offices in Luxor.

Collaborative Activity

Collaborate with your classmates to answer the following question:

What means of transportation is used to go to school?

Organize the data you collect in a table, and present it using a suitable graphical chart. Analyze the data to reach some conclusions.

Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

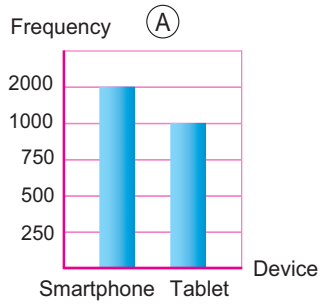
Stems	Leaves
0	9
1	0 2 2 2 3 4 5 6 6
2	0 1 1 5 7 8 9
3	1 2 3

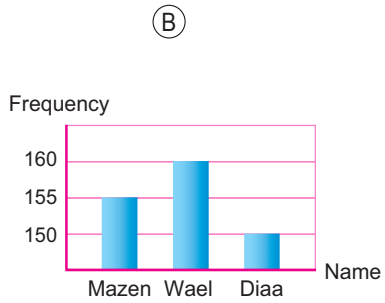
Key | 3|1 means 31 hours

1. From the opposite stem-and-leaf plot, what is the median?
   (a) 16 (b) 17
   (c) 18 (d) 20

2. Which of the following charts does not display the actual data?
   (a) Dot plot (b) Histogram
   (c) Stem-and-leaf plot (d) Bar graph

3. One of the following graphs represents the heights of three friends, and the other represents the number of tablets and smartphones owned by the employees of a company.
   Which of them is considered misleading?





(a) Only A is misleading (b) Only B is misleading
(c) Both are misleading (d) Both are not misleading

Second Applying Scientific Concepts

4. The following data represents the number of toys sold by a store over 30 days:

13	32	12	33	27	37	44	8	26	32
36	41	45	9	38	16	46	48	29	15
13	32	33	14	18	28	34	25	7	18

Represent this data using a stem-and-leaf plot, then find the median and the mode.

5 Draw a histogram to represent the following table:

Intervals	1 –	11 –	21 –	31 –	41 –
Frequency	8	12	36	24	20

6 The following data represents the heights of 32 students in centimeters:

134	152	140	134	130	142	131	144
144	132	147	143	135	135	145	137
148	151	133	142	136	138	132	146
140	139	141	148	130	144	149	139

1 Form a frequency table with intervals (130 – , 135 – , ...), then represent it using a histogram.

2 How many students have heights less than 145 cm?

Third Analysis and Subjects Integration

7 **Business Administration:** The following table demonstrates the sales of a computer shop last year:

Product	Frequency
PC	60
Laptop	100
Tablet	80
Spare parts	240

Represent this data using a suitable graphical chart.

8 **Saving:** The following table illustrates the savings of the first preparatory students at one school:

Intervals	Frequency
1 –	6
101 –	10
201 –	18
301 –	12
401 –	8

Represent these savings using a histogram.

Creative Thinking

9 **Industry:** The opposite stem-and-leaf plot displays the average battery lives of 25 smartphones.

Stems	Leaves
0	8 9
1	0 1 1 2 2 2 3 4 5 6 7 8 9
2	0 2 5 6 7 8 9 9
3	1 2
Key	3|2 means 32 hours

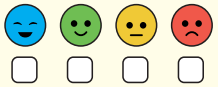
1 Draw the box plot of this data.

2 How many smartphones have an average battery life more than 17 hours?

3 Calculate the percentage of smartphones whose average battery life is less than 12 hours.

***Evaluate** your understanding!*

How well do you understand organizing data?
Tick the right box



Lesson

3 - 2

Arithmetic Mean

Learning Outcomes

- Learn the measures of central tendency.
- Learn how to calculate the arithmetic mean of a set of values.
- Learn how to calculate the arithmetic mean of a frequency table.
- Understand what is meant by misleading statistics.

Vocabulary

- Arithmetic Mean
- Median
- Mode

Helpful Hint

The arithmetic mean (mean) sometimes called the average.

Previous Knowledge

Measures of central tendency are the values that describe the central value around which a set of data points tends to cluster. The arithmetic mean is the most commonly used measure of central tendency.

Get Ready!

The following frequency table shows the internet consumption of one family in GB over a month. What is the average daily internet consumption of this family?

Internet consumption in GB	12	13	5	4	20
No. of days	5	7	4	12	2

In this lesson, you will learn how to calculate the arithmetic mean of a frequency distribution, which will enable you to solve such real-life problems.

Think & Discuss!

If you have four numbers a, b, c, and d, the arithmetic mean of the first two numbers is 21, and the arithmetic mean of the other two numbers is 30, what is the arithmetic mean of the four numbers?

Learn!

You have previously learnt the measures of central tendency of a set of values, which are :

1 Arithmetic Mean (Average)

The arithmetic mean $= \frac{\text{Sum of values}}{\text{Their number}}$

For example:
The arithmetic mean of the values 7, 4, 11, 8, 5 equals

$$\frac{7+4+11+8+5}{5} = 7$$

2 Median

The median is the value in the middle of the values after arranging.

For example:
- The median of the values 2, 2, 3, 4, 6, 9, 9 is 4
- The median of the values 2, 3, 4, 6, 9, 17 equals

$$\frac{4+6}{2} = 5$$

3 Mode

The mode is the most common or frequent value in a set.

For example:
- The mode of the values: 8, 7, 3, 7, 6, 0, 9, 7 is 7
- but, for the values: 7, 8, 3, 7, 6, 0, 5, 8 both 7 and 8 are modes (bimodal)
- Meanwhile, the values: 7, 8, 3, 5, 0, 2 do not have a mode.

Example 1

Sarah took five tests, each with full marks of 100. If her marks in three tests are 94, 88, and 81, what is the least mark she could get in one of the other two tests to achieve an average of 83 marks in the five tests?

The sum of Sarah's marks in the three tests = 94 + 88 + 81 = 263 marks.
To get an average of 83 marks in the five tests, the sum of the marks in the five tests should be = 83 × 5 = 415 marks

Thus, the sum of the marks that Sarah needs to get in the other two tests is the difference between 415 and 263

$$415 - 263 = 152$$

Since the full mark that Sarah can get in one of the two tests is 100, the least marks she could possibly get is 52.

The Arithmetic Mean of a Frequency Distribution

You can calculate the arithmetic mean of a frequency distribution using the following mathematical formula:

$$\text{Arithmetic mean } (\bar{x}) = \frac{\Sigma (f.x)}{\Sigma f}$$

Where: Σf is the sum of the frequencies, and $\Sigma (f.x)$ is the sum of the products of f and x.

Example 2

The opposite frequency table shows the daily pocket money of a student over one week. Find the average daily pocket money of this student.

Pocket money in LE (x)	No. of days (f)
16	2
20	3
25	1
30	1

x	f	$f.x$
16	2	32
20	3	60
25	1	25
30	1	30
Sum	7	147

The average daily pocket money

$$= \frac{\Sigma (f.x)}{\Sigma f} = \frac{147}{7} = 21 \text{ LE.}$$

Your pocket money is helping you how to be responsible and how to make your own choices.

Self-Evaluation (1)

55, 75, 55, 42, and 47 represent the numbers of children who took the infantile paralysis vaccine over five days.

If the number of children who took the vaccine on the sixth day is 95, which of the following statements is true?

(a) The median decreases
(b) The mode increases
(c) The average decreases
(d) The average increases

Hint:
Use the number sense to exclude some choices, or find the measures before and after adding 95, then compare the results.

Self-Evaluation (2)

If the masses of the school's Karate team players in kilograms are given in the following table:

Mass (Kg)	Frequency
71	1
72	2
73	4
74	3
75	5
76	3
77	2

calculate the arithmetic mean of the masses of these players.

Example 3

The following table shows the number of working hours for a group of workers:

No. of hours (x)	5	6	7	8	9	10
No. of workers (f)	12	n	30	39	16	9

Find the number of workers who work 6 hours such that the arithmetic mean of the number of working hours is 7.5

Nations are built through hard work.

x	f	$f.x$
5	12	60
6	n	6 n
7	30	210
8	39	312
9	16	144
10	9	90
Sum	$n + 106$	$6n + 816$

$$\bar{x} = \frac{\Sigma (f.x)}{\Sigma f}$$

$$\frac{15}{2} = \frac{6n + 816}{n + 106}$$

$$15\ (n + 106) = 2\ (6n + 816)$$

$$15n + 1590 = 12n + 1632$$

$$15n - 12n = 1632 - 1590$$

$$3n = 42$$

$$n = \frac{42}{3} = 14$$

Self-Evaluation (3)

The following table shows the number of children in each family for a group of families.

No. of children	No. of families
0	20
1	40
2	60
3	80
4	m

Find the number of families that have 4 children, such that the arithmetic mean of the number of children is 3.

Misleading Statistical Data

Using the arithmetic mean regardless of the overall distribution of the data can be misleading. If there are some extreme values (very large value or very small value), this can significantly affect the arithmetic mean. Therefore, in such cases, it is better to use the median or the mode.

Beware of Misleading Data

You can avoid misleading statistics by learning, training, verifying sources, and comparing them, so that you become a more intelligent consumer and critic of the data you encounter in your daily life.

Example 4

A restaurant displayed the menu of the lunch meals with prices in LE. The restaurant announced that the average meal price is 132 LE. Explain why the average meal price is misleading.

Menu

Item	Price
Kofta ($\frac{1}{2}$ kg)	180
$\frac{1}{2}$ Chicken	150
Shawarma	160
Fish ($\frac{1}{2}$ kg)	150
Water Bottle	20

The arithmetic mean of the meal prices is

$$\frac{180 + 150 + 160 + 150 + 20}{5} = 132 \text{ LE.}$$

This means that the restaurant owner used the arithmetic mean to measure the average meal price. This measure (arithmetic mean) is misleading because there is an extreme value (20), which is not a price of any meal. However, the restaurant exploited this value to announce that the average meal price is 132 LE, which is not true. So, it is important to find another more accurate measure in the presence of extreme values, which is the median. The median in this case, is 150 LE, and it represents the meal prices more accurately.

Self-Evaluation (4)

A student got the following marks in five tests:
40, 50, 100, 45, 40.
If the full marks for each test is 100, find the arithmetic mean and the median of the student's marks. Which of these measures is more accurate?

■ Technological Evaluation

Write down the ages of ten of your classmates in years , and then calculate their arithmetic mean, median , and mode using Excel.

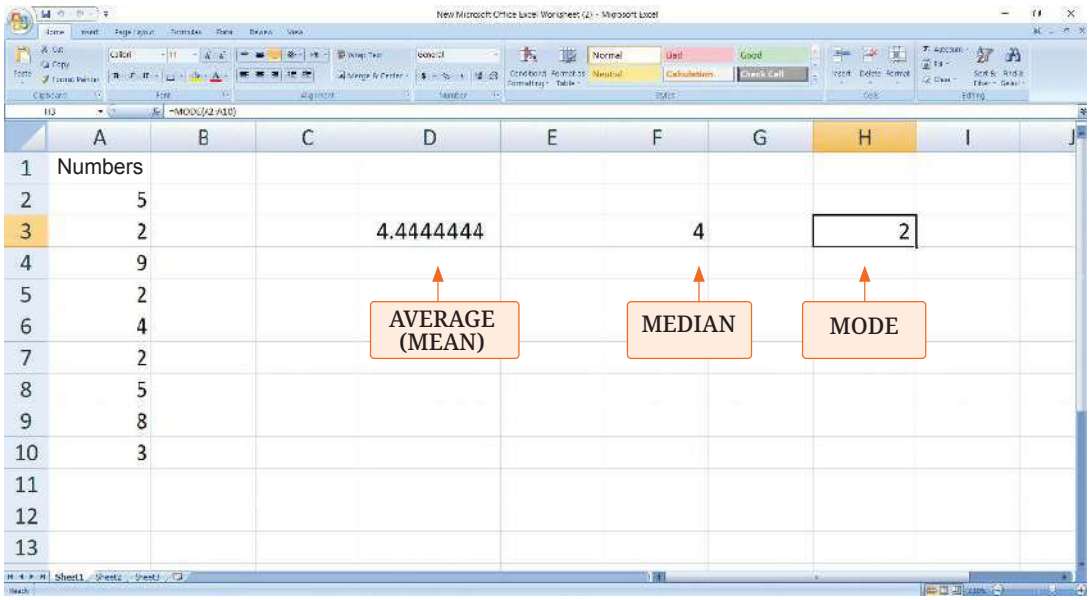
Technology Using Excel to Find Arithmetic Mean, Median, and Mode

Use Excel to calculate the arithmetic mean, the median, and the mode of the following set of values:

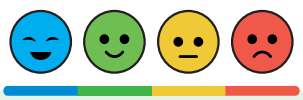
5 , 2 , 9 , 2 , 4 , 2 , 5 , 8 , 3

Steps:

1. Insert the numbers in one column, say, column **A**.
2. Select any cell, and from the **"Formulas"** menu, select fx Insert Function
3. **To Calculate the Arithmetic Mean:** - Select **"AVERAGE"**, then press **OK**.
   - Select the cells from A2 to A10, and then press **OK**.
4. **To Calculate the Median:** Repeat steps 2 and 3 but select **"MEDIAN"**.
5. **To Calculate the Mode:** Repeat steps 2 and 3 but select **"MODE"**.



Lesson Assessment



First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) For a set of values:
If $\Sigma f = 10$ and $\Sigma (f \, . \, x) = 40$,
what is the value of $\bar{x}$?
(a) 4 (b) 30 (c) 50 (d) 400

(2) For a set of values:
If $\bar{x} = 20$ and $\Sigma (f \, . \, x) = 1{,}500$,
what is the value of Σf?
(a) 75 (b) 150 (c) 3,000 (d) 30,000

3 In five mathematics tests, Sandy got 18, 17, 16, 15, and 18 marks. If the teacher removes the lowest mark, which of the following is true?

(a) The average decreases (b) The median decreases

(c) The average increases (d) The median does not change

Second Applying Scientific Concepts

4 If the arithmetic mean of the numbers 16 , $n + 3$, 14 , $2n - 2$ is 15.25 , find the median of these numbers.

5 The opposite table shows the number of minutes a group of people spend on phone calls. Calculate the average number of minutes a person spends on one phone call.

No. of minutes	2	3	4	5	6
Frequency	12	20	36	20	12

Third Analysis and Subjects Integration

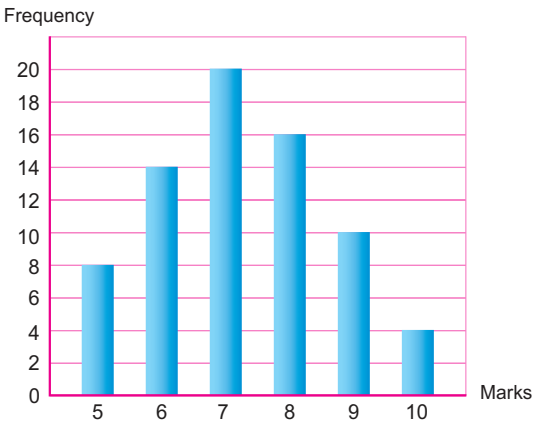
6 **Business Administration:** The opposite table shows the salaries of a group of company employees in LE. The company owner asserted that the average salary is 6,500 LE.
Explain why the average salary is misleading and gives the impression that salaries are higher than their actual values for most employees.

Salaries in LE	
Employee (1)	25,000
Employee (2)	4,000
Employee (3)	3,000
Employee (4)	2,500
Employee (5)	2,500
Employee (6)	2,000

7 **Physical Activity:** A group of first-year preparatory students were asked about the number of hours they spend in sports each week, and the answers are shown in the opposite table.
Calculate the arithmetic mean of the number of hours these students spend in sports.

No. of hours	8	9	10	11	12
Frequency	6	8	14	8	4

8 The opposite figure shows the marks of 72 students in a mathematics test represented by a bar graph. Find the average mark of those students.



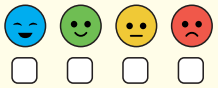
Creative Thinking

9 **Open-ended Problem:** Write down five numbers whose arithmetic mean is 11, median is 12, and mode is 14.

10 **Critical Thinking:** If the arithmetic mean of a set of five different positive integers is 13, and the median is 17, find the maximum value of the greatest integer in this set.

***Evaluate** your understanding!*

How well do you understand artithmetic mean?
Tick the right box



Lesson

3 - 3

Pie Charts

Learning Outcomes

- Learn the concept of pie charts.
- Learn how to represent data using pie charts.

Vocabulary

- Pie Chart
- Circular Sector
- Central Angle

Responsibility

Classroom election makes you take responsibility and respect others through fair competition.

Get Ready!

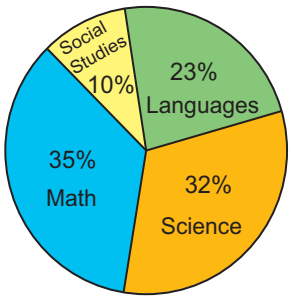
Our planet Earth is referred to as the "Water Planet" or the "Blue Planet" because water covers most of its surface; about 70% of surface of Earth is covered by water, while land represents 30% of its surface.

Can you represent these percentages with a pie chart to demonstrate the relationship between the water and land percentages relative to the overall area of Earth?

In this lesson, you will learn how to represent data using pie charts, which will enable you to solve such problems.

Think & Discuss!

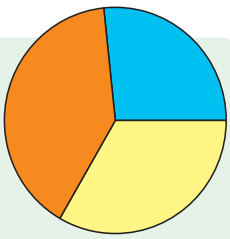
The opposite pie chart illustrates the results of a survey on the most favorite school subject for a group of preparatory students, in which math was the most favorite subject for 525 students. Can you find out the number of students who took the survey? How many students prefer social studies?



Learn!

A pie chart is a circle divided into sectors based on the percentage of frequencies in each group of the distribution. It is a simplified graphical method to show parts-to-whole relationship and visually comparing the areas of the sectors.

Note that: The sum of percentages of circular sectors = 100%



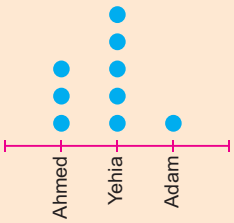
Example 1

Elections: The opposite table represents the number of votes each of Khaled, Anas, and Hamza gained in the classroom election.
Use a pie chart to represent this data.

Name	Votes
Khaled	8
Anas	12
Hamza	10

■ Self-Evaluation (1)

The dot plot below illustrates the contributions of three friends (in thousands) to a project. Represent the share of each one in the project using a pie chart.



■ Geometric Tools

A compass and a protractor are used to draw circles and pie charts.

■ Self-Evaluation (2)

A shopping market observed the selling activity during one day, and the results were as follows:

Items	Percentages
Foodstuffs	45 %
Dairy	35 %
Beverages & Juices	.....
Sweets	5 %

• Represent this data using a pie chart.

- Calculate the total number of votes: $8 + 12 + 10 = 30$.
- Calculate the measure of the central angle that represents the number of votes for each candidate:

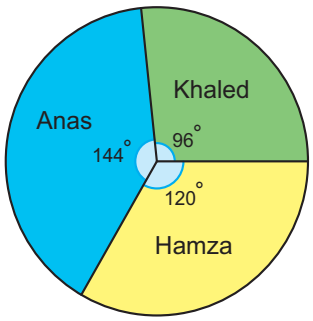
$$\text{Measure of the central angle} = \frac{\text{Number of votes for each candidate}}{\text{Total number of votes}} \times 360°$$

"Khaled" $\frac{8}{30} \times 360° = 96°$

"Anas" $\frac{12}{30} \times 360° = 144°$

"Hamza" $\frac{10}{30} \times 360° = 120°$

- Draw a suitable circle, and then draw the central angle representing the number of votes for each candidate.



Drawing Steps:

- Draw a circle by using a compass.
- Draw a radius in the circle.
- Draw a central angle of measure 96°, where this sector represents the votes for Khaled.
- Use the new radius to create a central angle of measure 144°, where this sector represents the votes for Anas.
- The remaining part is a circular sector with a central angle of measure 120°, It represents the votes for Hamza.

Example 2

Sports: In a survey, a group of people were asked their favorite sport. The results were as follows:

Sports	Handball	Basketball	Football	Volleyball
Percentages	15 %	25 %	45 %	.....

Represent the survey results using a pie chart.

- Find the percentage for volleyball:

$100\ \% - (45\ \% + 25\ \% + 15\ \%) = 15\ \%$

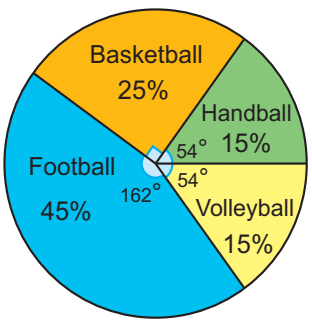
- Calculate the measure of the central angle representing each sport:

"Handball" $\frac{15}{100} \times 360° = 54°$

"Basketball" $\frac{25}{100} \times 360° = 90°$

"Football" $\frac{45}{100} \times 360° = 162°$

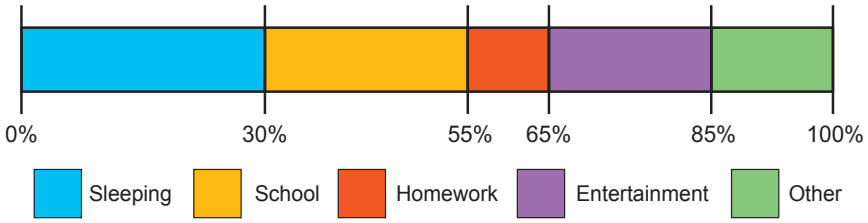
"Volleyball" $\frac{15}{100} \times 360° = 54°$



- Draw a suitable circle, and then draw the central angle for each sport.

Example 3

Daily Life: The figure below illustrates how Bassem spends his entire day :



Draw a pie chart expressing this routine.

- From the above figure, create the following table:

Practice	Sleeping	School	Homework	Entertainment	Other
Percentage	30 %	25 %	10 %	20 %	15 %

- Calculate the measure of the central angle representing each practice:

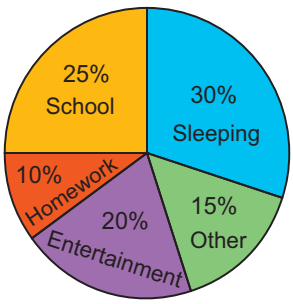
"Sleeping" $\frac{30}{100} \times 360° = 108°$

"School" $\frac{25}{100} \times 360° = 90°$

"Homework" $\frac{10}{100} \times 360° = 36°$

"Entertainment" $\frac{20}{100} \times 360° = 72°$

"Other" $\frac{15}{100} \times 360° = 54°$

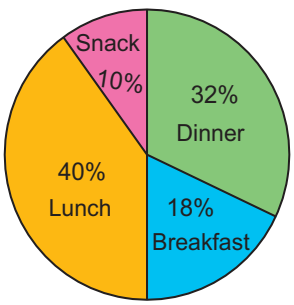


- Draw a suitable circle, and then draw the central angle for each practice.

Example 4

Nutrition: The opposite pie chart shows the percentage of calories Sally had in her meals on one day.

If Sally had 2,500 calories that day, how many calories did she have at dinner? Find the measure of the central angle for the breakfast sector.



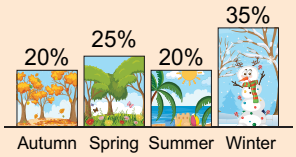
Dinner Calories: $2{,}500 \times 32\ \% = 2{,}500 \times 0.32 = 800$

Measure of the central angle for the breakfast sector :

$\frac{18}{100} \times 360° = 64.8° \approx 65°$

Self-Evaluation (3)

In a survey of a group of people about their preferred season, the results were as follows:



Represent the survey results using a pie chart.

Enriched Information

Normal human sleeping hours range between 7 and 8 hours daily. In other words, humans spend one third of their lives sleeping.

Self-Evaluation (4)

The graph below represents the broadcasting of a TV channel for 10 hours on one day.

- Calculate the percentage of the broadcasting hours for entertainment shows.
- Find the measure of the central angle for the cultural shows' sector.

■ Self-Evaluation (5)

A teacher recorded the marks of 30 students in the first-term exam, and the results were as follows.

Marks	Students
1 –	6
9 –	16
17 –	8

Represent these results using a pie chart.

Example 5

Social Life: The table below shows the number of children and youth members of one club according to their ages. Represent the results using a pie chart.

Intervals (Age-group)	1 –	11 –	21 –	31 –
Members (Frequency)	900	1,200	2,700	2,400

Since the sum of the measures of central angles in a circle is 360°

the sum of frequencies = 900 + 1,200 + 2,700 + 2,400 = 7,200,

measure of a central angle = $\frac{\text{frequency}}{\text{sum of frequencies}} \times 360°$

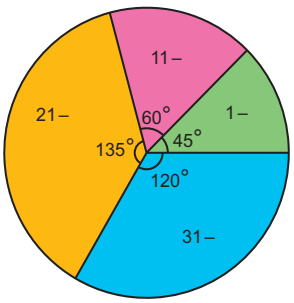
▸ Calculate the measure of the central angle representing each interval.

"Interval 1- " $\frac{900}{7,200} \times 360° = 45°$

"Interval 11- " $\frac{1,200}{7,200} \times 360° = 60°$

"Interval 21- " $\frac{2,700}{7,200} \times 360° = 135°$

"Interval 31- " $\frac{2,400}{7,200} \times 360° = 120°$



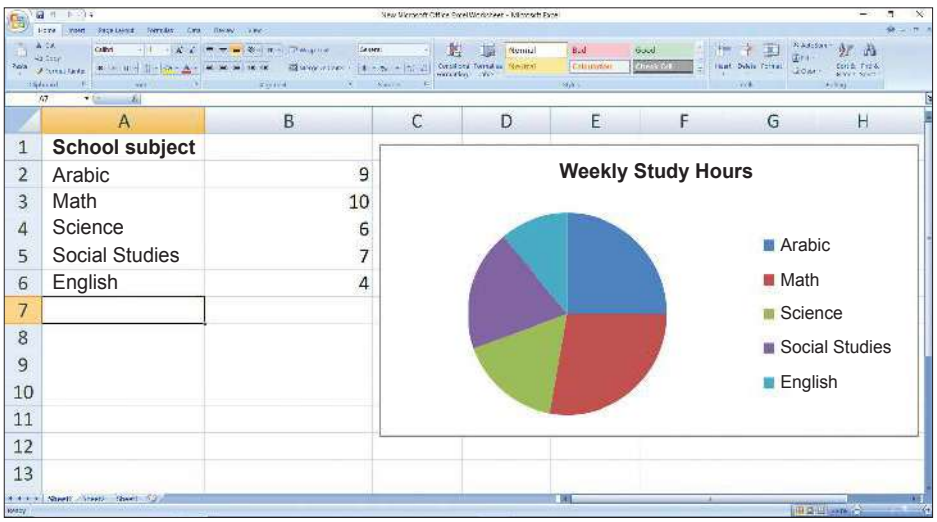
Technology Using Excel to Draw Pie Charts

The opposite table shows a student's weekly study hours for each school subject. Use **Excel** to represent the table using a pie chart.

School subject	Hours
Arabic	9
Math	10
Science	6
Social Studies	7
English	4

Steps :

1. Insert the data into two columns, Say, Column **A** and Column **B**.
2. Select the two columns, then from the **Insert** menu, click **Pie**, and from the upper ribbon, pick the pie chart you want.
3. Determine the heading of the chart.

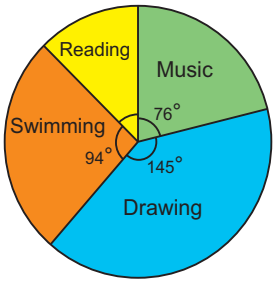


Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

In a survey of 2,000 girls about their preferred hobby, as shown in the opposite pie chart.



1. Which is the most preferred hobby?
   (a) Drawing (b) Music
   (c) Swimming (d) Reading

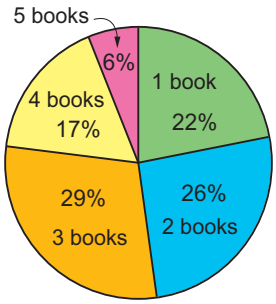
2. What is the central angle corresponding to the reading sector?
   (a) 35° (b) 45° (c) 86° (d) 104°

3. Which hobby do approximately $\frac{1}{4}$ of the girls prefere?
   (a) Drawing (b) Music (c) Swimming (d) Reading

Second Applying Scientific Concepts

▸ **Choose the correct answer from the given ones:**

4. The opposite pie chart shows the number of books read by 300 students at a school. How many students read fewer than 4 books?
   (a) 51 (b) 77
   (c) 231 (d) 282



5. When representing the opposite table in a pie chart, what is the measure of the central angle corresponding to the coffee sector?
   (a) 45° (b) 90°
   (c) 120° (d) 150°

Drink	Coffee	Tea	Juice
People	150	350	100

▸ **Answer the following questions:**

6. **Sports:** In a tennis match, a player hit 15 smashes, distributed as shown in the opposite table.
   Draw a pie chart for this distribution.

Set	First	Second	Third
No. of hits	7	3	5

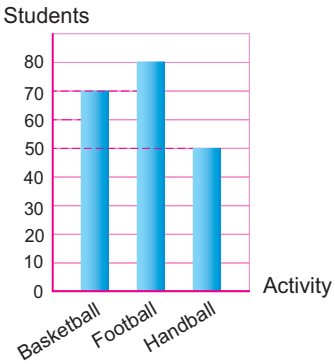
7. In a survey of a group of first-year preparatory students about their favorite color, the results were recorded in the opposite table.
   Draw a pie chart to represent this table.

Favorite color	Percentage
Red	25%
Blue	30%
Green	10%
Yellow	35%

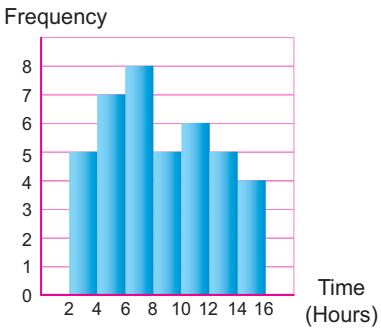
8 The bar graph below shows the distribution of students in summer activities according to their preferences. Fill in the table below:

Activity	Basketball	Football	Handball
Percentages	..... %	..... %	..... %

And then represent the data using a pie chart.

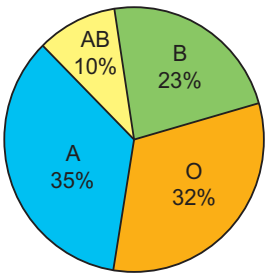


9 The following histogram shows the time that 40 students spend using their smartphones each week.
Represent this data using a pie chart.



Third Analysis and Subjects Integration

10 **Blood Type Distribution:** The opposite pie chart illustrates the blood type distribution of a group of people.
2,800,000 persons have blood type A.
How many persons were included in the survey?
How many persons have blood type AB ?

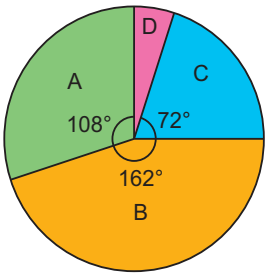


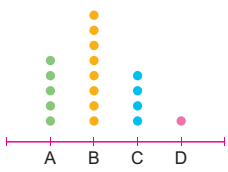
11 **Online Shopping:** Online retail sales are growing rapidly. The highest online retail sales categories are 50% for travel, 20% for clothes and footwear, 15% for computer devices and software, 10% for automobiles and spare parts, and 5% for home furnishings.
Draw a pie chart to represent the online sales.

Creative Thinking

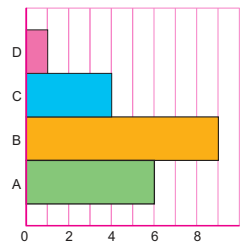
12 Represent, by using a pie chart, the sales of three different types of pens from brands x , y and z in a stationery store where the ratio of $x : y : z = 3 : 5 : 10$

13 Which of the following distributions is represented by the opposite pie chart?





(A)

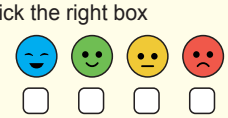


(B)

(C)

***Evaluate** your understanding!*

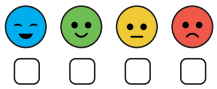
How well do you understand pie charts?
Tick the right box



Unit Three Assessment

Evaluate your understanding!

How well do you understand Unit Three? Tick the right box



▸ Choose the correct answer from the given ones:

1. If the arithmetic mean of the numbers: $x+2$, $x-5$, $2x+4$, 8 , $x+1$ is 7, what is the value of x?

   (a) 3 (b) 4 (c) 5 (d) 6

2. If the arithmetic mean of a student's marks in five exams is 94, and his marks in the first four exams are 97, 92, 94, and 91, how many marks did he score in the fifth exam?

   (a) 90 (b) 93 (c) 96 (d) 98

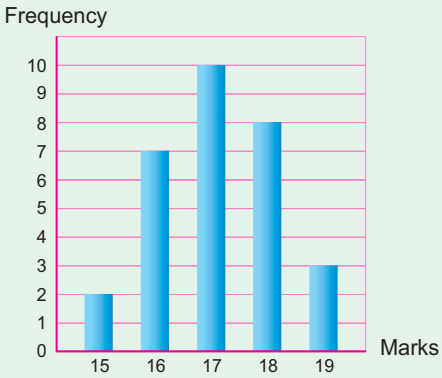
3. The figure below shows the marks of 30 students, what is the arithmetic mean of these marks?

   (a) 17

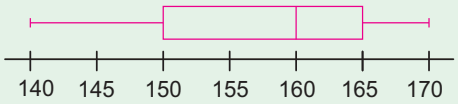
   (b) 17.1

   (c) 17.6

   (d) 18



4. The figure below represents the box plot of the heights of 50 students in centimeters. What is the value of the first quartile?



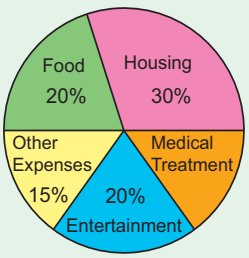
   (a) 140 cm (b) 150 cm

   (c) 160 cm (d) 165 cm

▸ Complete each of the following with the correct answer :

5. The arithmetic mean of the marks of 10 students is 87 marks. If the marks of two more students are added to these marks, the arithmetic mean increases to 89 marks. Thus, the arithmetic mean of the two added students is .....

6. The opposite figure shows a pie chart for the expenditure of a family whose monthly income is 10,000 LE. The monthly medical treatment expenses are ..... LE



7. If the arithmetic mean of five integers is 14, the median is 15, and the mode is 11, then the greatest of these integers, is .....

8. The opposite figure represents the temperatures recorded in a city over two weeks. The range of these temperatures is .....

Stems	Leaves
2	9
3	3 4 4 5 6 6 7 8 9
4	0 0 1 4

Key	2|9 represents 29 degrees

▸ **Answer the following questions :**

9 The opposite table shows the masses of 30 students in a classroom (in kilograms). Find the arithmetic mean of the students' masses.

Mass	44	46	47	49	50	51	52	53
Frequency	2	5	2	8	6	3	2	2

10 If the numbers of water bottles sold every day over two weeks are given as follows, represent this data using a stem-and-leaf plot.

6 42 36 33 12 17 21
19 34 45 27 25 14 18

11 If the marks of 40 students in a mathematics test are as follows, represent this data using a histogram.

7 11 31 30 51 8 12 50 32 43
49 22 48 9 39 28 17 44 18 41
26 34 25 52 35 13 42 40 27 39
34 41 37 42 24 55 23 38 59 39

12 If the sports preferred by 80 of the first-year preparatory students are given in the opposite table, use a pie chart to represent this data.

Sports	Football	Handball	Tennis	Swimming
Frequency	48	8	8	16

Unit Three Activity — Exploring Pie Charts in Our Daily Lives

▸ **Activity Aim :**

- To enhance the students' understanding of pie charts through educational games.
- To foster the skill of gathering and organizing data for the students.
- To enhancement cooperation and positive competition among students.

▸ **Implementation Steps :**

1. Divide the students into competing teams (2-3 students in each team) to improve teamwork and positive competition.
2. Each team should search for pie chart examples in our daily lives (e.g. slices of pizza, pieces of cake, round plates, etc.).
3. One team phrases questions regarding the example that has been chosen, and another team answers these questions within a specified time. These answers should be represented into a figure using pie charts.
4. Teams take turns carrying out steps 2 and 3.
5. The winner is the team who answers the greatest number of questions correctly in the least time and draws the most accurate pie chart.

UNIT 4

Geometry & Measurement

Unit Lessons

AI engineering aims to achieve significant advancements in the field of technological systems and smart robotics.

▶ **Can AI engineering be developed to address the many challenges we encounter in various fields of our daily lives?**

Issues and Life Skills

- Information Technology
- Globalisation
- Mathematical Understanding
- Creative thinking

Values

- Accuracy
- Persistence
- Responsibility
- Respect

Lesson

4 - 1

Types of Angles and Relations between Angles

Learning Outcomes

- Learn the concept of angles and their units of measurement.
- Identify and differentiate between various types of angles.
- Learn the adjacent angles.
- Learn the complementary angles.
- Learn the supplementary angles.
- Learn the vertically opposite angles.
- Learn about the accumulative angles at a point.
- Identify angles pairs in geometric shapes and the relationships between each pair.
- Apply the relationships between angles pairs to solve problems.

Vocabulary

- Straight Angle
- Reflex Angle
- Adjacent Angles
- Complementary Angles
- Supplementary Angles
- Vertically Opposite Angles
- Accumulative Angles at a Point

Get Ready!

Egypt has significant potential for generating clean, renewable energy from wind. The wind farm in Gabal El Zeit, south of Ras Gharib city, is one of the largest power plants to generate electricity in the world.

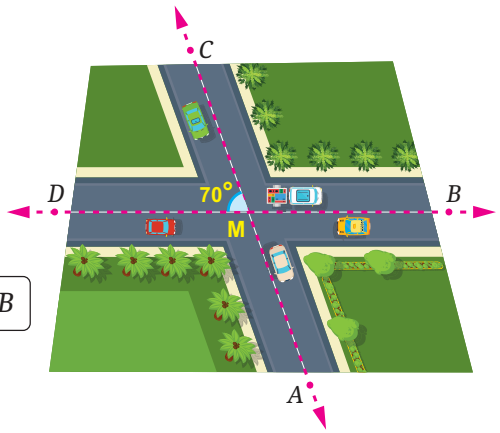
The opposite figure shows a wind turbine used to generate electricity. Can you determine the value of x?

In this lesson, you will learn about the types of angles and the relationships between them, which will enable you to solve such real-life problems.

Think & Discuss!

The opposite figure represents two intersecting roads. If the measure of $\angle CMD$ is 70°, can you determine the measures of the following three angles?

$\angle AMD$, $\angle AMB$ and $\angle CMB$



Learn!

Concept of Angle: an angle is the union of two rays with the same initial point.

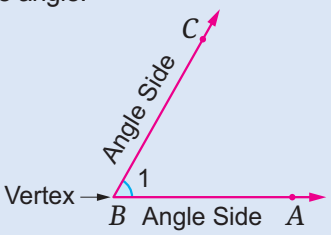
- The initial point of the two rays is called the vertex of the angle.
- Each one of the two rays is called side of the angle.

Example :

In the opposite figure:

$\overrightarrow{BA} \cup \overrightarrow{BC} = \angle ABC$

$\angle ABC$ can also be referred to as $\angle CBA$, $\angle B$ or $\angle 1$



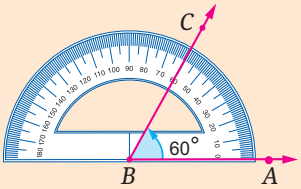
Units of Measurement for Angles:

Units of measurement for angles are degrees, minutes, and seconds, such that:

- A degree is equal to 60 minutes (1° = 60').
- A minute is equal to 60 seconds (1' = 60").

■ Geometric Tools

Protractor is used to measure angles

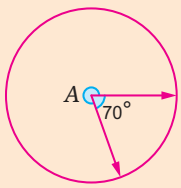


The measure of $\angle ABC$ is 60°.

- Use a protractor to measure each of the adjacent angles.
- Can you use this protractor in measuring the reflex angle?

■ Note that

The measure of a circle is 360°. Therefore, the sum of the measures of $\angle A$ and the reflex of $\angle A$ is 360°.

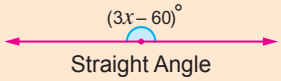


If the measure of $\angle A$ is 70°, then the measure of the reflex $\angle A$ is:
$360° - 70° = 290°$

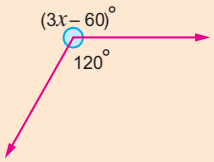
■ Self-Evaluation (1)

Find the value of x in each of the following:

1



2

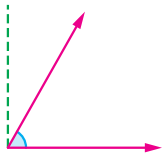


Types of Angles

1 Zero Angle

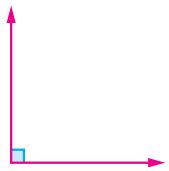
An angle whose measure is 0° and whose sides coincide.

2 Acute Angle



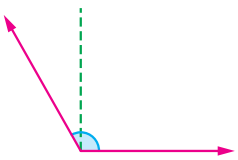
An angle whose measure is more than 0° and less than 90°.

3 Right Angle



An angle whose measure is 90°.

4 Obtuse Angle

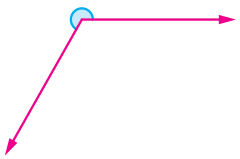


An angle whose measure is more than 90° and less than 180°.

5 Straight Angle

An angle whose measure is 180°, its sides are on opposite directions and form a straight line.

6 Reflex Angle

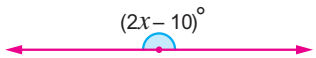


An angle whose measure is more than 180° and less than 360°.

Example 1

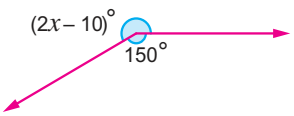
Find the value of x in each of the following:

1



Straight Angle

2



1

$2x - 10° = 180°$

$2x = 180° + 10°$

$2x = 190°$

$x = \frac{190°}{2}$

$x = 95°$

2

$2x - 10° = 360° - 150°$

$2x - 10° = 210°$

$2x = 210° + 10°$

$2x = 220°$

$x = \frac{220°}{2}$

$x = 110°$

■ Verify the Solution Yourself:

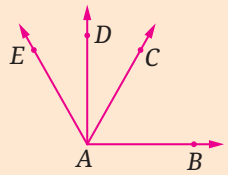
In problem (1):
Substitute x with 95°
$2x - 10° = 2 \times 95° - 10° = 190° - 10° = 180°$
This means that the solution is correct, since the angle is straight.

- Can you verify the solution for problem (2)?

Relation between Angles

Critical Thinking

Write all the pairs of adjacent angles in the figure below.

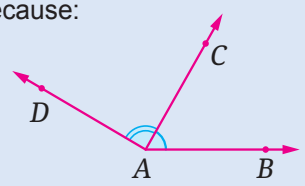


1 Adjacent Angles: They are two angles that lie in the same plane, share the same vertex, have a common side, and their outer sides lie on different sides of the common side.

Example:

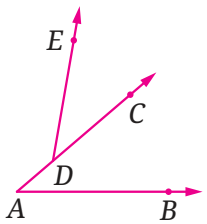
$\angle BAC$ and $\angle CAD$ are adjacent angles, because:

- They share the same vertex A, and have a common side $\overrightarrow{AC}$
- The outer sides $\overrightarrow{AD}$ and $\overrightarrow{AB}$ lie on different sides of the common side $\overrightarrow{AC}$.

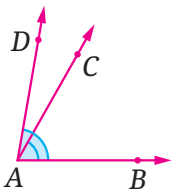


Think with your classmates

- Are $\angle BAC$ and $\angle CDE$ adjacent angles?



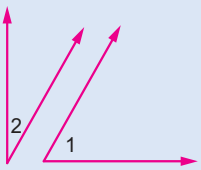
- Are $\angle BAD$ and $\angle BAC$ adjacent angles?

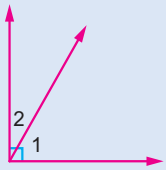


Notation in Mathematics

The measure of an angle is denoted by the letter (m). Thus, the measure of angle (ABC) is written as follows: m ($\angle ABC$).

2 Complementary Angles: They are two angles whose measures have a sum of 90°.





Example:

If $m(\angle 1) = 60°$, and $m(\angle 2) = 30°$

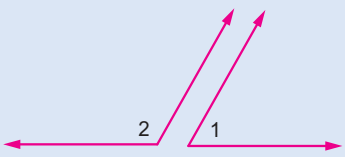
Then $\angle 1$ and $\angle 2$ are complementary angles, because:

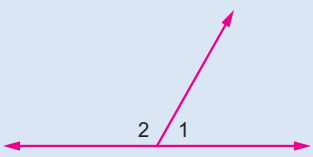
$m(\angle 1) + m(\angle 2) = 60° + 30° = 90°$

Think

Is it necessary for two angles to share the same vertex to be identified as complementary or supplementary angles?

3 Supplementary Angles: They are two angles whose measures have a sum of 180°.





Example:

If $m(\angle 1) = 60°$, and $m(\angle 2) = 120°$

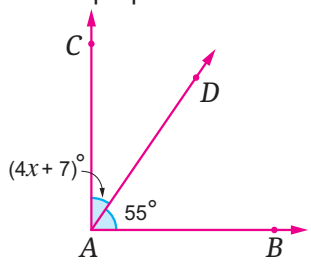
Then $\angle 1$ and $\angle 2$ are supplementary angles, because:

$m(\angle 1) + m(\angle 2) = 60° + 120° = 180°$

Example 2

Find the value of x in each of the following:

1 If $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$



1 The two angles form a right angle.

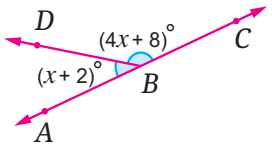
$55° + 4x + 7° = 90°$

$4x + 62° = 90°$

$4x = 90° - 62° = 28°$

$x = \frac{28°}{4} = 7°$

2 If A, B, and C are on a straight line.



2 The two angles form a straight angle

$4x + 8° + x + 2° = 180°$

$5x + 10° = 180°$

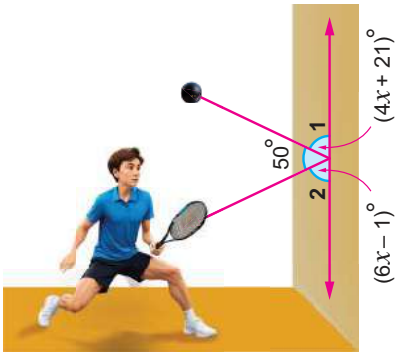
$5x = 180° - 10° = 170°$

$x = \frac{170°}{5} = 34°$

Example 3

Sports: While Amgad was playing squash, he hits the ball against the wall, and it bounced back.

Find the value of x, and substitute it to find: $m(\angle 1)$ and $m(\angle 2)$.



$4x + 21° + 50° + 6x - 1° = 180°$

$10x + 70° = 180°$

$10x = 180° - 70° = 110°$

$x = \frac{110°}{10} = 11°$

$m(\angle 1) = 4x + 21° = 4 \times 11° + 21° = 44° + 21° = 65°$

$m(\angle 2) = 6x - 1° = 6 \times 11° - 1° = 66° - 1° = 65°$

Did you know?

The world squash champions, both male and female, are Egyptians.

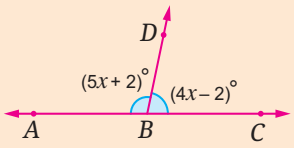
Complementary and Supplementary Adjacent Angles:

1 If two adjacent angles are complementary, then their outer sides are perpendicular.

2 If two adjacent angles are supplementary, then their outer sides are on the same straight line.

Self-Evaluation 2

If the points A, B, and C are on a straight line, calculate the value of x.

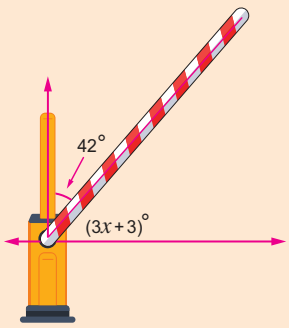


Note that

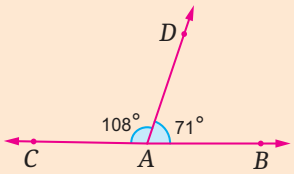
The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are supplementary.

Self-Evaluation 3

The pass gate arm moves at an angle of measure 42° from the vertical position. What is the measure of the angle the arm should move to become horizontal? What is the value of x?

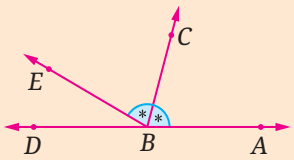


■ Self-Evaluation (4)



Do $\overrightarrow{AB}$ and $\overrightarrow{AC}$ form a straight line? Explain why.

■ Self-Evaluation (5)

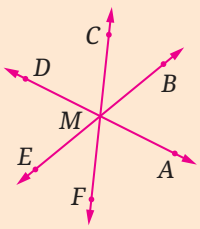


If $B \in \overleftrightarrow{AD}$, $\overrightarrow{BC}$ bisects $\angle ABE$ and m $(\angle EBA) = 150°$, what is the measure of $\angle DBC$?

(a) 150° (b) 105°
(c) 75° (d) 30°

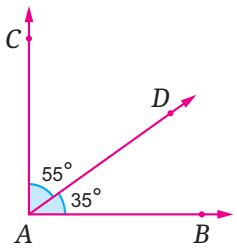
■ Critical Thinking

In the figure below, there are three straight lines intersecting at one point. Write down all the pairs of vertically opposite angles.



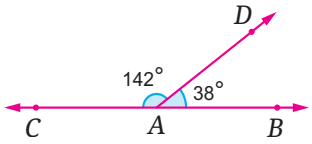
Example 4

(1)



Is $\overrightarrow{AB} \perp \overrightarrow{AC}$? Give a reason.

(2)



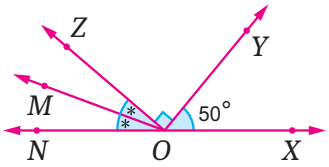
Do $\overrightarrow{AB}$ and $\overrightarrow{AC}$ form a straight line? Give a reason.

(1) $\overrightarrow{AB} \perp \overrightarrow{AC}$
Because:
$m(\angle BAD) + m(\angle DAC) = 90°$

(2) $\overrightarrow{AB}$ and $\overrightarrow{AC}$ form a straight line
Because:
$m(\angle BAD) + m(\angle DAC) = 180°$

Example 5

In the opposite figure:
$O \in \overleftrightarrow{XN}$, $\overrightarrow{OM}$ bisects $\angle NOZ$.
Calculate $m(\angle MOX)$.



$m(\angle NOZ) + 90° + 50° = 180°$

$m(\angle NOZ) = 180° - 90° - 50° = 40°$

$m(\angle NOM) = m(\angle MOZ) = \frac{40°}{2} = 20°$

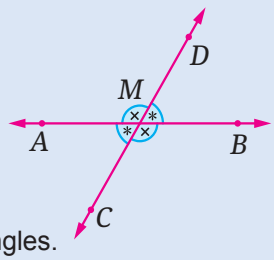
$m(\angle MOX) = 20° + 90° + 50° = 160°$

Angle Bisector

It is a ray that divides an angle into two congruent angles (equal in measure).

(4) Vertically Opposite Angles:

The two vertically opposite angles are non-adjacent angles formed by two intersecting straight lines.



Examples:

- $\angle AMC$ and $\angle BMD$ are vertically opposite angles.
- $\angle AMD$ and $\angle BMC$ are vertically opposite angles.

The two vertically opposite angles are congruent (equal in measure).

In other words:

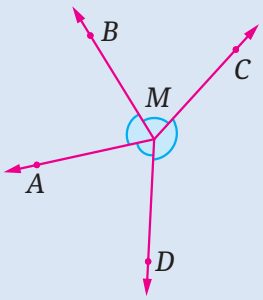
$m(\angle AMC) = m(\angle BMD)$ and $m(\angle AMD) = m(\angle BMC)$

5 Accumulative Angles at a Point:

The sum of measures of accumulative angles at a point is 360°.

Example:

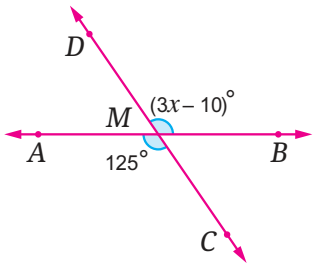
$m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DMA) = 360°$



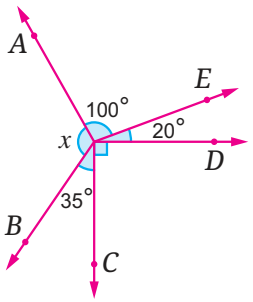
Example 6

Calculate the value of $\mathcal{X}$ in each of the following:

1 $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$



2



1 $3\mathcal{X} - 10° = 125°$

$3\mathcal{X} = 125° + 10° = 135°$

$\mathcal{X} = \frac{135°}{3} = 45°$

2 $\mathcal{X} + 100° + 20° + 90° + 35° = 360°$

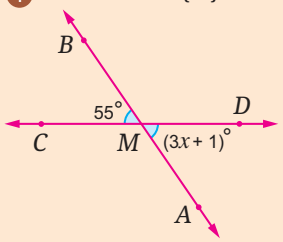
$\mathcal{X} + 245° = 360°$

$\mathcal{X} = 360° - 245° = 115°$

■ Self-Evaluation (6)

Calculate the value of $\mathcal{X}$ in each of the following:

1 $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$



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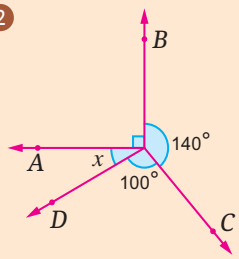
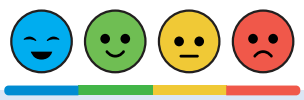


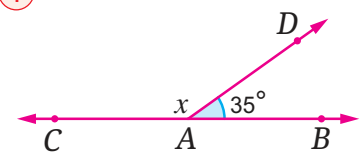
Lesson Assessment



First **Measuring Conceptual Understanding**

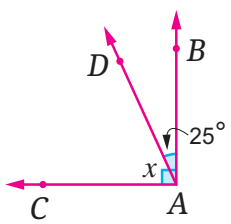
▸ **Calculate the value of $\mathcal{X}$ in each of the following figures:**

1

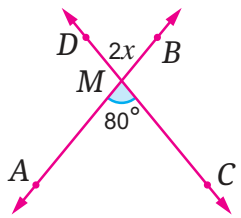


$A \in \overleftrightarrow{BC}$

2



3



$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

▸ **Choose the correct answer from the given ones:**

4 What is the type of the angle supplements an acute angle?

(a) Acute (b) Obtuse

(c) Straight (d) Reflex

5 What is the type of the angle complements a right angle?

(a) Acute (b) Obtuse

(c) Zero (d) Straight

6 If the angles A and B are complementary, and $m(\angle A) = 40°$, what is the measure of $\angle B$?

(a) 40° (b) 50°

(c) 90° (d) 140°

7 What is the measure of the angle that supplements an angle of measure 34° 60'?

(a) 55° (b) 56°

(c) 145° (d) 146°

▸ **Discuss:**

8 Mariam and Sandy were asked the following question:

What is the measure of one of two complementary angles if the difference between their measures is 12°?

Which of the two solutions is correct?

Explain why the results are different.

Mariam's Solution:

$x + (x - 12°) = 90°$

$2x - 12° = 90°$

$2x = 90° + 12° = 102°$

$x = \frac{102°}{2} = 51°$

Sandy's Solution:

$(90° - x) - x = 12°$

$90° - 2x = 12°$

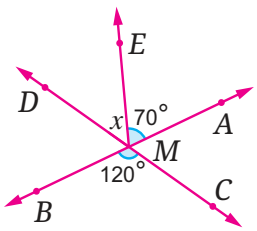
$2x = 90° - 12° = 78°$

$x = \frac{78°}{2} = 39°$

Second Applying Scientific Concepts

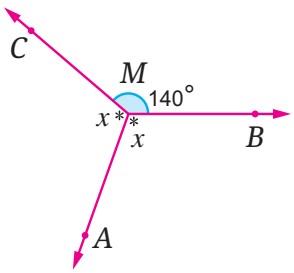
▸ **Find the value of x in each of the following:**

9

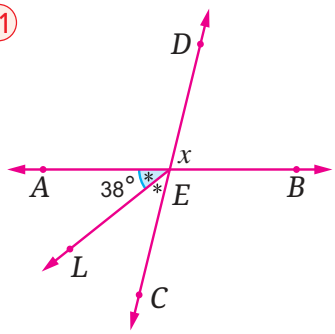


$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

10

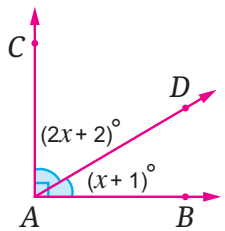


11

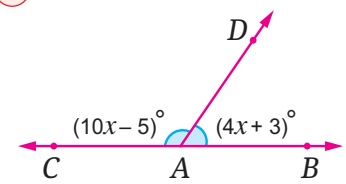


$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

12

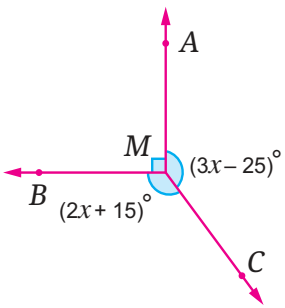


13

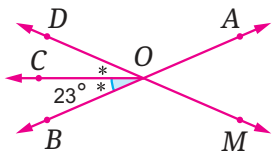


$A \in \overleftrightarrow{BC}$

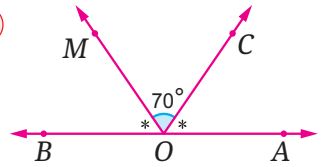
14



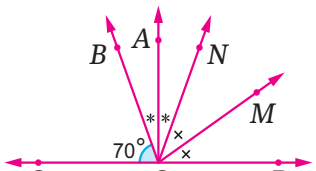
▸ **In each of the following, calculate m($\angle AOM$):**

15 

$\overleftrightarrow{AB} \cap \overleftrightarrow{DM} = \{O\}$

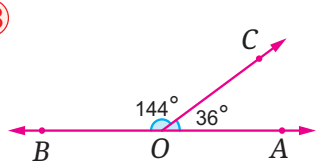
16 

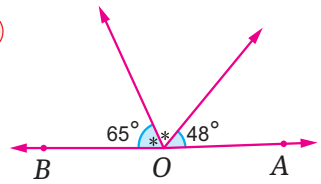
$O \in \overleftrightarrow{AB}$

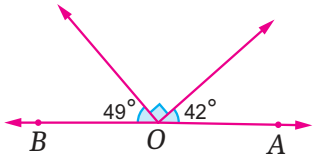
17 

$O \in \overleftrightarrow{CD}$

▸ **In each of the figures below, do $\overrightarrow{OA}$ and $\overrightarrow{OB}$ form a straight line or not? Why?**

18 

19 

20 

Third Analysis and Subjects Integration

▸ **Answer the following:**

21 Two vertically opposite angles: the measure of one of them is $(2x)°$ and the measure of the other is $(x + 28)°$. Find the measure of one of them.

22 Two complementary angles: the ratio between their measures is 5: 7. Find the measure of the smaller angle.

23 **Sports:**
In the opposite billiard table, if the measure of $\angle 1$ equals the measure of $\angle 3$, and the measure of $\angle 1$ is 43°, calculate measure of $\angle 2$.

Creative Thinking

24 **Critical Thinking:** If the measures of both angles between the blades of a pair of scissors are $(3x - 18)°$ and $(2x + 12)°$, as shown in Figure (1), and the measure of one angle is reduced by $(x + 16)°$ as shown in Figure (2), find the value of y.

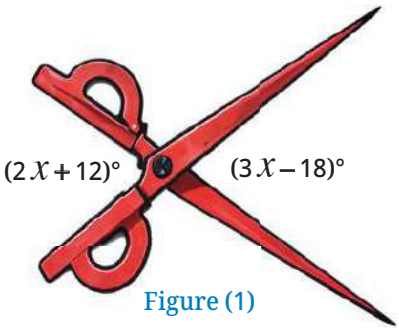


Figure (1)

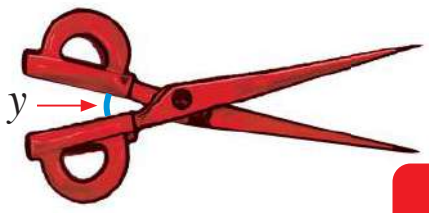


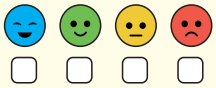
Figure (2)

25 Two supplementary angles, the sum of their measures is 74° more than the difference between their measures. What are the measures of both angles?

(a) 74° , 106°
(b) 16° , 74°
(c) 37° , 53°
(d) 37° , 143°

***Evaluate** your understanding!*

How well do you understand types of angles and relations between angles?
Tick the right box.



Lesson

4 - 2

Parallelism

■ Learning Outcomes

- Learn the concept of parallelism.
- Identify the angles formed when a straight line intersects two other straight lines.
- Learn about the corresponding angles.
- Learn about the alternating angles.
- Learn about the interior angles on the same side of a transversal.
- Identify the relationship between angles formed when a straight line intersects two parallel straight lines.
- Prove that two straight lines are parallel.
- Learn how to write a geometric proof.

■ Vocabulary

- Parallelism
- Corresponding Angles
- Alternating Angles
- Interior Angles
- Transversal

■ Notation in Mathematics

- The symbol ($\perp$) is used to indicate that two straight lines are perpendicular, such that ($m \perp n$) means that m is a straight line perpendicular to the straight line n.
- The symbol (//) is used to indicate that two straight lines are parallel, such that ($p \,//\, q$) means that p is a straight line parallel to the straight line q.

Get Ready!

Parallel lines are used in engineering, architectural, decorative designs, and in the construction of roads and bridges.

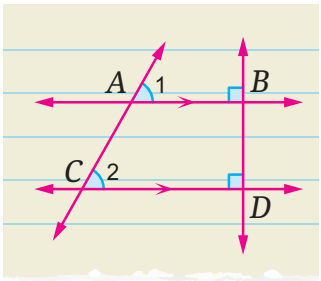
In the given picture, a garage was planned by drawing several parallel and intersecting lines. If the measure of one of the angles formed by the intersection of two lines is 65°, can you find the other angles have the same measure? Can you find angles measuring 115°?

In this lesson, you are going to learn the relationship between angles formed when a straight line intersects two parallel straight lines, which enable you to solve such real-life problems.

Think & Discuss!

Ibrahim drew two parallel straight lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. When he drew a straight line $\overleftrightarrow{BD}$, perpendicular to one of them, he found out that $\overleftrightarrow{BD}$ also perpendicular to the other. Ibrahim draws an inclined straight line $\overleftrightarrow{AC}$, to intersect both $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

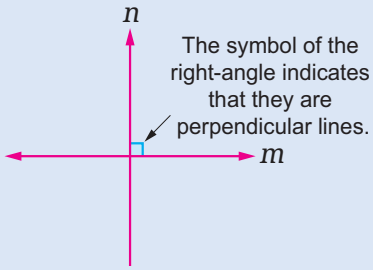
Does $m(\angle 1) = m(\angle 2)$?



Learn!

Two perpendicular straight lines

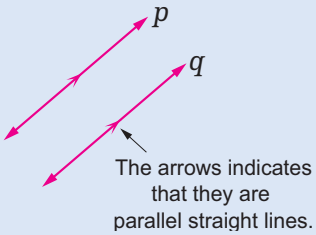
Two perpendicular straight lines are two straight lines that intersect and form 4 right angles.



$m \perp n$

Two parallel straight lines

Two parallel straight lines are two straight lines that never intersect.



$p \,//\, q$

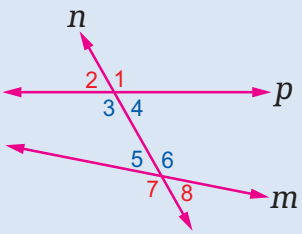
Individual Activity

On a lined paper, draw two parallel lines and then draw a transversal. Identify the interior and exterior angles.

Transversal

A transversal is a straight line that intersects two or more straight lines.
If a straight line intersects two straight lines, eight angles are formed:

- Four angles are called interior angles, which lie between the two straight lines.
- The other four angles are called exterior angles, which lie outside the two straight lines .



n is a straight line that intersects two straight lines m and p

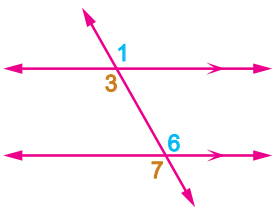
- ∠3, ∠4, ∠5, and ∠6 are interior angles.
- ∠1, ∠2, ∠7, and ∠8 are exterior angles.

Relationships between pairs of angles formed when a straight line intersects two parallel straight lines

Corresponding Angles

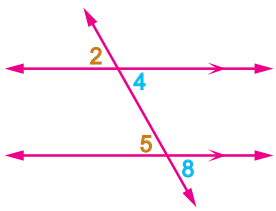
They are a pair of non-adjacent angles that lie on the same side of a transversal, one of them is an exterior angle and the other is an interior angle.

1 If a straight line intersects two parallel straight lines, then each two corresponding angles are equal in measure.



∠ 1 and ∠ 6 are corresponding angles
∠ 3 and ∠ 7 are corresponding angles

$m(\angle 1) = m(\angle 6)$
$m(\angle 3) = m(\angle 7)$



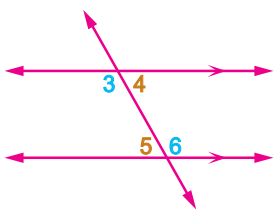
∠ 2 and ∠ 5 are corresponding angles
∠ 4 and ∠ 8 are corresponding angles

$m(\angle 2) = m(\angle 5)$
$m(\angle 4) = m(\angle 8)$

Alternating Angles

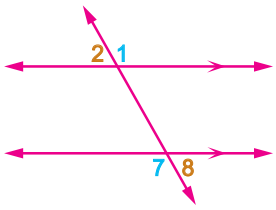
They are a pair of non-adjacent interior or exterior angles that lie on different sides of the transversal.

2 If a straight line intersects two parallel straight lines, then each two alternating angles are equal in measure.



∠ 3 and ∠ 6 are alternating interior angles
∠ 4 and ∠ 5 are alternating interior angles

$m(\angle 3) = m(\angle 6)$
$m(\angle 4) = m(\angle 5)$



∠ 1 and ∠ 7 are alternating exterior angles
∠ 2 and ∠ 8 are alternating exterior angles

$m(\angle 1) = m(\angle 7)$
$m(\angle 2) = m(\angle 8)$

■ Practical Activity

Use a protractor to measure each of the eight angles formed when a transversal intersects two parallel straight lines and verify the relationships between each pairs of angles yourself.

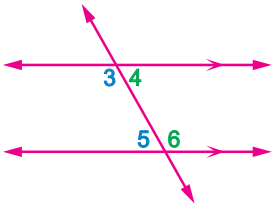
3 If a straight line intersects two parallel straight lines, then each two interior angles on the same side of the transversal are supplementary.

∠ 3 and ∠ 5 are interior angles on the same side of the transversal

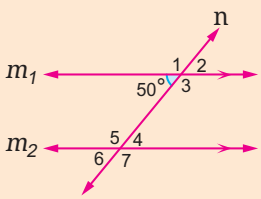
∠ 4 and ∠ 6 are interior angles on the same side of the transversal

$m(\angle 3) + m(\angle 5) = 180°$

$m(\angle 4) + m(\angle 6) = 180°$



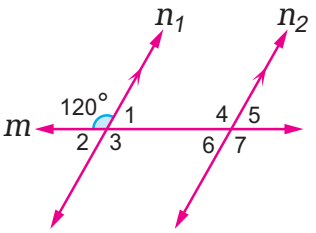
■ Self-Evaluation (1)



Find the measures of the numbered angles in the figure and explain your reasoning.

Example 1

In the opposite figure, there are three angles, the measure of each is 120°. Identify these angles and state the reason. Given that n_1 // n_2 and m is the transversal.



$m(\angle 3) = 120°$ (Vertically opposite angles)

$m(\angle 4) = 120°$ (Corresponding angles)

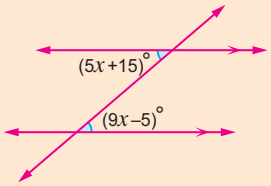
$m(\angle 7) = 120°$ (Alternating exterior angles)

i.e., the three angles are: ∠ 3 , ∠ 4 and ∠ 7

■ Think

Try to solve the example using different methods yourself.

■ Self-Evaluation (2)

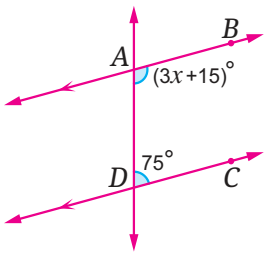


Find the value of $\mathcal{X}$ and verify the solution yourself.

Example 2

In the opposite figure:

$\overleftrightarrow{AB}$ // $\overleftrightarrow{DC}$. What is the value of $\mathcal{X}$?



$\overleftrightarrow{AB}$ // $\overleftrightarrow{DC}$, ∠ BAD and ∠ ADC are interior angles on the same side of the transversal, thus:

$m(\angle BAD) + m(\angle ADC) = 180°$

$3\mathcal{X} + 15° + 75° = 180°$

$3\mathcal{X} + 90° = 180°$

$3\mathcal{X} = 180° - 90°$

$3\mathcal{X} = 90°$

$\mathcal{X} = \frac{90°}{3} = 30°$

■ Verifying the Solution

Substitute $\mathcal{X}$ with 30° in the expression $(3\mathcal{X} + 15)°$

$3\mathcal{X} + 15° = 3 \times 30° + 15°$

$= 90° + 15°$

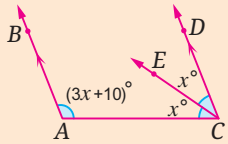
$= 105°$

$m(\angle BAD) + m(\angle ADC)$

$= 105° + 75° = 180°$

The solution is correct because the two angles are supplementary.

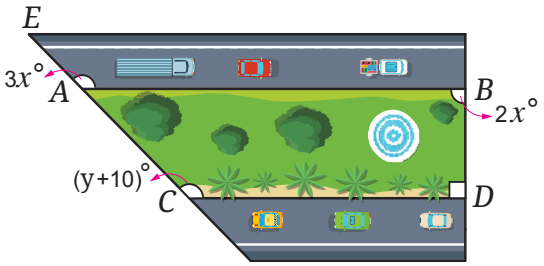
Self-Evaluation (3)



Find the value of x.

Example 3

The opposite figure shows a garden between two parallel roads. Find the values of x and y.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$\angle ABD$ and $\angle CDB$ are supplementary angles, because they are interior and lie on the same side of the transversal.

$2x + 90° = 180°$

$2x = 180° - 90° = 90°$

$x = \frac{90°}{2} = 45°$

$\angle ACD$ and $\angle BAE$ are equal in measure, because they are corresponding angles.

$(y + 10)° = 3x$

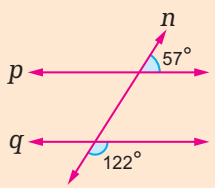
$y + 10° = 3 \times 45° = 135°$

$y = 135° - 10° = 125°$

Think

In each of the following: Is $q \parallel p$? Why?

1



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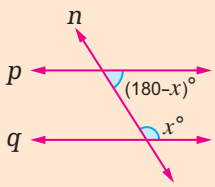
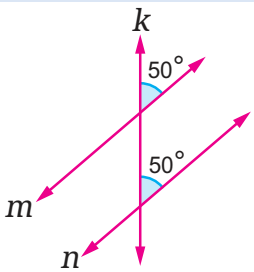


Proof parallelism of two straight lines

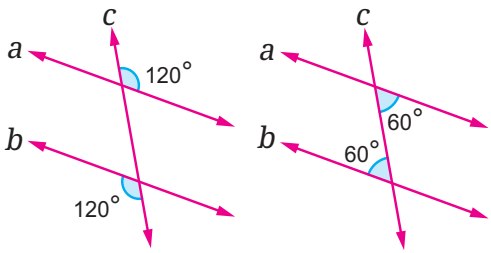
The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases has occurred:

1 Two Corresponding Angles are Equal in Measure



Straight line m is parallel to straight line n ($m \parallel n$), because there are two corresponding angles that are equal in measure.

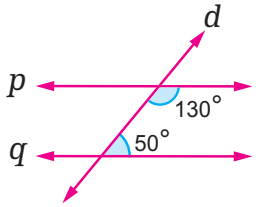
2 Two Alternating Angles are Equal in Measure



Straight line a is parallel to straight line b ($a \parallel b$), because there are two alternating angles that are equal in measure.

3 Two Interior Angles on the Same Side of the Transversal are Supplementary

Straight line p is parallel to straight line q ($p \parallel q$), because there are two interior angles that are supplementary on the same side of the transversal.



How do you write a proof in geometry?

When you want to write a geometric proof, you write a series of logical steps go from the givens to the conclusion to prove the validity of what you want to prove.

Givens ▶ Definitions, Theorems, Properties, Postulates ▶ Conclusion

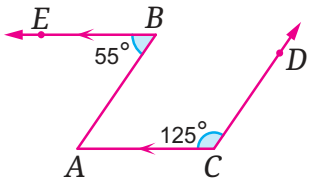
Example 4

In the opposite figure :

$\overline{CA} \parallel \overrightarrow{BE}$

$m(\angle B) = 55°$, $m(\angle C) = 125°$

Prove that: $\overline{AB} \parallel \overrightarrow{CD}$



Proof Tactical Plan:

Given
$\overline{CA} \parallel \overrightarrow{BE}$
$\overleftrightarrow{AB}$ is a transversal

Conclusion
$m(\angle A) = m(\angle B) = 55°$
Alternating interior angles

Required to Prove
$\overline{AB} \parallel \overrightarrow{CD}$

Conclusion
$m(\angle A) + m(\angle C) = 55° + 125° = 180°$
They are interior angles on the same side of the transversal

Given: $\overline{CA} \parallel \overrightarrow{BE}$, $\overleftrightarrow{AB}$ is a transversal.

Required to Prove: $\overline{AB} \parallel \overrightarrow{CD}$

Proof:

$\because \overline{CA} \parallel \overrightarrow{BE}$, and $\overleftrightarrow{AB}$ is a transversal.

$\therefore m(\angle A) = m(\angle B) = 55°$

Alternating interior angles.

$\because m(\angle A) + m(\angle C) = 55° + 125° = 180°$,

and they are interior angles on the same side of the transversal.

$\therefore \overline{AB} \parallel \overrightarrow{CD}$ **(Required to Prove "RTP")**

■ Hint

When writing a proof in geometry, it is important to provide logical reasoning for each step.

■ Steps to Write a Proof

1. Read the problem carefully.
2. Identify the givens.
3. Identify the requirements (Required to Prove).
4. Think of a plan for the proof, their could be many solutions methods.
5. Use your plan to write a proof.
6. Check your answer.
7. If your answer is incorrect, start over and choose another method.

■ Self-Evaluation (4)

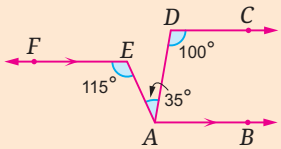
In the following figure:

$\overrightarrow{AB} \parallel \overrightarrow{EF}$

$m(\angle D) = 100°$

$m(\angle E) = 115°$

$m(\angle DAE) = 35°$



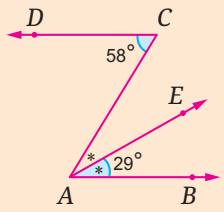
Prove that: $\overrightarrow{AB} \parallel \overrightarrow{DC}$

▶ **Think:** Is $\overrightarrow{DC} \parallel \overrightarrow{EF}$?

■ Notation in Mathematics

- $(\because)$ is used for "since" and is usually placed before a given information, a fact, or a theorem.
- $(\therefore)$ is used for "therefore" and is placed before the concluded mathematical statement.

■ Self-Evaluation (5)



Prove that: $\overrightarrow{AB} // \overrightarrow{CD}$

■ Note that

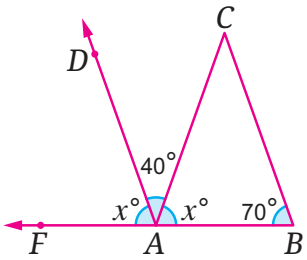
You can identify the givens and requirements in a problem while reading it, you don't need to write them again in the solution.

Example 5

In the opposite figure: $F \in \overleftrightarrow{BA}$

$m(\angle CAD) = 40°$, $m(\angle B) = 70°$

Prove that: $\overrightarrow{AD} // \overrightarrow{BC}$



$\because \angle BAF$ is a striaght angle

$\therefore m(\angle BAF) = 180°$

$\therefore 2x + 40° = 180°$

$\therefore 2x = 180° - 40° = 140°$

$\therefore x = \frac{140°}{2} = 70°$

$\therefore m(\angle FAD) = 70°$

$\therefore m(\angle FAD) = m(\angle B)$, and they are corresponding angles.

$\therefore \overrightarrow{AD} // \overrightarrow{BC}$

Collaborative Activity

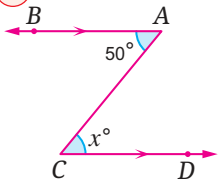
Collaborate a classmate and use **GeoGebra** to draw a parallelogram. use what you have learned to conclude the relationship between the measures of its angles.

Lesson Assessment

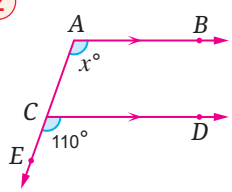
First **Measuring Conceptual Understanding**

▸ **Find the value of x in each of the following figures:**

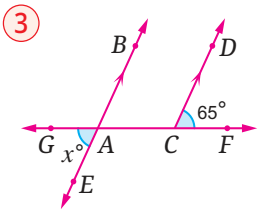
(1)



(2)

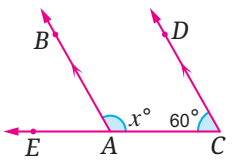


(3)



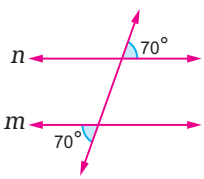
$\overleftrightarrow{FG} \cap \overleftrightarrow{BE} = \{A\}$

(4)

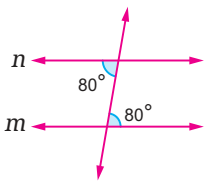


▸ **In each of the following figures, prove that $m // n$:**

(5)



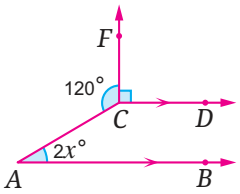
(6)



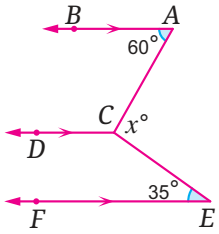
Second Applying Scientific Concepts

▸ **Calculate with proof the value of x in each of the following figures:**

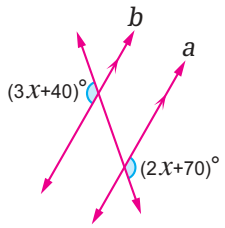
⑦



⑧



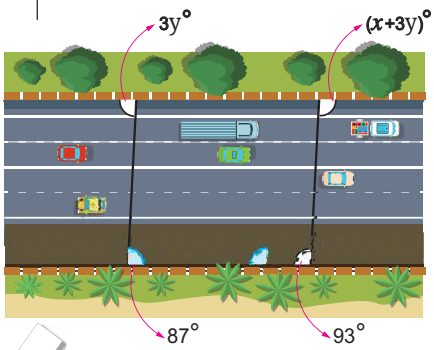
⑨



Third Analysis and Subjects Integration

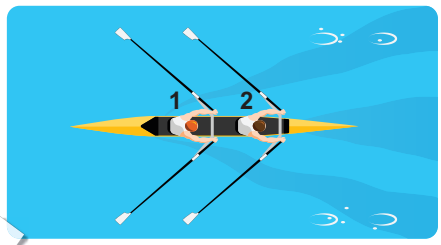
⑩ **Road Safety:**

The opposite figure shows a pedestrian road next to a car road. What is the value of x?



Ensure you cross the road using the pedestrian lane to avoid accidents.

⑪ **Sports:** Some rowing competitions take place in the River Nile. At a certain moment $m(\angle 2) = (3x - 29)°$, and $m(\angle 1) = (2x - 6)°$, If $x = 23°$, are the left oars parallel or not?



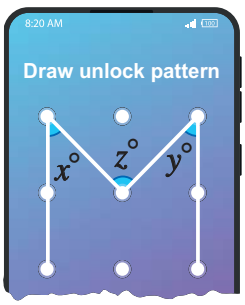
In rowing, you must keep the oars on both sides parallel during the race for optimum performance.

Creative Thinking

⑫ **Critical Thinking:** Mostafa created a pattern to unlock his smartphone, as shown in the opposite figure. What is the relationship between x, y and z?

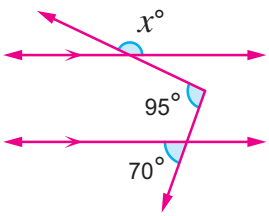
(a) $x = y + z$ (b) $y = x + z$

(c) $z = x + y$ (d) $x + y + z = 360°$

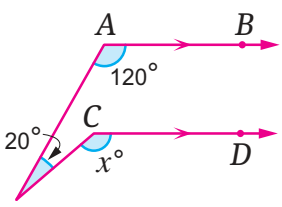


▸ **Calculate the value of x in each of the following figures:**

⑬

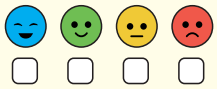


⑭



***Evaluate** your understanding!*

How well do you understand parallelism? Tick the right box



Lesson

4 - 3

Triangle

Learning Outcomes

- Deduce the sum of the measures of the interior angles of a triangle.
- Learn the concept of the exterior angle of a triangle.
- Deduce the relationship between the exterior angle of a triangle and its interior angles.
- Learn the concept of the triangle inequality

Vocabulary

- Interior Angle
- Exterior Angle
- Triangle Inequality

Previous Knowledge

Triangles are classified based on the measures of their angles into:

- Acute-angled triangle (has 3 acute angles)
- Right-angled triangle (has 1 right angle)
- Obtuse-angled triangle (has 1 obtuse angle).

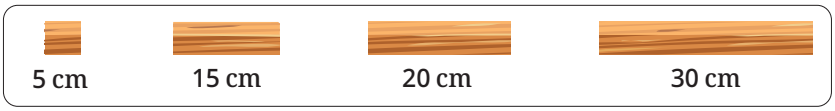
Get Ready!

Triangles are used in the construction of numerous projects, including building houses, crafting boat sails, and more. Demonstrate how to calculate the value of 'x' in the opposite figure.

In this lesson, you will learn about some properties of triangles, which will enable you to solve such real-life problems.

Think & Discuss!

If you have four wood planks of lengths of 30 cm, 20 cm, 15 cm, and 5 cm, which three can be used to make a triangle?

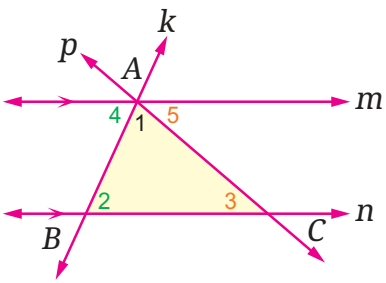


Try all the possible combinations yourself.

Learn!

Sum of Measures of Interior Angles of a Triangle

Straight lines m and n are parallel, straight line k intersects them at A and B, and straight line p intersects them at A and C.



$\angle 4$, $\angle 1$ and $\angle 5$ form a straight angle

$\therefore\ m(\angle 4) + m(\angle 1) + m(\angle 5) = 180°$

$\because\ m(\angle 4) = m(\angle 2)$ Alternating Angles

$\because\ m(\angle 5) = m(\angle 3)$ Alternating Angles

$\therefore\ \boxed{m(\angle 2) + m(\angle 1) + m(\angle 3) = 180°}$

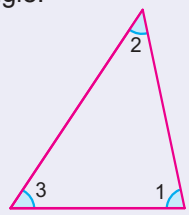
Rule:

The sum of the measures of the interior angles of any triangle is 180°.

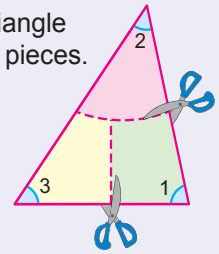
Collaborative Activity

Collaborate with your classmate and follow the steps below:

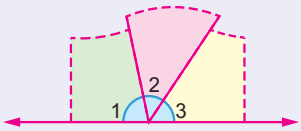
1 Draw a triangle.



2 Cut the triangle into three pieces.



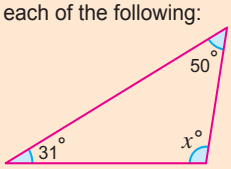
3 Rearrange the angles of the triangle to be adjacent.

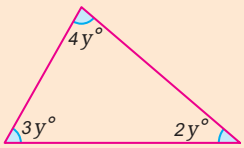


- What is the type of the angle formed by the three angles in step 3?
- Can you deduce the rule that states the sum of the measures of the interior angles of a triangle?

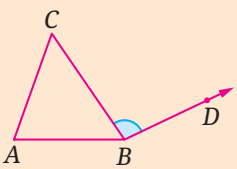
Self-Evaluation (1)

Find the value of the variable in each of the following:





Think

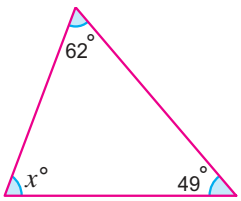


Is $\angle CBD$ an exterior angle of the triangle ABC? Why?

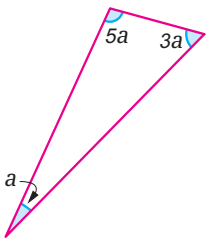
Example 1

Determine the value of the variable in each of the following figures:

1



2



(1) $62° + 49° + x = 180°$

$x = 180° - 111°$

$x = 69°$

(2) $5a + 3a + a = 180°$

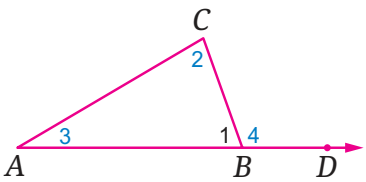
$9a = 180°$

$a = \frac{180°}{9} = 20°$

▸ **Think:** How can you verify the solution?

The Exterior Angle of a Triangle

If ABC is a triangle, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, then $\angle 4$ is called an exterior angle of the triangle ABC, $\angle 1$, $\angle 2$, and $\angle 3$ are the interior angles of the triangle.



$m(\angle 1) + m(\angle 2) + m(\angle 3) = 180°$ (1)

$\angle 1$ and $\angle 4$ form a straight angle.

$m(\angle 1) + m(\angle 4) = 180°$ (2)

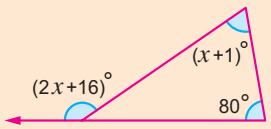
From (1) and (2), you will find that: $m(\angle 4) = m(\angle 2) + m(\angle 3)$

Rule:

The measure of the exterior angle of any triangle is equal to the sum of the measures of the two non-adjacent interior angles.

Self-Evaluation (1)

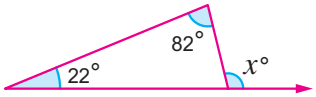
Find the value of x



Example 2

Find the value of the variable in each of the following:

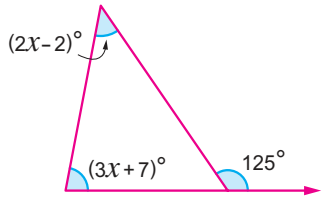
1



(1) $x = 82° + 22°$

$x = 104°$

2



(2) $2x - 2° + 3x + 7° = 125°$

$5x + 5° = 125°$

$5x = 120°$ $\quad x = 24°$

▸ **Think:** How can you verify the solution?

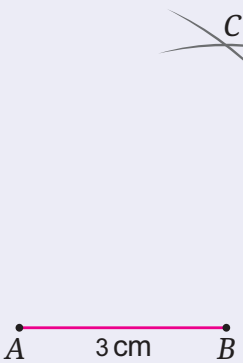
Triangle Inequality

Practical Activity

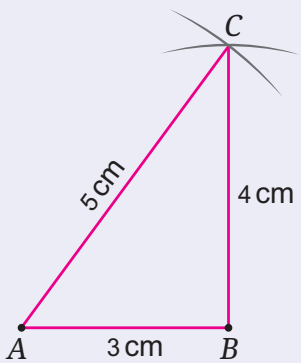
Draw a triangle with side lengths 3 cm, 4 cm, and 5 cm, using a ruler and a compass.

1 Use the ruler to draw a line segment $\overline{AB}$ of length 3 cm.

2 Set the compass to a length of 4 cm, place the needle at point B, and draw an arc. Then, set the compass to a length of 5 cm, place the needle at point A, and draw another arc that intersects the first arc at point C.



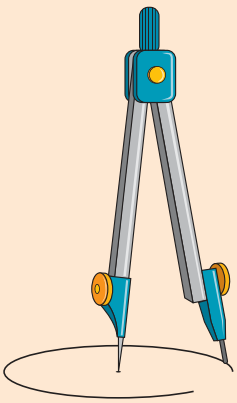
3 Draw $\overline{AC}$ and $\overline{BC}$ to form triangle ABC with side lengths of 3 cm, 4 cm, and 5 cm.



- Try to draw a triangle the lengths of its sides are 7 cm, 5 cm, and 5 cm. Can you draw this triangle? Discuss.
- Try to draw a triangle the lengths of its sides are 8 cm, 4 cm, and 3 cm. Can you draw this triangle? Discuss.
- Try to draw a triangle the lengths of its sides are 6 cm, 4 cm, and 2 cm. Can you draw this triangle? Discuss.

Geometric Tools

- A compass is a geometric tool used for drawing circles. It can also be used to determine dimensions in a geometric drawing.



- Use a protractor to measure the angles of the triangle that you have drawn, and determine its type based on the measures of its angles.

■ Remember

Triangles are classified according to the lengths of their sides as follows:
- A scalene triangle.
- An isosceles triangle.
- An equilateral triangle.

■ Self-Evaluation (3)

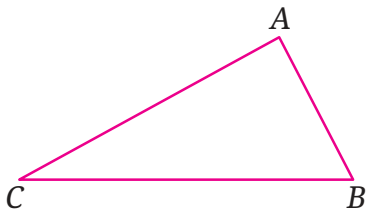
Which of the following sets of side lengths can be used to form a triangle? Why?

1. 9 cm, 5 cm, 4 cm
2. 7 cm, 6 cm, 5 cm
3. 10 m, 5 m, 2 m

Triangle Inequality:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$AB + BC > AC$$
$$AB + AC > BC$$
$$AC + BC > AB$$



Example 3

Can you draw a triangle with each of the following sets of side lengths:

1. 10 cm, 6 cm, 5 cm.
2. 8 cm, 4 cm, 4 cm.
3. 12 cm, 3 cm, 6 cm.

Compare the sum of the lengths of the two shortest sides to the length of the third side

1. $5 + 6 = 11$
   $11 > 10$
   The triangle can be drawn
2. $4 + 4 = 8$
   $8 = 8$
   The triangle cannot be drawn
3. $6 + 3 = 9$
   $9 < 12$
   The triangle cannot be drawn

Note:

The length of any side of a triangle is greater than the difference between the lengths of the other two sides and smaller than their sum.

■ Self-Evaluation (4)

In triangle ABC, the length of $\overline{AB}$ is 5 cm, and the length of $\overline{BC}$ is 7 cm. What is the smallest possible integer value for the length of $\overline{AC}$?

Example 4

If the lengths of two sides of a triangle are 5 cm and 2 cm, what is the largest integer that can represent the length of the third side?

The length of the third side is greater than the difference between 5 cm and 2 cm, and smaller than the sum of 5 cm and 2 cm.

i.e. the length of the third side is greater than 3 cm and smaller than 7 cm. Since the length of the required side must be an integer, the possible lengths for the third side could be 4 cm, 5 cm, or 6 cm.

Therefore, the largest integer length for the third side is 6 cm.

Lesson Assessment

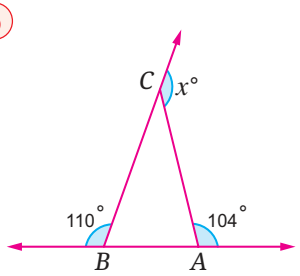
First Measuring Conceptual Understanding

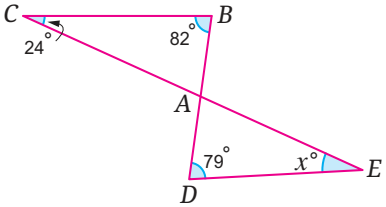
▸ **Choose the correct answer from the given ones:**

1. If the sum of the measures of two angles in a triangle is 130°, what is the measure of the third angle?
   (a) 20° (b) 30° (c) 50° (d) 60°
2. If the measures of two angles in a triangle are 30° and 70°, which of the following cannot be the measure of an exterior angle of the triangle?
   (a) 150° (b) 130° (c) 110° (d) 100°
3. Which of the following sets of numbers cannot be used as side lengths of a triangle?
   (a) 4 cm, 7 cm, 7 cm (b) 3 cm, 4 cm, 7 cm
   (c) 7 cm, 7 cm, 7 cm (d) 9 cm, 7 cm, 5 cm
4. The lengths of two sides of an isosceles triangle are 3 cm and 7 cm. What is the length of the third side?
   (a) 3 cm (b) 4 cm (c) 5 cm (d) 7 cm
5. If ABC is a scalene triangle in which the length of $\overline{AC}$ is 3 cm, and the length of $\overline{BC}$ is 5 cm. How many integers could be the length of $\overline{AB}$?
   (a) 2 (b) 3 (c) 4 (d) 5

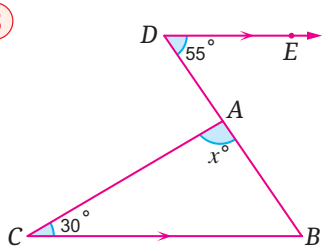
Second Applying Scientific Concepts

▸ **Find with proof the value of x in each of the following figures:**

6.


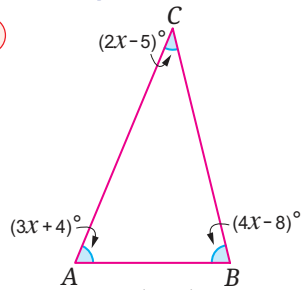
7.


$\overline{BD} \cap \overline{CE} = \{A\}$

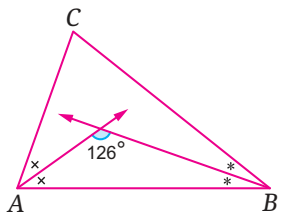
8.


9. In the given picture, a wire of length 48 cm. The ends of this wire are bent at two points to form a triangle. Which of the following lengths could be bent to form a triangle with the remaining part?
   (a) 12 cm and 16 cm (b) 12 cm and 12 cm

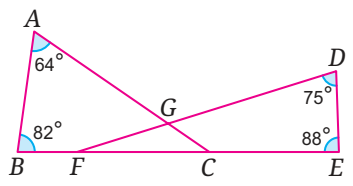
▸ **Find with proof the value of the required angles in each of the following figures:**

10.


$m(\angle B)$

11.


$m(\angle C)$

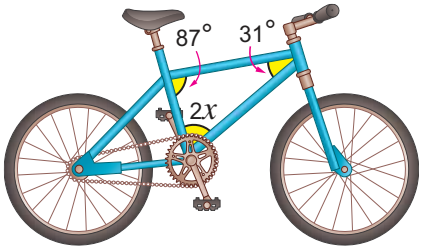
12.


$m(\angle FGC)$

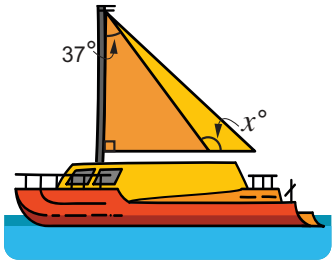
Third Analysis and Subjects Integration

▸ **Calculate the value of x in each of the figures below:**

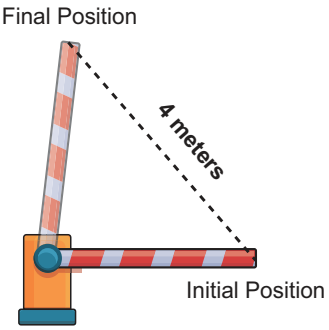
(13)

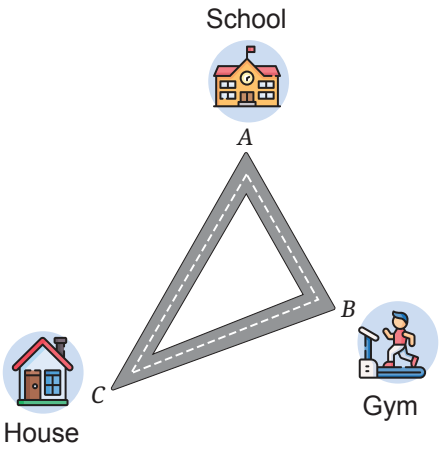


(14)



(15) **Mental Math:** At the entrance of a garage, when a car barrier is opened at an angle of less than 90°, the distance between the end point of the barrier in the initial and final positions is 4 meters. What is the smallest integer for the barrier's length?





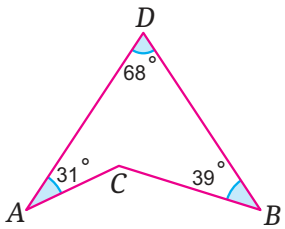
(16) **Roads:** Mohamed wakes up early to go to his school, which is 300 meters away from his house. After the school day, Mohamed goes to the gym, which is 197 meters away from his school, and he leaves it after finishing his training and returns to his house as shown in the figure. What is the smallest integer for the distance travelled by Mohamed from his house until he returns?

Creative Thinking

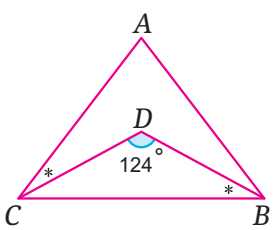
(17) **Critical Thinking :** If ABC is a triangle in which the length of $\overline{BC}$ is 9 cm. Find the smallest integer for the perimeter of ABC.

▸ **In each of the following figures, determine with proof: m ($\angle ACB$)**

(18)

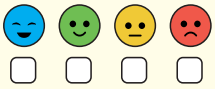


(19)



***Evaluate** your understanding!*

How well do you understand triangle? Tick the right box.



Lesson

4 - 4

Quadrilaterals

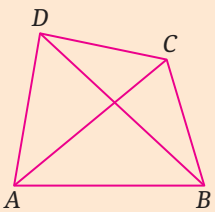
Learning Outcomes

- Learn the concept of quadrilaterals.
- Deduce the sum of measures of the interior angles of a quadrilateral.
- Distinguish between the special quadrilaterals.
- Identify the relationships among the quadrilaterals in the quadrilateral family tree.

Vocabulary

- Quadrilateral
- Trapezium
- Parallelogram
- Rectangle
- Rhombus
- Square

Note that



A diagonal of a quadrilateral is the line segment that joins two non-consecutive vertices. Therefore, $ABCD$ is a quadrilateral that has two diagonals, $\overline{AC}$ and $\overline{BD}$.

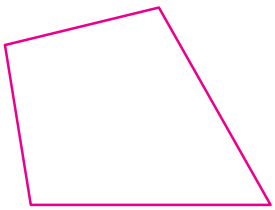
Get Ready!

There are many buildings with unique designs around the world. In the given picture, a building with a front in the shape of a parallelogram, one of its angles of measure 25°. Can you determine the measures of the other three angles?

In this lesson, you will learn about quadrilaterals in general, as well as some special quadrilaterals and their properties, which enable you to solve such real-life problems.

Think & Discuss!

- Use a protractor to measure each angle in the given quadrilateral. What is the sum of the measures of the four angles?
- Draw a diagonal from one vertex to the opposite one. What are the shapes formed by splitting the quadrilateral?
- Can you use the sum of the measures of the interior angles of a triangle to find the sum of the measures of the interior angles of a quadrilateral?



Learn!

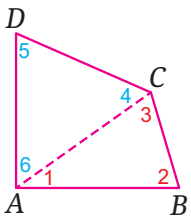
The sum of the measures of the interior angles of a quadrilateral

In the opposite figure:

ABCD is a quadrilateral in which a diagonal $\overline{AC}$ is drawn to split it into two triangles:

$m(\angle 1) + m(\angle 2) + m(\angle 3) = 180°$ (1)

$m(\angle 4) + m(\angle 5) + m(\angle 6) = 180°$ (2)



From (1) and (2), you get:

$m(\angle 1) + m(\angle 2) + m(\angle 3) + m(\angle 4) + m(\angle 5) + m(\angle 6) = 360°$

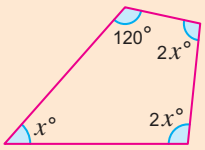
Thus, $m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360°$

i.e., the sum of the measures of the interior angles of the quadrilateral $ABCD$ is 360°.

Rule:

The sum of the measures of the interior angles of any quadrilateral is 360°.

■ Self-Evaluation (1)

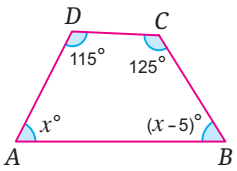


Find the value of $\mathcal{X}$.

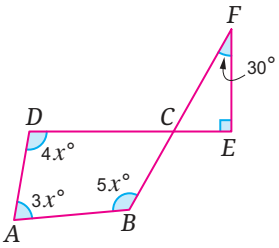
Example 1

Find the value of $\mathcal{X}$ in each of the following figures:

1



2



(1) $\mathcal{X} + \mathcal{X} - 5° + 115° + 125° = 360°$

$2\mathcal{X} + 235° = 360°$

$2\mathcal{X} = 360° - 235°$

$2\mathcal{X} = 125°$

$\mathcal{X} = \frac{125°}{2} = 62.5°$

(2) In the triangle CEF:

m ($\angle ECF$) = $180° - 120° = 60°$

m ($\angle BCD$) = m ($\angle ECF$) = $60°$

(Vertically opposite angles)

In the quadrilateral $ABCD$:

$4\mathcal{X} + 3\mathcal{X} + 5\mathcal{X} + 60° = 360°$

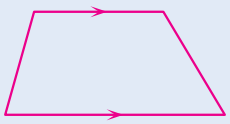
$12\mathcal{X} = 360° - 60° = 300°$

$\mathcal{X} = \frac{300°}{12} = 25°$

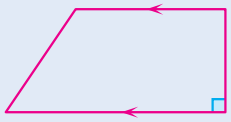
The special quadrilaterals

1 Trapezium

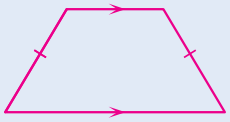
It is a quadrilateral that has only two parallel sides.



Trapezium



Right Trapezium

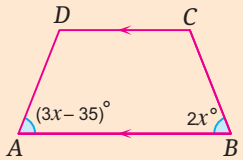


Isosceles Trapezium

■ Self-Evaluation (2)

$ABCD$ is a trapezium

$m(\angle A) = m(\angle B)$

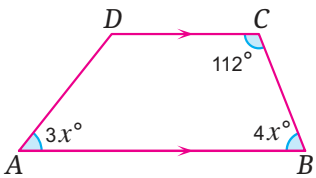


Find: $m(\angle D)$

Example 2

In the opposite figure: $ABCD$ is a trapezium

Find with proof: $m(\angle D)$



$\because \overline{AB} // \overline{DC}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle B) + m(\angle C) = 180°$ (Interior angles on the same side of the transversal)

$\therefore 4\mathcal{X} + 112° = 180°$

$\therefore 4\mathcal{X} = 68°$

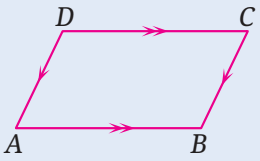
$\therefore \mathcal{X} = 17°$

$\therefore m(\angle A) = 3\mathcal{X} = 3 \times 17° = 51°$

$\therefore m(\angle D) = 360° - (112° + 68° + 51°) = 129°$

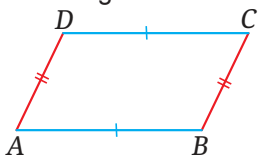
2 Parallelogram

A parallelogram is a quadrilateral in which each two opposite sides are parallel.



▸ Parallelogram Properties

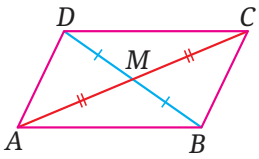
1. Each two opposite sides are equal in length.



$AB = DC$

$BC = AD$

2. The diagonals bisect each other.



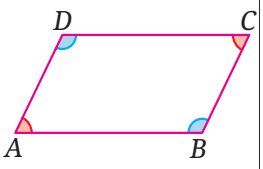
$BM = DM$

$AM = CM$

3. Each two opposite angles are equal in measure.

$m(\angle A) = m(\angle C)$

$m(\angle B) = m(\angle D)$



4. Each two consecutive angles are supplementary.

$m(\angle A) + m(\angle B) = 180°$, $m(\angle B) + m(\angle C) = 180°$

$m(\angle C) + m(\angle D) = 180°$, $m(\angle D) + m(\angle A) = 180°$

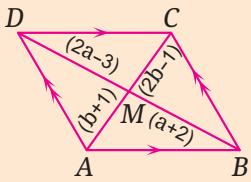
■ When does a quadrilateral become a parallelogram?

If one of the following conditions has occurred:

1. Each two opposite sides are parallel.
2. Each two opposite sides are equal in length.
3. Two opposite sides are parallel and equal in length.
4. Diagonals bisect each other.
5. Each two opposite angles are equal in measure.

■ Self-Evaluation (3)

***ABCD* is a parallelogram.**



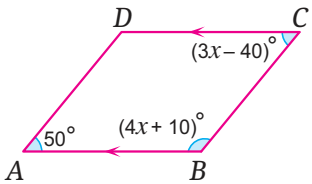
Find the length of each of:

1. $\overline{MB}$
2. $\overline{BD}$
3. $\overline{MA}$
4. $\overline{MC}$

Example 3

In the opposite figure:

Prove that $ABCD$ is a parallelogram.



$\because \overline{AB} \parallel \overline{DC}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle B) + m(\angle C) = 180°$ (Interior angles on the same side of the transversal)

$\therefore 4\mathcal{X} + 10° + 3\mathcal{X} - 40° = 180°$

$\therefore 7\mathcal{X} - 30° = 180°$

$\therefore 7\mathcal{X} = 180° + 30° = 210°$

$\therefore \mathcal{X} = \frac{210°}{7} = 30°$

$\therefore m(\angle B) = 4 \times 30° + 10° = 130°$

$\therefore m(\angle A) + m(\angle B) = 50° + 130° = 180°$,

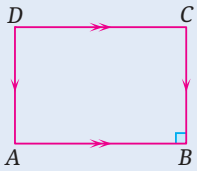
and they are interior angles on the same side of the transversal.

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore ABCD$ is a parallelogram **(Required to Prove)**

3 Rectangle

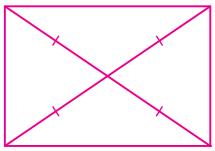
A rectangle is a parallelogram that has one of its angles is a right angle.



▸ **A rectangle has all the properties of a parallelogram, in addition to:**

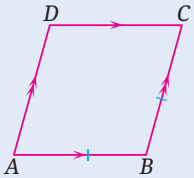
1. All its angles are right angles.

2. Its diagonals are equal in length.



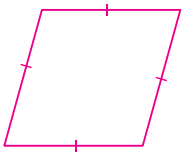
4 Rhombus

A rhombus is a parallelogram in which two adjacent sides are equal in length.

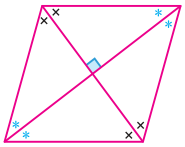


▸ **A rhombus has all the properties of a parallelogram, in addition to:**

1. All its sides are equal in length.

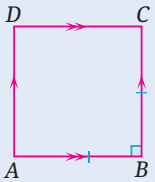


2. Its diagonals are perpendicular and bisect its interior angles.



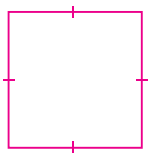
5 Square

A square is a parallelogram that has one right angle and two adjacent sides are equal in length.



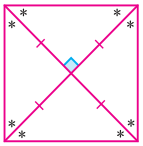
▸ **A square has all the properties of a parallelogram, in addition to:**

1. All its sides are equal in length.



2. All its angles are right angles.

3. Its diagonals are equal in length, perpendicular, and bisect its interior angles.

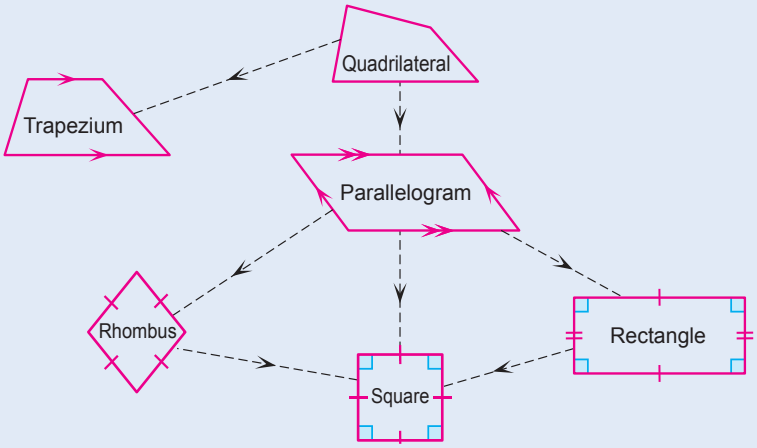


■ **Enrichment Information**

The Great Pyramid of Giza (Khufu Pyramid)

The Great Pyramid of Giza, is one of the Seven Wonders of the Ancient World, located in Giza. Its base is a square of side length 230.4 meters, and its current height is 138.8 meters, after it was 146.5 meters high originally. Find out why.

Quadrilateral Family Tree:

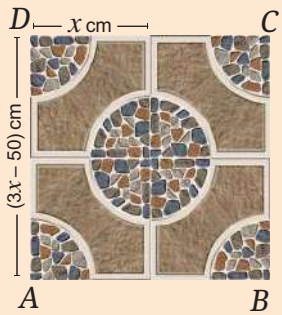


Think

What are the common properties in each of the following:

1. A rectangle and a square.
2. A rhombus and a square.

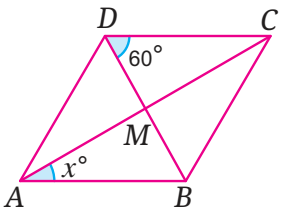
Self-Evaluation (4)



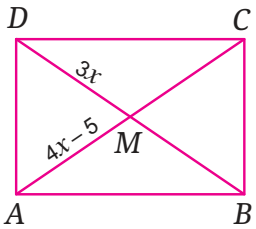
The figure above shows a decorative design composed of four-square ceramic tiles. If the side length of one tile is $\mathcal{X}$ cm, and the length of $\overline{AD}$ equals $(3\mathcal{X} - 50)$ cm, find the value of $\mathcal{X}$.

Example 4

1 In the following figure:
If $ABCD$ is a rhombus, find the value of $\mathcal{X}$.



2 In the following figure:
If $ABCD$ is a rectangle, find the value of $\mathcal{X}$.



1 ∵ $ABCD$ is a rhombus.
∴ Its diagonals are perpendicular.
∴ $m(\angle DMC) = 90°$
From the triangle DMC:
$m(\angle DCM) = 180° - (90° + 60°) = 30°$
∵ $\overline{DC} \parallel \overline{AB}$, and $\overleftrightarrow{AC}$ is a transversal.
∴ $m(\angle CAB) = m(\angle DCA)$ (Alternating interior angles)
∴ $\mathcal{X} = 30°$

2 ∵ $ABCD$ is a rectangle
∴ Its diagonals are equal in length and bisect each other.
∴ $AM = MD$
∴ $4\mathcal{X} - 5 = 3\mathcal{X}$
∴ $4\mathcal{X} - 3\mathcal{X} = 5$
∴ $\mathcal{X} = 5$

Think

- Can a square be considered as a rhombus that has one right angle?
- Can a square be considered as a rectangle that has two adjacent sides are equal in length?

When does a parallelogram become a rectangle, a rhombus, or a square?

A parallelogram becomes as:

1 A Rectangle

If:

- It has one right angle,

Or

- Its diagonals are equal in length.

2 A Rhombus

If:

- It has two adjacent sides equal in length,

Or

- Its diagonals are perpendicular.

3 A Square

If:

- It has one right angle and two adjacent sides equal in length,
- It has one right angle and its diagonals are perpendicular,
- Its diagonals are perpendicular and equal in length,

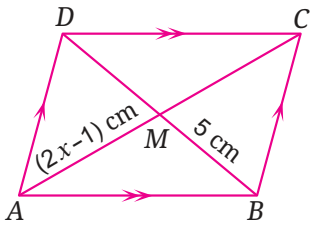
Or

- It has two adjacent sides equal in length and its diagonals are equal in length.

Example 5

In the opposite figure:

Find the value of x that makes the parallelogram $ABCD$ is a rectangle.



The parallelogram $ABCD$ is a rectangle if: $AC = BD$

$2(2x - 1) = 2 \times 5$

$2x - 1 = 5$

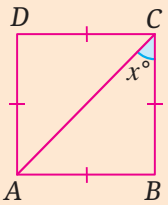
$2x = 5 + 1 = 6$

$x = \frac{6}{2} = 3$

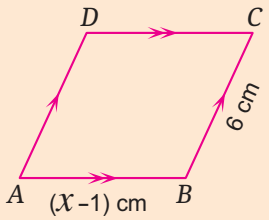
Self-Evaluation 5

Find the value of x that makes:

1 $ABCD$ is a square.



2 The parallelogram $ABCD$ is a rhombus.



Collaborative Activity

Collaborate with a group of your classmates. Each one of you should draw a quadrilateral on a graph paper, so that you will have a variety of different quadrilaterals. Each time, use a protractor to measure each angle in each quadrilateral and verify that the sum of the measures of the interior angles is 360°.

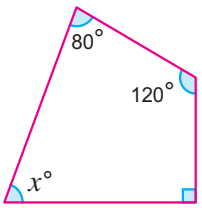
Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

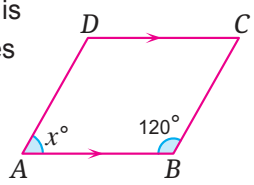
(1) In the given figure, what is the value of $\mathcal{X}$?

(a) 70° (b) 80°
(c) 90° (d) 120°



(2) In the given figure, what is the value of $\mathcal{X}$ that makes $ABCD$ a parallelogram?

(a) 109° (b) 120°
(c) 80° (d) 60°

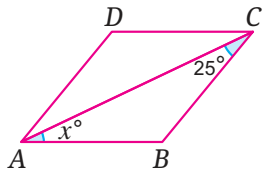


(3) Which of the following sets of quadrilaterals have all their sides are equal in length?

(a) {Square, Rectangle}
(b) {Trapezium, Rhombus}
(c) {Square, Rhombus}
(d) {Rectangle, Rhombus}

(4) In the given figure, $ABCD$ is a rhombus, what is the value of $\mathcal{X}$?

(a) 25° (b) 50°
(c) 100° (d) 130°



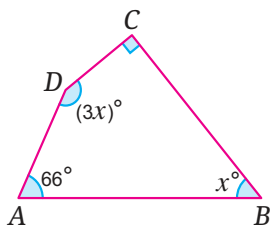
(5) If $ABCD$ is a parallelogram in which: $AC = BD$, $\overline{AC} \perp \overline{BD}$, then $ABCD$ is:

(a) a trapezium. (b) a rhombus. (c) a rectangle (d) a square.

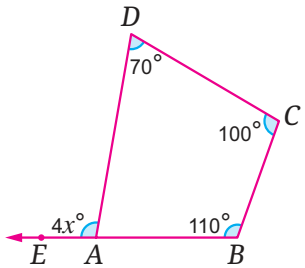
Second Applying Scientific Concepts

▸ **In each of the following figures, calculate with proof the value of $\mathcal{X}$:**

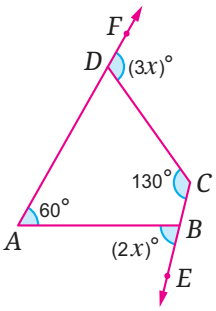
(6)



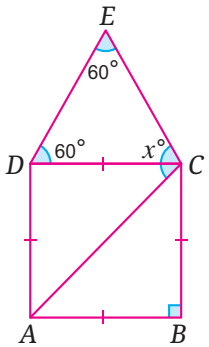
(7)



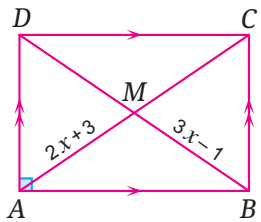
(8)



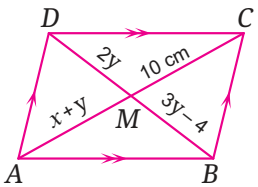
(9)



(10)



(11)

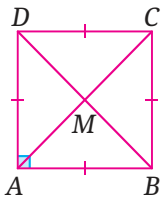


(12) In the opposite figure:

$ABCD$ is a square.

$BD = 5a - 4$, $MC = 2a - 1$.

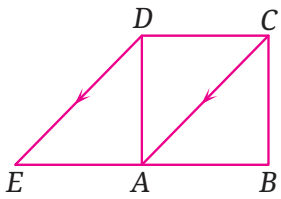
Find the value of a, and then find the length of $\overline{AC}$.



(13) In the following figure:

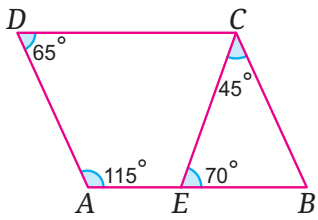
$ABCD$ is a square , $\overline{AC} \parallel \overline{ED}$, $E \in \overrightarrow{BA}$

Prove that: $AE = AB$



(14) In the following figure:

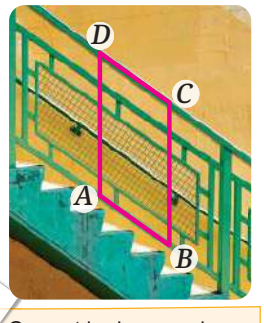
Prove that $ABCD$ is a parallelogram.



Third Analysis and Subjects Integration

(15) Architecture:

If $ABCD$ is a parallelogram, $m(\angle B) = (3x + 37)°$, and $m(\angle D) = (9x + 1)°$, what is the measure of $\angle C$?

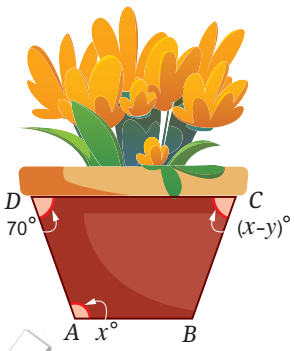


Geometric shapes enhance the aesthetics of artworks, architectural designs, and decorations.

(16) Agriculture:

A flowerpot that has one of its faces in the shape of a trapezium, $m(\angle C) = m(\angle D)$

Find the value of y.



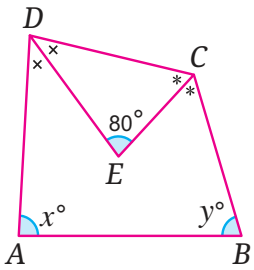
Share in solving the climate change by planting even just one flower.

Creative Thinking

(17) In the opposite figure:

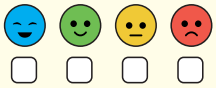
$\overrightarrow{DE}$ bisects $\angle ADC$, $\overrightarrow{CE}$ bisects $\angle BCD$.

Find with proof: the value of $x + y$.



***Evaluate** your understanding!*

How well do you understand quadrilaterals?
Tick the right box.



Lesson

4 - 5

Polygons

Learning Outcomes

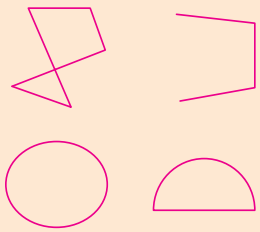
- Learn the concept of polygon.
- Distinguish between convex and concave polygons.
- Deduce the sum of the measures of the interior angles of a polygon.
- Learn the regular polygon.
- Learn the concept of the axis of symmetry.
- Determine the axes of symmetry in polygons.

Vocabulary

- Polygon
- Regular
- Irregular
- Convex
- Concave

Note that

None of the following shapes represents a polygon.



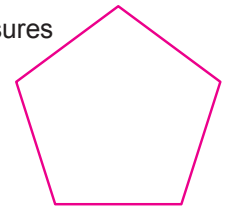
Get Ready!

Bees build their hive in the form of a pattern of polygons. Can you identify the polygon and calculate the measures of its interior angles?

In this lesson, you will learn about various types of polygons, how to determine the sum of the measures of the interior angles of a polygon and explore the regular polygons and the measure of each angle.

Think & Discuss!

Both Bassem and Islam calculated the sum of the measures of the interior angles of a shape with five sides.



Bassem:

The sum of the angles' measures: $3 \times 180° = 540°$

Islam:

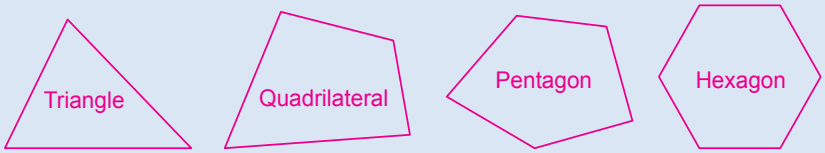
The sum of the angles' measures: $180° + 360° = 540°$

How did each of them calculate the sum of the measures of the interior angles of this polygon?

Learn!

Polygon: **A polygon is a plan, closed shape formed from a union of three or more line segments, such that:**

- The line segments are known as the polygon's sides.
- These line segments intersect at their endpoints, which are known as the polygon's vertices.

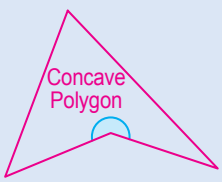


3 sides **4 sides** **5 sides** **6 sides**

Convex and Concave Polygons

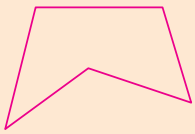
Convex Polygon:
Does not contain any reflex interior angles.

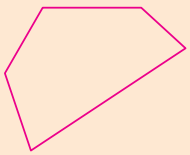
Concave Polygon:
Contains at least one reflex interior angle.

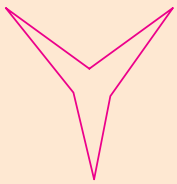


Self-Evaluation (1)

Which of the following polygons are convex, and which are concave?

1 

2 

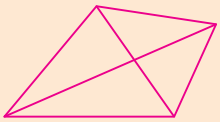
3 

Remember

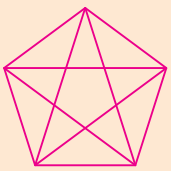
The diagonal of a polygon is the line segment joining two non-consecutive vertices of the polygon.

For example:

- A quadrilateral has two diagonals.



- A pentagon has five diagonals.



Example 1

$ABCD$ is a quadrilateral in which $m(\angle A) = 4\mathcal{X}°$, $m(\angle B) = 5\mathcal{X}°$, $m(\angle C) = 7\mathcal{X}°$, and $m(\angle D) = 20\mathcal{X}°$. Find the value of $\mathcal{X}$, then determine whether the quadrilateral is convex or concave.

$\because$ $ABCD$ is a quadrilateral.

$\therefore m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360°$

$\therefore 4\mathcal{X}° + 5\mathcal{X}° + 7\mathcal{X}° + 20\mathcal{X}° = 360°$

$\therefore 36\mathcal{X}° = 360°$

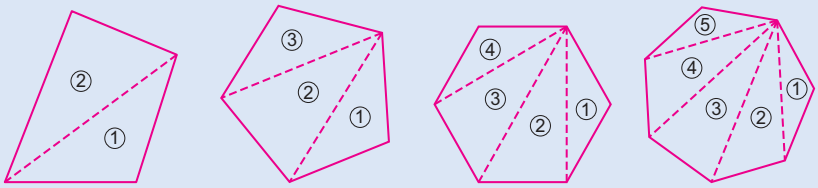
$\therefore \mathcal{X}° = \frac{360°}{36} = 10°$

$\therefore m(\angle D) = 20 \times 10° = 200°$ (Reflex Angle)

$\therefore ABCD$ is a concave polygon.

The sum of the measures of the interior angles of a polygon

To calculate the sum of the measures of the interior angles of a convex polygon, draw all possible diagonals from one of its vertices, dividing the polygon into number of triangles, as shown in the following figures.

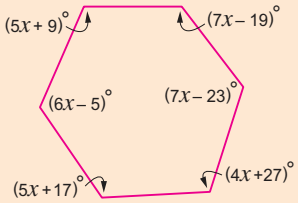


Polygon	Sides	Number of Triangles	Sum of the measures of the Interior Angles
Quadrilateral	4	2	$2 \times 180° = 360°$
Pentagon	5	3	$3 \times 180° = 540°$
Hexagon	6	4	$4 \times 180° = 720°$
Heptagon	7	5	$5 \times 180° = 900°$
:	:	:	:
Decagon	10	8	$8 \times 180° = 1440°$

- Find out the relationship between the number of sides of a polygon and the number of the triangles formed by drawing all possible diagonals from one of its vertices.

■ Self-Evaluation (2)

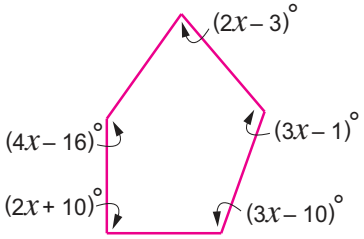
Find the value of x in the following figure:



Example 2

In the opposite figure:

Calculate the value of x.



$\because$ The sum of the measures of the interior angles of a pentagon is 540°

$\therefore 2x - 3° + 3x - 1° + 3x - 10° + 2x + 10° + 4x - 16° = 540°$

$\therefore 14x - 20° = 540°$

$\therefore 14x = 540° + 20° = 560°$

$\therefore x = \frac{560°}{14} = 40°$

■ Think

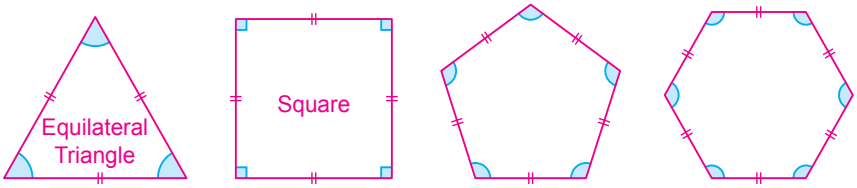
Can a regular polygon be concave?

Regular Polygons

A regular polygon is a polygon that has both of the following properties:

1. All its sides are equal in length.
2. All its interior angles are equal in measure.

Examples of Regular Polygons:



Regular Triangle | Regular Quadrilateral | Regular Pentagon | Regular Hexagon

Note:

The measure of each interior angle of a regular polygon $= \frac{\text{sum of the measures of the interior angles}}{\text{number of interior angles}}$

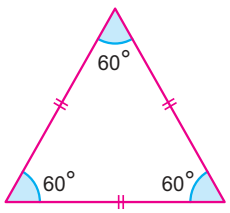
■ Activity

Calculate the measure of each interior angle of a regular polygon that has:

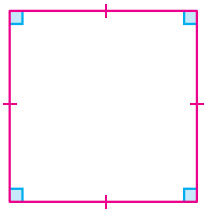
1. 8 sides
2. 10 sides

For example :

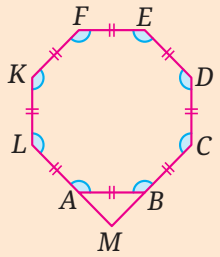
- The measure of each interior angle of an equilateral triangle $= \frac{180°}{3} = 60°$



- The measure of each interior angle of a square $= \frac{360°}{4} = 90°$



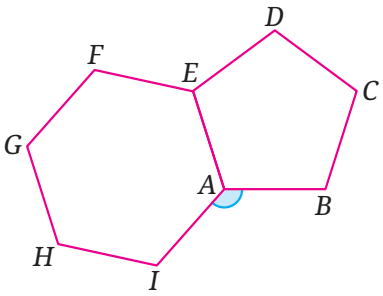
Self-Evaluation (3)



Find: $m\,(\angle AMB)$

Example 3

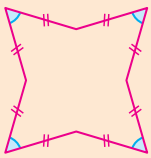
The opposite figure consists of a regular pentagon and a regular hexagon.
Calculate $m\,(\angle IAB)$



- $ABCDE$ is a regular pentagon
  $m\,(\angle EAB) = \frac{540°}{5} = 108°$
- $AEFGHI$ is a regular hexagon
  $m\,(\angle EAI) = \frac{720°}{6} = 120°$
- The sum of the measures of the accumulative angles at a point is 360°
  $m\,(\angle IAB) = 360° - (108 + 120°) = 132°$

Activity

How many axes of symmetry does the following shape have?



Think

Do all irregular polygons have no axes of symmetry?

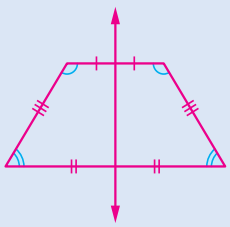
Note that

The number of axes of symmetry in a regular polygon is equal to the number of its sides.

Axes of symmetry in Polygons

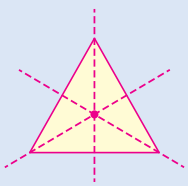
Axis of Symmetry:

The axis of symmetry of a shape is a straight line that divides the shape into two identical parts. When the shape is folded along the axis of symmetry, both parts coincide. A shape may have one or more axes of symmetry, or no axes of symmetry.

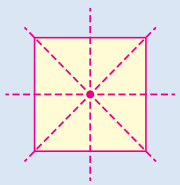


Axes of Symmetry of a Regular Polygon:

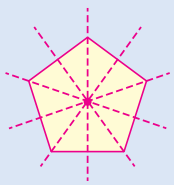
The axis of symmetry of a regular polygon is a straight line that passes through the center of the polygon and divides it into two identical polygons.



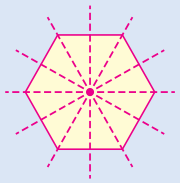
Equilateral Triangle
(3 axes of symmetry)



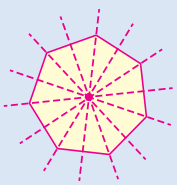
Square
(4 axes of symmetry)



Regular Pentagon
(5 axes of symmetry)



Regular Hexagon
(6 axes of symmetry)

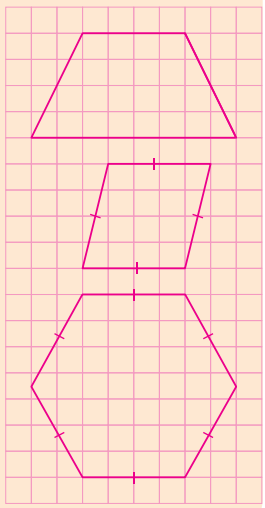


Regular Heptagon
(7 axes of symmetry)

■ Self-Evaluation (4)

Draw the axes of symmetry for each of the following shapes and state their number.

1

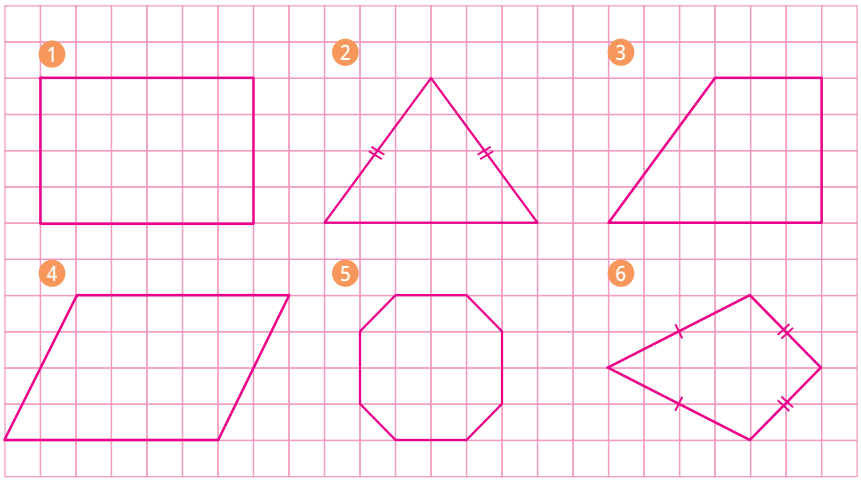


2

3

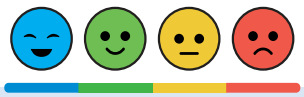
Example 4

How many axes of symmetry are there for each of the following shapes?



(1) 2 (2) 1 (3) 0 (4) 0 (5) 4 (6) 1

Lesson Assessment



First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) Which of the following angles could be one of the interior angles of a polygon to be concave?

(a) straight (b) acute
(c) right (d) reflex

(2) How many axes of symmetry does a regular polygon with 9 sides have?

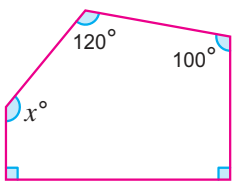
(a) 9 (b) 7
(c) 18 (d) 11

(3) What is the measure of the interior angle of a regular polygon with 10 sides?

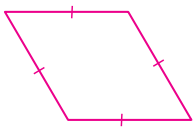
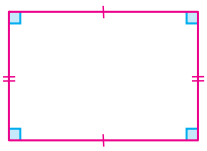
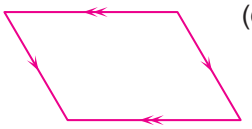
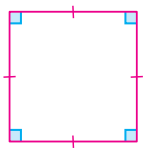
(a) 108° (b) 120°
(c) 135° (d) 144°

(4) In the given figure:
What is the value of x ?

(a) 120° (b) 140°
(c) 150° (d) 135°



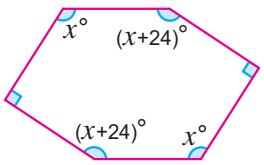
(5) Which of the following figures does not have an axes of symmetry?

(a) 
(b) 
(c) 
(d) 

Second Applying Scientific Concepts

▸ **In each of the following figures, find the value of x:**

6



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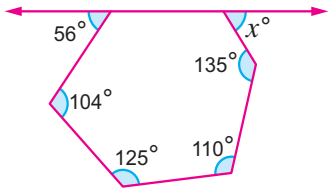

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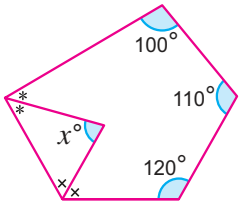
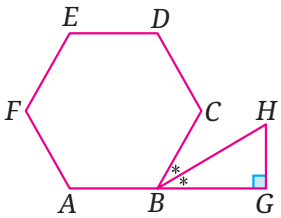
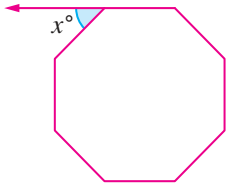


9 In the following figure:
$ABCDEF$ is a regular hexagon.
Find with proof $m(\angle H)$



10 Each of the following is a regular polygon, find the value of x, and determine the number of axes of symmetry for each polygon.

1



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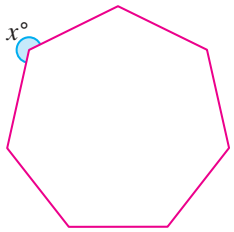
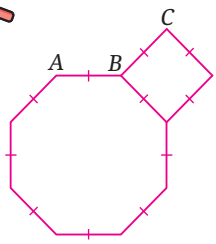
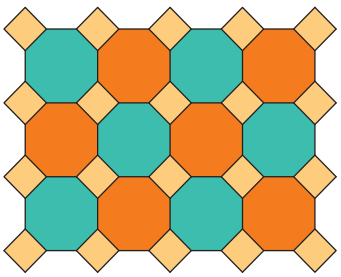


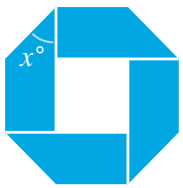
Third Analysis and Subjects Integration

11 **Decoration**: Sometimes regular polygons are used as units for decoration, like the regular octagon and square shown in the figure. What is the measure of $\angle ABC$?



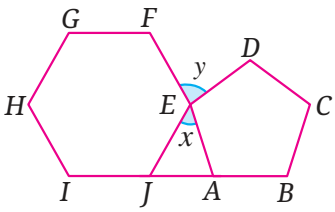


12 **Design:** A designer used a right-angled trapezium to create a company logo, and produced the opposite regular polygon. What is the value of x in the figure?



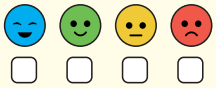
Creative Thinking

13 **In the opposite figure:**
$ABCDE$ is a regular pentagon,
$EFGHIJ$ is a regular hexagon,
$A \in \overline{IB}$, and $J \in \overline{IB}$
Find the value of each of: x and y



***Evaluate** your understanding!*

How well do you understand polygons?
Tick the right box.



Lesson

4 - 6

Coordinates

Learning Outcomes

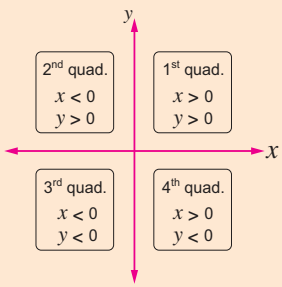
- Represent the coordinates of a point in a coordinate plane.
- Determine the quadrant in which a point is located in a coordinate plane.
- Determine the projection of a point on the coordinate axes.
- Determine the projection of a line segment on the coordinate axes.
- Determine the coordinates of the midpoint of a line segment.

Vocabulary

- x-axis
- y-axis
- x-coordinate
- y-coordinate
- The Origin
- 1st Quadrant
- 2nd Quadrant
- 3rd Quadrant
- 4th Quadrant
- Midpoint of a Line Segment

Previous Knowledge

Note the signs of the points in each quadrant of the coordinate plane.



Get Ready!

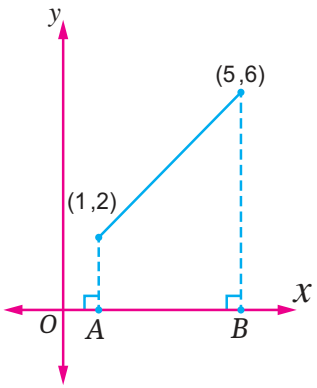
There are many scenarios where you need to determine the midpoint of a distance between two points, such as placing a cell-phone tower to provide optimal coverage between two areas.

How can you determine the best location to set up this tower?

In this lesson, you will learn how to plot points in the coordinate plane and how to find the midpoint of a line segment, Which enable you to solve such real-life problems.

Think & Discuss!

In the opposite figure,
what is the distance between points A and B?

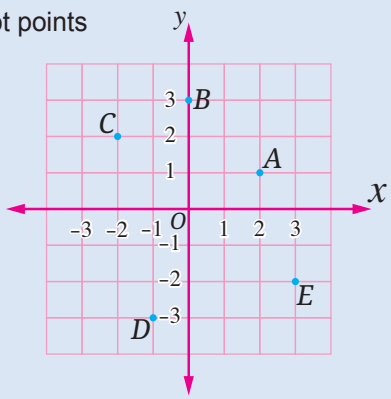


Learn!

You have previously learned how to plot points on the coordinate plane.

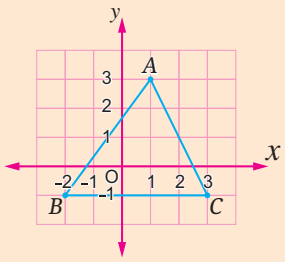
For example, in the opposite figure:

- Point A is (2 , 1)
- Point B is (0 , 3)
- Point C is (– 2 , 2)
- Point D is (– 1, – 3)
- Point E is (3 , – 2)
- Point O is (0 , 0)



Self-Evaluation (1)

From the following graph,



Determine the coordinates of the vertices of the triangle ABC, then calculate its area.

Self-Evaluation (2)

If point $A\ (-3\ k\ ,\ 2\ k-4)$ lies on the x-axis, determine the quadrant in which point $B\ (k-6\ ,\ -3\ k)$ is located.

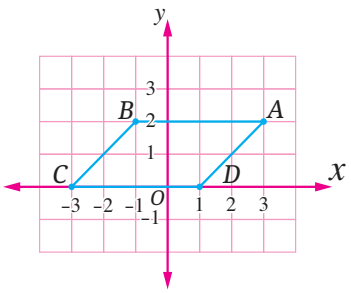
Note that

- If a point lies on the x-axis, its projection on the x-axis is the point itself.
- If a point lies on the y-axis, its projection on the y-axis is the point itself.

Example 1

In the coordinate plane, plot the following points:
$A\ (3\ ,\ 2)$, $B\ (-1\ ,\ 2)$, $C\ (-3,\ 0)$, $D\ (1\ ,\ 0)$.
Then, calculate the area of the figure $ABCD$.

$ABCD$ is a parallelogram.
Its area = Base length × The length of the corresponding height
$= 4 \times 2 = 8$ square units.



Example 2

If point $A\ (4\ k+4\ ,\ -k+3)$ lies on the y-axis, determine the quadrant in which point $B\ (-2\ k\ ,\ 4\ k+1)$ is located.

$\because A\ (4\ k+4\ ,\ -k+3)$ lies on the y-axis
$\therefore$ The x-coordinate is equal to 0
$\therefore\ 4\ k+4=0\ \ \therefore\ 4\ k=-4\ \ \ \therefore\ k=-1$
By substituting the value of k in point B,
the coordinates of B are $B\ (2\ ,\ -3)$
$\therefore$ B is located in the 4th quadrant.

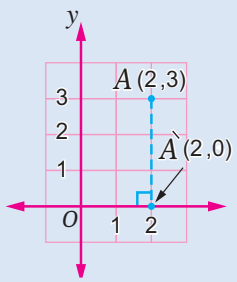
Note that

- The point $(x\ ,\ 0)$ lies on the x-axis.
- The point $(0\ ,\ y)$ lies on the y-axis.

Projection of a Point on the Coordinate Axes

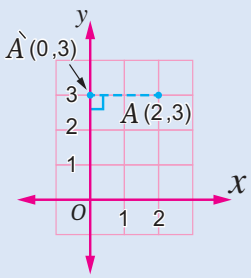
To find the projection of a point, such as $A\ (2\ ,\ 3)$, on each of the x-axis and y-axis:

1 Projection on the x-axis:
From point A, draw a line perpendicular to the x-axis, intersecting it at point $A'\ (2\ ,\ 0)$.



Therefore, $A'\ (2,0)$ is the projection of point $A\ (2\ ,\ 3)$ on the x-axis.

2 Projection on the y-axis:
From point A, draw a line perpendicular to the y-axis, intersecting it at point $A'\ (0\ ,\ 3)$.



Therefore, $A'\ (0,3)$ is the projection of point $A\ (2\ ,\ 3)$ on the y-axis.

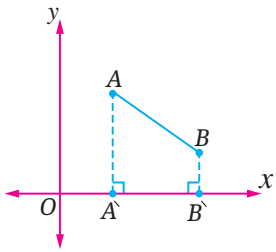
Projection of a Line Segment on the Coordinate Axes

To determine the projection of a line segment on either axis, find the projection of its endpoints on that axis. In each of the following examples, notice the projection of $\overline{AB}$ on both the x-axis and y-axis:

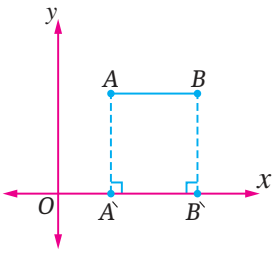
Think

What is the relationship between the length of a line segment and the length of its projection?

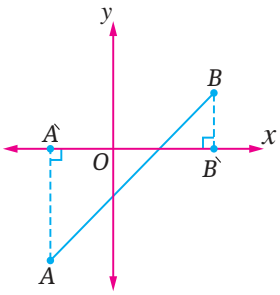
1. $\overline{A`B`}$ is the projection of $\overline{AB}$ on the x-axis.



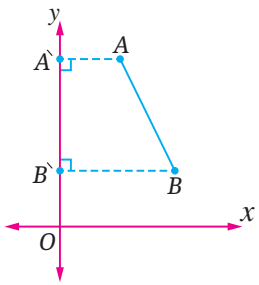
2. $\overline{A`B`}$ is the projection of $\overline{AB}$ on the x-axis.



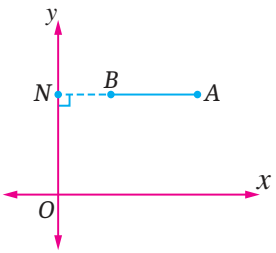
3. $\overline{A`B`}$ is the projection of $\overline{AB}$ on the x-axis.



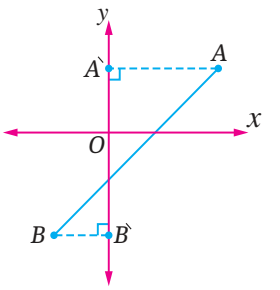
4. $\overline{A`B`}$ is the projection of $\overline{AB}$ on the y-axis.



5. Point N is the projection of $\overline{AB}$ on the y-axis.



6. $\overline{A`B`}$ is the projection of $\overline{AB}$ on the y-axis.



Self-Evaluation (3)

In the coordinate plane, Draw $\overline{AB}$, where the coordinates of points A and B are A (5, 6) and B (3, 2), and determine the following on the plane:

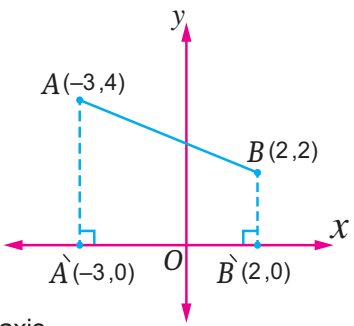
1. The projection of $\overline{AB}$ on the x-axis.
2. The projection of $\overline{AB}$ on the y-axis.

Then, calculate the length of each projection.

Example 3

Determine the length of the projection of the line segment $\overline{AB}$ on the x-axis, where the coordinates of points A and B are A (– 3, 4) and B (2, 2).

- Draw the line segment $\overline{AB}$ on the coordinate plane.
- From both points A and B, draw lines perpendicular to the x-axis, as shown in the figure.



Thus, point $A`$ (– 3, 0) is the projection of point $A$ on the $x$-axis, and point $B`$ (2,0) is the projection of point B on the x-axis.

So, line segment $\overline{A`B`}$ is the projection of the line segment $\overline{AB}$ on the x-axis.

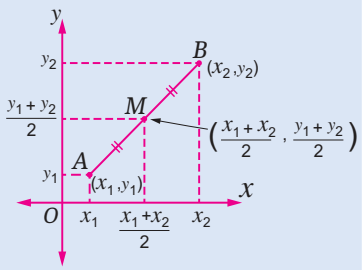
$A`B` = |2| + |– 3| = 5$.

Therefore, the length of $\overline{A`B`}$ is 5 length units.

Midpoint of a Line Segment

The midpoint of the line segment $\overline{AB}$ is a point that lies on the line segment $\overline{AB}$ and is equidistant from both endpoints.
If M is the midpoint of $\overline{AB}$,
where $A(x_1, y_1)$, $B(x_2, y_2)$,

Then $\mathrm{M} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



Example 4

Determine the coordinates of the midpoint of $\overline{AB}$, where the coordinates of points A and B are $A(2, -2)$ and $B(-6, 8)$.

Let that the midpoint of $\overline{AB}$ be M

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + (-6)}{2}, \frac{-2 + 8}{2}\right) = \left(\frac{-4}{2}, \frac{6}{2}\right) = (-2, 3)$$

Self-Evaluation (4)

Determine the coordinates of the midpoint of $\overline{AB}$, where $A(1, -6)$ and $B(5, 2)$.

Example 5

If the point $M(0, -3)$ is in the middle of the distance between points $A(x, -10)$ and $B(7, y)$, calculate the values of x and y.

$\because (0, -3) = \left(\frac{7 + x}{2}, \frac{y - 10}{2}\right)$

$\therefore \frac{7 + x}{2} = 0 \quad \therefore 7 + x = 0 \quad \therefore x = -7$

$\frac{y - 10}{2} = -3 \quad \therefore y - 10 = -6 \quad \therefore y = 4$

Self-Evaluation (5)

If point $M(3, 5)$ is in the middle of the distance between points $A(x, 5)$ and $B(4, y)$, find the values of x and y.

Example 6

If $ABCD$ is a parallelogram where $A(-1, 1)$, $B(4, 2)$, and $D(1, 4)$, determine the coordinates of the following:

1. The point of intersection of its diagonals.
2. The vertex C.

Let that M be the point of intersection of the diagonals

$\therefore M$ is the midpoint of $\overline{BD}$ $\quad \therefore M = \left(\frac{4 + 1}{2}, \frac{2 + 4}{2}\right) = (2.5, 3)$

, $\because M$ is also the midpoint of $\overline{AC}$. $\quad$ Let $C = (x_1, y_1)$

$\therefore \left(\frac{-1 + x_1}{2}, \frac{1 + y_1}{2}\right) = (2.5, 3)$

$\therefore \frac{-1 + x_1}{2} = 2.5 \quad \therefore -1 + x_1 = 5 \quad \therefore x_1 = 6$

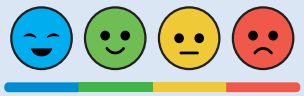
$\frac{1 + y_1}{2} = 3 \quad \therefore 1 + y_1 = 6 \quad \therefore y_1 = 5 \quad \therefore C = (6, 5)$

Self-Evaluation (6)

If $ABCD$ is a parallelogram where $A(-4, 2)$, $B(4, -2)$, and $C(9, 1)$, determine the coordinates of the following:

1. The point of intersection of its diagonals.
2. The vertex D.

Lesson Assessment



First Measuring Conceptual Understanding

▸ **Choose the correct answer from the give ones :**

(1) In which quadrant is the point (3, – 4) located?
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

(2) If the origin is the midpoint of $\overline{AB}$, and A is located in the 2nd quadrant, in which quadrant is point B located?
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

(3) Which of the following points does not lie on the y-axis?
(a) (0 , – 5) (b) (3 , 0)
(c) (0 , 0) (d) (0 , 2)

(4) If $x < 0$ and $y > 0$, in which quadrant is the point $(x, -y)$ located?
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

(5) If the point (3, k – 2) lies on the x-axis, what is the value of k?
(a) – 3 (b) – 2
(c) 2 (d) 3

(6) What is the point representing the projection of the point (– 3, 5) on the x-axis?
(a) (0, 5) (b) (– 3, 0)
(c) (3, – 5) (d) (– 3, 5)

(7) What is the projection of the point (– 3, 5) on the y-axis?
(a) (0, 5) (b) (– 3, 0)
(c) (3, – 5) (d) (– 3, 5)

(8) If the point (a , b) in the 3rd quadrant, then the quadrant which the point $(-2a , b-6)$ lies in is:
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

▸ **Discuss :**

(9) A teacher asked his students about the position of the point (x, y), such that $xy < 0$. Ahmed answered that the point was located in the 2nd quadrant, while Hend answered that the point was located in the 4th quadrant. Who is correct?

Second Applying Scientific Concepts

(10) If the point $(a - 2, 3a + 9)$ lies on the x-axis, determine the quadrant in which the point $(a , 6 - a)$ is located.

(11) Determine the length of the projection of the line segment $\overline{AB}$ on the x-axis in each of the following cases:
① A (– 2, 1), B (3, 6) ② A (– 4, 3), B (– 2, 3) ③ A (5, 5), B (5, – 1)

(12) If the point C (– 2, 7) is the midpoint of $\overline{AB}$, where the coordinates of A and B are A (4, y) and B (x, – 2), find the value of each of x and y.

(13) If $ABCD$ is a rhombus where A (3, 5), B (12, – 3), and C (13, 9), determine the coordinates of the following:

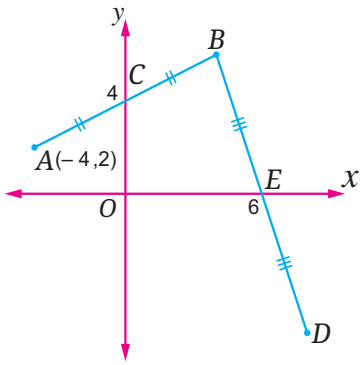
① The point of intersection of its diagonals. ② The vertex D.

(14) If A (– 7, 13) and B (3, 5), find the coordinates of the points that divide $\overline{AB}$ into four equal parts.

(15) If A (3, – 1) and B (– 1, – 1) Draw the square $ABCD$ such that point C is located in the second quadrant.

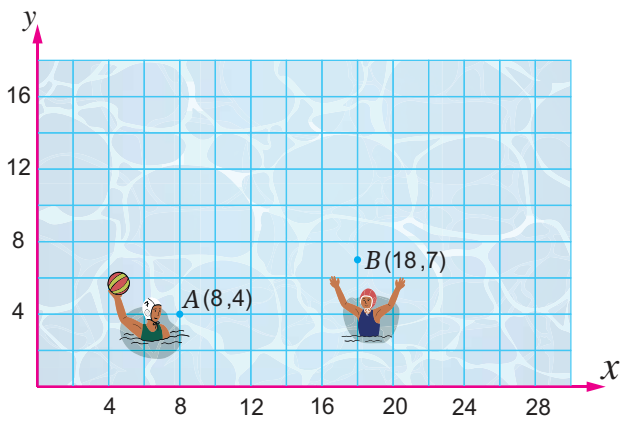
(16) **In the opposite figure:**

If C and E are the midpoints of $\overline{AB}$ and $\overline{BD}$ respectively, determine the coordinates of point D.



Third Analysis and Subjects Integration

(17) **Sports:** The following figure shows the positions of players A and B during a water polo game. Determine the coordinates of player C, such that player B is in the middle of the distance between players A and C.



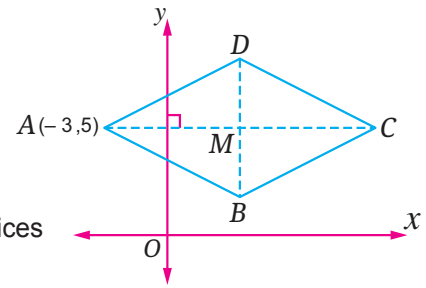
Creative Thinking

(18) **In the opposite figure:**

$ABCD$ is a rhombus in which:

$AC = 12$ and $BD = 6$.

Find the coordinates of the other vertices of the rhombus



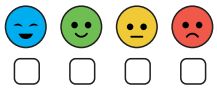
***Evaluate** your understanding!*

How well do you understand coordinates? Tick the right box.

Unit Four Assessment

Evaluate your understanding!

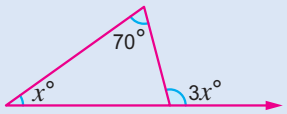
How well do you understand Unit Four? Tick the right box



▸ Choose the correct answer from the given ones:

(1) In the opposite figure:

What is the value of x?



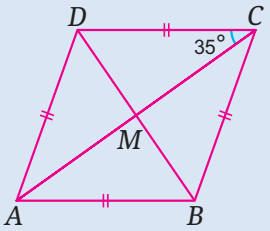
(a) 70° (b) 140° (c) 35° (d) 100°

(2) If A (3,1) and B (3, -1), which of the following points is the midpoint of $\overline{AB}$?

(a) (0, 3) (b) (3, 0) (c) (6, 0) (d) (0, 6)

(3) In the opposite figure:

What is the measure of $\angle CBD$?



(a) 35° (b) 45°

(c) 55° (d) 65°

(4) The lengths of two sides in an isosceles triangle are 4 cm and 8 cm. What is the length of the third side?

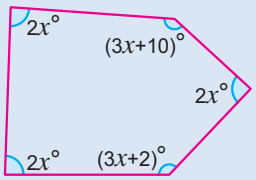
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

▸ Complete each of the following:

(5) The measure of each interior angle in a regular octagon is .....

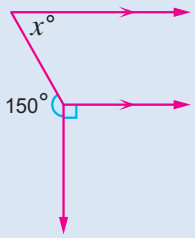
(6) The projection of point A (– 4, 0) on the y-axis is point .....

(7) In the opposite figure:



$x =$ .....°

(8) In the opposite figure:



$x =$ .....°

▸ Answer the following questions:

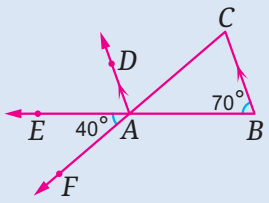
(9) $ABCD$ is a parallelogram whose diagonals intersect at M.

If A (3, 4) and M (–1, 5), find the coordinates of C.

10 In the opposite figure:

$\overrightarrow{BE} \cap \overrightarrow{CF} = \{A\}$

Prove that: $\overrightarrow{AD}$ bisects $\angle EAC$.

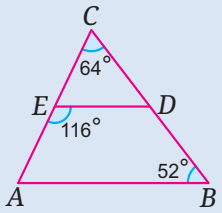


11 $ABCD$ is a rectangle whose diagonals are intersecting at M.

If $AC = (3\mathcal{X} - 4)$ cm, and $BM = (\mathcal{X} + 1)$ cm, what is the value of $\mathcal{X}$?

12 In the opposite figure:

Prove that: $\overline{ED} \,//\, \overline{AB}$



Unit Four Activity — Tangram

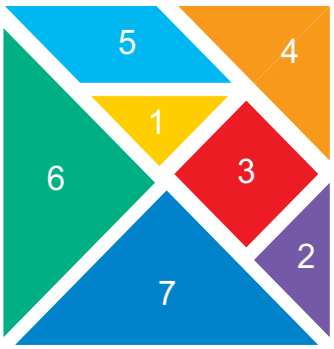
Tangram is a renowned puzzle that involves arranging geometric pieces to form various shapes.

Activity Aims:

- To enhance the understanding of geometric shapes among students.
- To develop geometric skills students have.
- To develop the ability to focus and pay attention.

Implementation steps:

1. Students are divided into competing teams (2-3 students per team) to promote positive competitiveness.
2. Each team should select a specific geometric shape (square, rectangle, parallelogram, etc.).
3. The opposing team is challenged to form this shape multiple times using the Tangram pieces, starting with the fewest number of pieces till to use all the pieces within a time limit of (3-5) minutes.
4. Teams take turns carrying out steps 2 and 3.
5. Steps 2, 3, and 4 are repeated after each team chooses a different shape.
6. The winner is the team that manages to form the required shape in the shortest time.



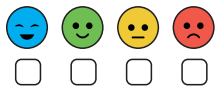
(Tangram)

Note: One of the teams can ask the other teams to form different shapes, such as numbers or various figures like a cat, dog, or house, and so on.

Final Assessment

Evaluate your understanding!

How well do you understand the lessons of first preparatory? Tick the right box



First Group of Questions

▸ **Choose the correct answer from the given ones:**

(1) If $\frac{9}{k+1} = \frac{3}{5}$, what is the value of k?

(a) 13 (b) 14 (c) 15 (d) 16

(2) What is the solution set of the equation:

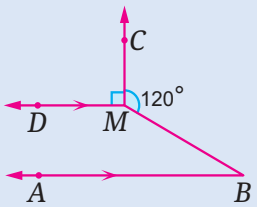
$4(2x + 7) = 12$ in N?

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{-4\}$ (d) ϕ

(3) In the opposite figure:

$\overrightarrow{BA} // \overrightarrow{MD}$, $\overrightarrow{MC} \perp \overrightarrow{MD}$

and $m(\angle BMC) = 120°$. What is the measure of $\angle B$?



(a) 20° (b) 30°

(c) 50° (d) 70°

(4) If a map scale is 1 : 200,000 and the distance between two points on the map is 3.5 cm, what is the real distance between these two points in kilometers?

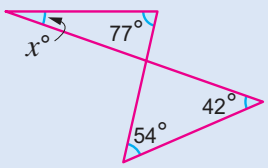
(a) 3.5 (b) 7 (c) 8.5 (d) 700

(5) $-3-(-2) =$ .....

(a) -5 (b) -1 (c) 1 (d) 5

(6) In the opposite figure:

What is the value of x?



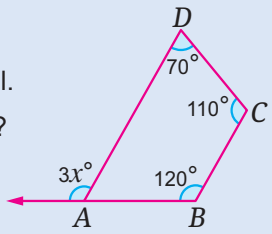
(a) 19° (b) 32°

(c) 48° (d) 60°

(7) In the opposite figure:

$ABCD$ is a quadrilateral.

What is the value of x?



(a) 40° (b) 50°

(c) 60° (d) 70°

Second Group of Questions

▸ **Choose the correct answer from the given ones:**

(1) Which of the equations below is not equivalent to $4x + 5 = 9$?

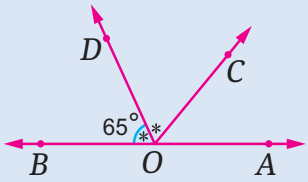
(a) $3x = 3$ (b) $4x + 1 = 5$ (c) $x - 1 = 5$ (d) $x + 1 = 2$

(2) In the opposite figure:

What is the measure of $\angle DOA$?

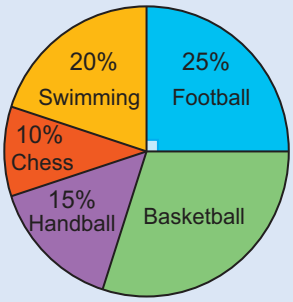
(a) 50° (b) 80°

(c) 115° (d) 130°



(3) The opposite pie chart illustrates the distribution of students' preferences for summer activities. If there are 200 students participating in all the activities, how many students chose basketball?

(a) 30 (b) 50 (c) 60 (d) 70



(4) How many axes of symmetry are there in a regular hexagon?

(a) 2 (b) 3 (c) 4 (d) 6

(5) If point M (4, 3) is the midpoint of $\overline{AB}$, where A (x , 5) and B (2 , y), what is the value of $(x + y)$?

(a) 3 (b) 5 (c) 7 (d) 9

(6) Which of the following is equal to 8 y?

(a) $5 + 3y$ (b) $3 + 5y$

(c) $8 + y$ (d) $3y + 5y$

(7) If the price of a product is reduced from 2,000 LE to 1,700 LE, what is the discount rate?

(a) 10% (b) 15% (c) 25% (d) 27%

Third Group of Questions

▸ **Answer the following questions:**

(1) Simplify the expression: $3(a - 2b) - 2(a + b)$, then find the value of the expression if $a = 5$ and $b = -1$.

(2) Three persons participated in a project with a capital of 750,000 LE. It was divided in the ratio of 4 : 5 : 3. Calculate the share of each person in the capital.

(3) If the masses of a group of students in kilogram are listed on the following table:

Mass (kg)	72	73	75	76	77	78
Frequency	1	3	5	3	6	2

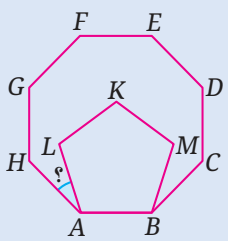
Calculate the arithmetic mean of the masses of these students.

(4) In the opposite figure:

$ABCDEFGH$ is a regular octagon, and $ABMKL$ is a regular pentagon.

Find with proof:

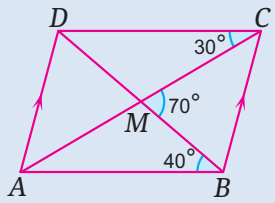
$m(\angle HAL)$



(5) In the opposite figure:

$\overline{AC} \cap \overline{BD} = \{M\}$

Prove that $ABCD$ is a parallelogram.



MATHEMATICS

First Preparatory
First Term
2025-2026

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FIRST PREPARATORY
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